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# PONDEROMOTIVE STABILIZATION OF INTERCHANGE MODES IN MIRROR PLASMAS AT ICRF HEATING

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# PONDEROMOTIVE STABILIZATION OF INTERCHANGE MODES IN MIRROR PLASMAS AT ICRF HEATING

by I.A.Kotel'nikov and S.G.Yakovchenko

The effect of a ponderomotive force produced by RF field at ICRF heating on MHD stability of a plasma in an axisymmetric mirror device is considered. It is assumed that (*i*) RF field is excited by a full-turn-loop antenna, (*ii*) its frequency is below the eigen frequency spectrum of the plasma column, and (*iii*) hence RF field is localized in near zone of the antenna. Dissipation of RF power that results in plasma heating is found to drive an instability that has a resonant-particle interaction in its origin. This instability turns out to be very strong if a minority ions scheme of ICRF heating is used. However at a sufficiently high level of RF power it can be suppressed due to quasilinear relaxation of the minority ions distribution. Conditions for the instability to be suppressed are discussed. In the case of the instability being suppressed, a simple criterion for the ponderomotive force to have a stabilizing effect on the plasma is obtained.

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## 1. Introduction

Stabilization of interchange modes by a radio-frequency (RF) field has been successfully demonstrated in a number of mirror experiments. Vast investigations performed at the Phaedrus tandem mirror facility are worth to be especially mentioned (see J.J.Browning et al., 1988, and references herein). They revealed for the ability of RF field to stabilize or, vice versa, to destabilize an interchange mode to be very sensitive to minor details of RF spectrum. This crucial observation feeds an impetus to developing theoretical interpretation of RF stabilization.

Theory of RF stabilization was dealt with by many authors. Having been advanced by D'Ippolito & Myra (1986), the theory is likely to acquire most up-to-dated form. It attributes the effect of RF field on MHD stability to a ponderomotive force (PMF) which is a nonlinear in RF amplitude, low frequency part of a total force acting on plasma in RF field. Provided that a perturbation of PMF induced by a MHD activity is found, subsequent study of the MHD stability is subject to a well known procedure. Thus, the task for the theory is to determine the perturbation of PMF[1]. The theory of D'Ippolito & Myra (1986) solves this task with a quasilinear approximation in use. Such approach is valid for the majority of experiments because RF spectrum have usually a large spread in $k_{\parallel}$. Last version of D'Ippolito's & Myra's (1986) quasilinear theory extends their earlier approaches (Myra & D'Ippolito 1984; D'Ippolito & Myra 1985) to incorporate an arbitrary RF field polarization, a self-consistent calculation of an electromagnetic plasma response (with the so called sideband modes coupling effect taken into account), and an absorption of the RF field spin and momentum by resonant particles.

In the present paper we shall discuss some aspects of PMF

[1] The term "perturbation of PMF" is sometimes used to indicate only that part of low frequency force which is calculated without taking into account perturbation of RF field due to a sideband mode coupling effect. We have no grounds, except for the historical ones, to follow this meaning of the term.



stabilization arising if RF field is designed primarily to heat a plasma and therefore a dissipation of the RF power in a plasma is sufficiently strong to be taken into account. To be more specific, we shall consider stability of a small-scale interchange mode in an axisymmetric two-ion-species plasma which is subject to ICRF heating at the cyclotron frequency $\Omega_a$ of a minority ions species. RF field is supposed to be excited by a full-turn-loop antenna with the frequency $\omega$ being lower than the eigenfrequency $\omega_A$ of a plasma column magnetosonic oscillation. By order of magnitude $\omega_A \approx V_A / a$ where $V_A$ is the Alfven velocity and $a$ is the plasma radius. Our choice of heating scheme to be discussed is motivated not so by its anticipated advantages (see Kotel'nikov & Yakovchenko 1990 for details) as mainly by that it makes simple analytical analysis being possible.

As we shall show, the quasilinear theory predicts for the RF-particles interaction to drive an instability which becomes especially strong when the minority ions scheme of ICRF heating is choosen. We note also that a quasilinear relaxation of minority ions distribution suppress the instability. For the case of the resonant-particles instability being suppressed, we shall obtain a simple criterion for PMF to have a stabilizing effect on plasma.

Our paper is organized as follows. The quasilinear theory of the RF stabilization is reviewed briefly in Sec.II. We modify the theory as to take into account an axial variation of the RF field. Sec.III is devoted to the calculation of RF field at an applied and sideband frequencies. In Sec.IV, the dispersion equation for small scale interchange modes is derived and analyzed. Our conclusions are summarized in Sec.V.

## 2. Basic equations

As the starting point for studying stability of a mirror plasma in an axisymmetric, paraxial geometry we take the equation

$$\int\frac{d\ell}{r^2B^3}\left[\omega_s^2\rho + 2\varkappa\nabla p\right]\frac{\partial}{\partial\varphi}r\xi_r B = \int dz\ \mathbf{e}_z\cdot\nabla\times\left(\frac{\delta\mathbf{f}_{rf}}{B^2}\right) \qquad (1)$$

where $\omega_s$ is the interchange mode frequency, $p$ and $\rho$ are the plasma pressure and density, $\xi_r$ is the radial component of perturbed displacement vector, and $\varkappa$ is the curvature of magnetic field line. Were the right-hand side omitted, (1) would be a well known dispersion equation for a small scale interchange mode (Rosenbluth & Longmire 1957). The right-hand side of (1) is the contribution of the PMF density $\mathbf{f}_{rf}$. The perturbed force $\delta\mathbf{f}_{rf}$ is linear in $\xi_r$. Hence $\delta\mathbf{f}_{rf}$ as well as $\xi_r$ can be considered as being proportional to $\exp(im\varphi-i\omega_s t)$. For a small scale perturbation $m\gg1$.

Since propagating waves are not generated at $\omega<\omega_A$, a RF field is localized near an antenna. We assume for the antenna to be placed at a solenoidal insert between the regions of enhanced magnetic field (between mirrors). Just for this reason the integration along the magnetic field line length $\ell$ is converted to the integration along the $z$ co-ordinate in the right-hand side of (1). Another simplification comes from the idea that, in the low plasma pressure limit $\beta\ll1$, the magnetic field B in the solenoidal insert is uniform. Therefore $B$ and the operator $\mathbf{e}_z\cdot\nabla\times$ can be taken out the integral sign in (1) so that

$$\int dz\ \mathbf{e}_z\cdot\nabla\times\left(\frac{\delta\mathbf{f}_{rf}}{B^2}\right) = \frac{1}{B^2}\ \mathbf{e}_z\cdot\nabla\times\int dz\ \delta\mathbf{f}_{rf}.$$



For what follows it is useful to represent a RF field as a superposition of partial waves[2] with different wave numbers $\ell'$, $k_\parallel'$ and frequency $\omega'$:

$$\mathbf{E}(\mathbf{r},t) = \sum_{\omega'}\sum_{\ell'}\int\frac{dk_\parallel'}{2\pi}\exp(ik_\parallel' z+i\ell' t-i\omega' t)\mathbf{E}_{q'}(r), \qquad (2)$$

where $q'$ denotes the set of quantities $\{\ell',k_\parallel',\omega'\}$. The summation over $\omega'$ includes waves at the applied frequency $\omega$ and at the two sidebands, $\omega'=\omega\pm\omega_s$. The reality condition on $\mathbf{E}$ implies

$$\mathbf{E}_{-q'}=\mathbf{E}^*_{q'}. \qquad (3)$$

The wave amplitude $\mathbf{E}_{q'}$ can be represented in several ways in terms of reciprocal bases:

$$\mathbf{E}_q=\mathbf{e}_j E^j_q = \mathbf{e}^j E_{qj} \qquad (4)$$

where the convention is adopted that repeated indices of the proper form (one down and one up) imply a summation over $j=1,2,3$. Defining the basis $\mathbf{e}_j$ by the equalities

$$\mathbf{e}_1=\mathbf{e}_L=\frac{\mathbf{e}_x-i\mathbf{e}_y}{\sqrt{2}},\quad \mathbf{e}_2=\mathbf{e}_R=\frac{\mathbf{e}_x+i\mathbf{e}_y}{\sqrt{2}},\quad \mathbf{e}_3=\mathbf{e}_\parallel=\mathbf{e}_z \qquad (5)$$

casts $\mathbf{E}_q$ into a superposition of left-hand $(j=1=L)$, right-hand $(j=2=R)$, and longitudinally polarized $(j=3=\parallel)$ waves. The reciprocal basis $\mathbf{e}^j=\mathbf{e}^*_j$ satisfy to the equation

$$\mathbf{e}^j\cdot\mathbf{e}_k=\delta^j_{\cdot k} \qquad (6)$$

where $\delta^j_{\cdot k}$ denotes Kronecker delta. It follows from (3)-(6) that

$$E^j_{q'}=\mathbf{e}^j\cdot\mathbf{E}_{q'},\quad E_{q'j}=\mathbf{e}_j\cdot\mathbf{E}_{q'},\quad E_{-q'j}=E^{j*}_{q'}. \qquad (7)$$

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[2] In the case under discussion these waves are not the eigen oscillations of the plasma column.

Left-hand, right-hand, and longitudinal waves differ in the eigen values $\sigma_j=\{-1,+1,0\}$ of spin projection on the $z$ direction. Using this notation we can recast (5) into

$$\mathbf{e}_\mu=\frac{\mathbf{e}_x+i\sigma_\mu\mathbf{e}_y}{\sqrt{2}}=\frac{\mathbf{e}_r+i\mathbf{e}_\mu\sigma_\varphi}{\sqrt{2}}\exp(i\sigma_\mu\varphi), \qquad \mathbf{e}_3=\mathbf{e}_\parallel=\mathbf{e}_z. \tag{5a}$$

Here and hereafter Greek index $\mu$, in contrast to Roman index $j$, takes only two values, $\mu=1=L$ and $\mu=2=R$. With the same notation in use $E^j$ can be expressed as

$$E^\mu=\frac{E_x-i\sigma_\mu E_y}{\sqrt{2}}=\frac{E_r-i\sigma_\mu E_\varphi}{\sqrt{2}}\exp(-i\sigma_\mu\varphi), \qquad E^3=E_3=E_z. \tag{8}$$

Constraining ourselves to the case where larmor radius $\rho_i$ of ions (and furthermore that of electrons) is much less than the perpendicular wavelength $2\pi/k_\perp$, $k_\perp\rho_i\ll 1$, we suppose for RF spectrum to be sufficiently narrow in the azimuthal wave numbers $\ell$,

$$\ell\ll a/\rho_i. \tag{9}$$

Then the linear plasma susceptibility tensor $\overleftrightarrow{\chi}$ as well as the dielectric tensor $\overleftrightarrow{\varepsilon}=\overleftrightarrow{I}+\overleftrightarrow{\chi}$ are approximately diagonal in helicity co-ordinates spanned onto the basis $\{\mathbf{e}_L,\mathbf{e}_R,\mathbf{e}_\parallel\}$. This means that

$$\chi^j_{.k}=\mathbf{e}^j\cdot\overleftrightarrow{\chi}\cdot\mathbf{e}_k.$$

is zero in leading order of small parameter $\rho_i\ell/a$ if $j\neq k$ and reciprocal bases are determined by (5) and (6). For a Maxwellian collisionless multi-species plasma the diagonal elements in $\overleftrightarrow{\chi}$ are equal to

$$\chi^\mu_{.\mu}=-\sum_s\frac{\omega_{ps}^2}{\omega(\omega+\sigma_\mu\Omega_s)}\,Z\left[\frac{\omega+\sigma_\mu\Omega_s}{|k_\parallel v_s|}\right],$$



$$\chi^3_{.3}=\sum_s\frac{2\omega_{ps}^2}{k_\parallel^2 v_s^2}\left[1-Z\left[\frac{\omega}{|k_\parallel v_s|}\right]\right], \tag{10}$$

where the summation is performed over all species, $\omega_{ps}$ and $\Omega_s$ are Langmuir and cyclotron frequency of the species number $s$, $v_s$ is its thermal velocity, and

$$Z(\xi)=\frac{\xi}{\sqrt{\pi}}\int_{-\infty}^{+\infty}\frac{dt}{\xi-t+i0}\exp(-t^2)=\xi\exp(-\xi^2)\left[2\int_0^\xi\exp(t^2)dt-i\sqrt{\pi}\right]$$

is the plasma dispersion function. In the case of a two-ion-species plasma, which we shall consider, the index $s$ takes three values $e$, $i$ and $a$ to mark electrons, ion majority and ion minority species respectively. The density $n_s$ of the species should satisfy to the quasineutrality condition, $e_e n_e+e_i n_i+e_a n_a=0$. In the cold plasma limit, $|k_\parallel v_s|\ll|\omega-\Omega_s|$, (10) reduces to

$$\chi^\mu_{.\mu}=\frac{\omega_{pi}^2}{\Omega_i(\Omega_i+\sigma_\mu\omega)}+\frac{\omega_{pa}^2}{\Omega_a(\Omega_a+\sigma_\mu\omega)}, \qquad \chi^3_{.3}=-\frac{\omega_{pe}^2}{\omega^2}.$$

Using aforementioned approximations and notations we found that integration of the steady-state ponderomotive force $\mathbf{f}_{rf}$ along a magnetic field line over the region where RF field is localized yields

$$\int dz\,\mathbf{f}_{rf}=\frac{1}{4\pi}\sum_{j=1}^{3}\sum_{\ell'}\int\frac{dk_\parallel}{2\pi}\Big[\mathrm{Re}\chi^j_{.j}\,\nabla_\perp|E^j_q|^2+ \tag{11}$$
$$+\,i\mathrm{Im}\chi^j_{.j}\,(E^j_q\nabla_\perp E^{j*}_q-E^{j*}_q\nabla_\perp E^j_q)-\sigma_j\,\mathbf{e}_z\times\nabla_\perp(\mathrm{Im}\chi^j_{.j}|E^j_q|^2)\Big].$$

In deriving Eq. (11) we have not used the approximation $\frac{\partial}{\partial z}|\mathbf{E}|^2=0$ that was adopted by D'Ippolito & Myra (1986). However, our result (11) is very similar to their formula (30) since all additional terms in $f_{rf}$, which arise when the approximation $\frac{\partial}{\partial z}|\mathbf{E}|^2=0$ is canceled, turn into zero after the integration over $z$ has been performed. This is also true for the perturbed ponderomotive force $\delta f_{rf}$.

The first term in (11) is the only one that survives in the cold plasma limit. In this limit it matches the result of Motz & Watson (1967):

$$f_{rf} = \frac{1}{4\pi}\left[\frac{\omega_{pi}^2\nabla_\perp|E_L|^2}{\Omega_i(\Omega_i-\omega)} + \frac{\omega_{pi}^2\nabla_\perp|E_R|^2}{\Omega_i(\Omega_i+\omega)} - \frac{\omega_{pe}^2}{\omega^2}\nabla_\perp|E_\parallel|^2\right]. \qquad (12)$$

As it was proved by D'Ippolito & Myra (1985), other two terms in (11) appear due to RF field momentum and spin absorption by the resonant particles. These terms become negligibly small if there are no resonant absorption in the plasma and, as a sequence, $\mathrm{Im}\chi^j_{.j}$ tends to zero.

Assuming an axial symmetry of RF field amplitude $|E^j|$ in addition to that of the plasma, we find that the first term in (11) produces a radial force whereas the other two terms give rise to an azimuthal force which cause the plasma to rotate in the azimuthal direction. Basic features of a rotating plasma equilibrium relevant to the problem of PMF stabilization have been reviewed by D'Ippolito & Myra (1985). Omitting some details we emphasize only that any azimuthal rotation is not taken into account in (1).

Having obtained (11), we can find an explicit expression for the integral $\int dz\delta f_{rf}$ in (1). In the long run, the procedure of how to do it comprises the following two steps. First, we should perturb every production of any factors in (11) so as to meet



the usual mathematic rules. For example,

$$\delta(\chi^j_{.j}|E^j_q|^2) = |E^j_q|^2\delta\chi^j_{.j} + \chi^j_{.j}\delta|E^j_q|^2.$$

The second step is to substitute the perturbed quantities by the explicit expressions given below:

$$\delta\chi^j_{.j} = -\vec{\xi}\cdot\nabla\chi^j_{.j},$$

$$\delta|E^j_q|^2 = \delta(E^j_q E^{j*}_q) = E^j_{q+}E^{j*}_q + E^j_q E^{j*}_{q-}, \qquad (13)$$

$$\delta(E^j_q\nabla E^{j*}_q - E^{j*}_q\nabla E^j_q) = E^j_{q+}\nabla E^{j*}_q + E^j_q\nabla E^{j*}_{q-} - E^{j*}_{q-}\nabla E^j_q - E^{j*}_q\nabla E^j_{q+}.$$

Here $E^j_{q+}$, $E^j_{q-}$ are the sideband mode amplitudes and

$$q_\pm \equiv q \pm q_s, \quad q_s \equiv (m, k_{\parallel s}, \omega_s) = (m, 0, \omega_s).$$

Every perturbed quantity in (13) is proportional to $\exp(im\varphi - i\omega_s t)$. The amplitudes $E^j_q$, $E^j_{q+}$, $E^j_{q-}$ are to be found from the equations

$$\nabla\times\nabla\times\mathbf{E}_q - \frac{\omega^2}{c^2}\left[\vec{I} + \vec{\chi}(q)\right]\cdot\mathbf{E}_q = \frac{4\pi i\omega}{c^2}\mathbf{j}_{Aq}, \qquad (14)$$

$$\nabla\times\nabla\times\mathbf{E}_{q+} - \frac{\omega^2}{c^2}\left[\vec{I} + \vec{\chi}(q)\right]\cdot\mathbf{E}_{q+} = \frac{4\pi i\omega}{c^2}\mathbf{j}_{Aq+} - \frac{\omega^2}{c^2}\vec{\xi}\cdot\nabla\vec{\chi}(q)\cdot\mathbf{E}_q, \qquad (15)$$

$$\nabla\times\nabla\times\mathbf{E}^*_{q-} - \frac{\omega^2}{c^2}\left[\vec{I} + \vec{\chi}^*(q)\right]\cdot\mathbf{E}^*_{q-} = -\frac{4\pi i\omega}{c^2}\mathbf{j}^*_{Aq-} - \frac{\omega^2}{c^2}\vec{\xi}\cdot\nabla\vec{\chi}^*(q)\cdot\mathbf{E}^*_q, \qquad (16)$$

where $\mathbf{j}_{Aq}$ and $\mathbf{j}_{Aq\pm}$ are the fundamental and the sideband harmonics of the antenna current density $\mathbf{j}_A$; they are related together by (2) with the substitution $\mathbf{j}_A$ and $\mathbf{j}_{Aq}$, instead of $\mathbf{E}$ and $\mathbf{E}_q$, respectively.

### 3. RF field

For the sake of simplicity we shall consider the case where RF field is excited by a full-turn-loop antenna. Such antenna can be modeled by a piece of a conductive cylinder with a radius $b>a$. RF field is excited by the current

$$j_A = \mathcal{I}(z)\delta(r-b)\cos(\omega t)e_\varphi \qquad (17)$$

supplied by a RF generator. Here $\mathcal{I}(z)$ is the surface current with a dependence on $z$ which we do not specify. It follows from (17) that the fundamental harmonic of the antenna current is

$$j_{Aq} = \frac{1}{2}\mathcal{I}_k\delta(r-b)e_\varphi, \quad \mathcal{I}_k = \int dz\,\mathcal{I}(z)\exp(-ik_\parallel z). \qquad (18)$$

To proceed further we firs introduce more customary notation for nonzero susceptibility tensor elements in cartesian co-ordinates:

$$\chi^\mu_{\cdot\mu} \equiv \varepsilon - \sigma_\mu g, \quad \chi^3_{\cdot 3} \equiv \eta,$$

where

$$\chi_{xx} = \chi_{yy} = \varepsilon, \quad \chi_{xy} = -\chi_{yx} = ig, \quad \chi_{zz} = \eta.$$

Next, we suppose for the basic frequency $\omega$ to be lower than the lowest frequency $\omega_A \approx V_A/a$ of the plasma column eigen magnetosonic oscillations. The condition $\omega \le \omega_A$ holds for plasmas in mirror devices at $\omega \approx \Omega_i$ provided that $a$ is as small as few centimeters.

For the case $\omega \ll \omega_A$ the solution of (14) has been found by Kotel'nikov & Yakovchenko (1990). Reproducing their result in helicity coordinates gives

$$E^\mu_q = -i\sigma_\mu\exp(-i\sigma_\mu\varphi)\,\frac{\varepsilon - N_\parallel^2 + \sigma_\mu g}{\varepsilon - N_\parallel^2}\,\frac{E_{q\varphi}}{\sqrt{2}} \qquad (19)$$

where $N_\parallel = k_\parallel c/\omega$ is the refractive index and $E_{q\varphi}$ is the vacuum RF field. If the plasma is enclosed by a conductive vacuum chamber of radius $R>b$ the vacuum RF field in the region $r \le b$ is .

$$E_{\varphi q} = \frac{2\pi i\omega b}{c^2}\,\mathcal{I}_k\Big[K_1(k_r b)I_1(k_r R) - K_1(k_r R)I_1(k_r b)\Big]I_1(k_r r)/I_1(k_r R) \qquad (20)$$



where $k_r \equiv \sqrt{k_\parallel^2 - \omega^2/c^2}$. In the case $k_r R \ll 1$, (17) reduces to the expression

$$E_{\varphi q} = \frac{\pi i\omega \mathcal{I}_k}{c^2}\left[1 - \frac{b^2}{R^2}\right]r \qquad (21)$$

being valid also for all $k_\parallel$ such, that $k_r$ is imaginary, because $\omega/cR \ll 1$. In deriving (20) we set that $E_z = 0$ which is true if

$$\left|\frac{\varepsilon - N_\parallel^2}{\varepsilon}\right| \gg \left|\frac{c^2}{\omega^2 a^2\eta}\right|^{1/3}.$$

This condition is most restrictive in the neighborhood of Alfven resonance where $\varepsilon \simeq N_\parallel^2$ and low limit of $\varepsilon - N_\parallel^2$ is determined by the imaginary part $\zeta$ in $\varepsilon$.

The solution of wave equations (15), (16) for the sideband modes $\mathbf{E}_{q\pm}$ can be easily found for a small-scale interchange perturbation. Since $m \gg 1$ in this case, the sideband fields are evanescent outside the plasma: $\mathbf{E}_{q\pm} \propto \exp(-rm/a)$. Hence the antenna current density $j_{Aq\pm}$ at the sideband frequencies $\omega_\pm = \omega \pm \omega_s$ is small too. We set $j_{Aq\pm} = 0$. An additional simplification is achieved from the ordering that the azimuthal dimension $r\Delta\varphi \simeq \pi r/m$ of most unstable interchange perturbation is small compared with its dimension $\Delta r$ in the radial direction, $r\Delta\varphi \ll \Delta r \ll a$.

Using these two approximations simplify the radial component

$$\frac{\partial}{r\partial\varphi}\left[\frac{\partial}{r\partial r}rE_{\varphi q+} - \frac{\partial}{r\partial\varphi}E_{rq+}\right] = \frac{\omega^2}{c^2}\left[(\varepsilon - N_\parallel^2)E_{rq+} + igE_{\varphi q+}\right] - \frac{\omega^2}{c^2}\left[\vec{\xi}\cdot\nabla\vec{\chi}(q)\cdot\mathbf{E}_{\varphi q}\right]_r$$

of the equation (15) to give

$$\left[\frac{\partial}{r\partial r}rE_{\varphi q+} - \frac{\partial}{r\partial\varphi}E_{rq+}\right] = O\left(\frac{1}{m}\right). \qquad (22)$$

Hence

$$E_{rq+} = -\frac{i\partial}{m\partial r}rE_{\varphi q+} = O\left[\frac{1}{m}\right]. \qquad (23)$$

Substituting (22) and (23) into the azimuthal component of (15)

$$-\frac{\partial}{\partial r}\left[\frac{\partial}{r\partial r}rE_{\varphi q+} - \frac{\partial}{r\partial\varphi}E_{rq+}\right] = \frac{\omega^2}{c^2}\left[(\varepsilon-N_\parallel^2)E_{\varphi q+} - igE_{rq+}\right] - \frac{\omega^2}{c^2}\left[\vec{\xi}\cdot\nabla\vec{\chi}(q)\cdot\mathbf{E}_{\varphi q}\right]_\varphi$$

yields

$$O\left[\frac{r}{m\Delta r}\right] = \frac{\omega^2}{c^2}\left[(\varepsilon-N_\parallel^2)E_{\varphi q+} + O\left[\frac{1}{m}\right]\right] - \frac{\omega^2}{c^2}\left[\vec{\xi}\cdot\nabla\vec{\chi}(q)\cdot\mathbf{E}_{\varphi q}\right]_\varphi.$$

Discarding small terms here we get

$$E_{rq+}=0, \quad E_{\varphi q+} = \left[\vec{\xi}\cdot\nabla\vec{\chi}(q)\cdot\mathbf{E}_{\varphi q}\right]_\varphi / (\varepsilon-N_\parallel^2), \quad E_{zq+}=0. \qquad (24)$$

Similar formulas can be obtained for $\mathbf{E}_{q-}$ from (16). In helicity co-ordinates the solution of (15) and (16) looks like

$$E^j_{q+} = \xi_r A^j_{\cdot\mu}E^\mu_q, \quad E^{j*}_{q-} = \xi_r A^{j*}_{\cdot\mu}E^{\mu*}_q, \qquad (25)$$

where

$$A^j_{\cdot\mu} = \frac{\sigma_j\sigma_\mu}{2(\varepsilon-N_\parallel^2)}\exp\left[-i(\sigma_j-\sigma_\mu)\varphi\right]\frac{\partial}{\partial r}\chi^\mu_{\cdot\mu}$$

and summation on $\mu=1,2$ is implied.

### 4. The dispersion relation

Having found $\mathbf{E}_q$ and $\mathbf{E}_{q\pm}$ we can now calculate the integral $\int\delta\mathbf{f}_{rf}dz$ that appears in the right-hand side of (1). The procedure of the calculation is described in Sec.II. Performing strait forwardly what it prescribes to do we get an explicit expression for the integral $\int\delta\mathbf{f}_{rf}dz$. Substituting it into (1) we find the final dispersion equation for a small-scale interchange perturbation:



$$M\omega_s^2 = K + K_R + iK_I. \qquad (26)$$

Here the coefficients

$$M = \int\frac{d\ell}{r^2B^3}\rho, \quad K = \int\frac{d\ell}{r^2B^3}\vec{\kappa}\nabla p \qquad (27)$$

refer to the plasma stability problem without any RF field applied while $K_R$ and $K_I$ appear in (26) due to the ponderomotive force $\mathbf{f}_{rf}$. We can express them in terms of the unperturbed RF field $\mathbf{E}_q$:

$$K_R = \frac{1}{4\pi r^2B^3}\sum_{j=1}^{3}\int\frac{dk_\parallel}{2\pi}\left\{2i\,\mathrm{Im}\chi^j_{\cdot j}\left[A^{j*}_{\cdot\mu}E^{\mu*}_q\frac{\partial}{\partial r}E^j_q - \text{c.c.}\right]\right.$$

$$\left. + \left[E^j_qA^{j*}_{\cdot\mu}E^{\mu*}_q\frac{\partial}{\partial r}\chi^j_{\cdot j} + \text{c.c.}\right] + \left[E^j_q\left(\frac{\partial}{\partial r}\chi^j_{\cdot j}\right)\frac{\partial}{\partial r}E^{j*}_q + \text{c.c.}\right]\right\}, \qquad (28)$$

$$K_I = \frac{m}{4\pi r^3B^3}\sum_{j=1}^{3}\sigma_j\int\frac{dk_\parallel}{2\pi}\left\{\mathrm{Im}\chi^j_{\cdot j}\left[E^j_qA^{j*}_{\cdot\mu}E^{\mu*}_q + \text{c.c.}\right] - \left(\frac{\partial}{\partial r}\mathrm{Im}\chi^j_{\cdot j}\right)|E^j_q|^2\right\}, \qquad (29)$$

where "c.c." means the complex conjugate. The imaginary term $iK_I$ in (26) leads to the resonant-particle instability first discussed by D'Ippolito & Myra (1985).

This instability has usually a small residual growth rate (see below for more details). However, ICRF heating at the cyclotron frequency $\Omega_a$ of a minority ions species is appeared to enhance it substantially.

Before to proceed to minority ions heating we shall prove that the resonant-particles instability does really have a small increment if an ICRF heating is performed not at the cyclotron frequency $\Omega_a$ of minority ions. Indeed, if an applied frequency $\omega$ does not coincide with the cyclotron frequency $\Omega_s$ of any plasma species then $|\mathrm{Im}\vec{\chi}| \ll |\mathrm{Re}\vec{\chi}|$ and hence $K_I \ll K_R$. At $\omega\approx\Omega_1$ $\mathrm{Im}\chi^1_{\cdot 1}$ may be compatible with $\mathrm{Re}\vec{\chi}$ while $\mathrm{Im}\chi^2{}_2$ and $\mathrm{Im}\chi^3{}_3$ are much less. However the left-hand polarized wave $E^1_q$ is shorted out to much extent precisely at $\omega = \Omega_i$. This can be seen from (19) if to note that $\varepsilon \cong g$ at $\omega = \Omega_i$ and $\varepsilon \gg N^2_\parallel$ inside suffisiently dense plasma. It is this disappearance of $E^1_q$ that limits an application of ICRF heating at the fundamental harmonic of cyclotron frequency (see for example England et al 1989). One of the ways to bypass this limitation is to apply ICRF heating at the cyclotron frequency $\Omega_a$ of minority ions. A calculation of RF power absorbed by minority ions for the case under consideration (full-turn-loop antenna and $\omega < V_A/a$) is provided by Kotel'nikov & Yakovchenko (1990). For a sufficiently low density $n_a$ of minority ions, such that $n_a \le n^*_a \approx (|k_\parallel v_a|/\omega_a) n_i$, left-hand wave is not shorted out and $E^1_q \approx E^2_q$ even at $\omega = \Omega_a$. Hence $K_I$ is at least no less than $K_R$ so that $K_I \ge K_R$ at $n_a \approx n^*_a$. The magnitude of $K_I$ decreases from a maximum at $n_a = n^*_a$ to a small amount as $n_a$ exceeds $n^*_a$ since $E^1_q$ decreases in inverse proportion to $n_a$. $K_I$ decreases also as $n_a$ decreases from $n^*_a$ to zero because of decrease in $\mathrm{Im}\vec{\chi}$.

Comparing terms $K_R + iK_I$ contributed to (26) by PMF with the term $K$ originated from a curvature of magnetic field lines we can find a condition for the PMF to be important in the plasma stability problem. In terms of a typical RF field amplitude $E$ this condition takes the form

$$\frac{E}{B} \ge \frac{v_i}{c}\left(\frac{a^2}{\lambda L_M}\right)^{1/2} \tag{30}$$

where $L_M$ is the length of magnetic mirrors and $\lambda$ is the dimension of the region where RF field is localized; the quantities $E$ and $\lambda$ are related by the equation



$$E^2\lambda = \int dz\,|\mathbf{E}|^2.$$

ICRF heating rebuilds the distribution function $f_a$ of minority ions. Up to this point we consider $f_a$ to be Maxwellian and neglect the effect of quasilinear relaxation on it. This supposition is valid provided that the time

$$\tau_{QL} \approx \left(\frac{m_a v_a}{eE}\right)^2 \frac{v_a}{\lambda} \tag{31}$$

of quasilinear relaxation is much greater than the transit time $\lambda/v_a$ of a minority ion through the region that RF field occupies. The condition $\tau_{QL} \gg \lambda/v_a$ is equivalent to

$$\frac{E}{B} \ll \frac{v_i}{c}\frac{\rho_a}{\lambda} \tag{32}$$

where $\rho_a$ is the larmor radius of the ion. The conditions (30) and (32) keep a gap where our calculations are valid and, at the same time, RF field is sufficiently large in the aforementioned sense. This gap disappears at $\lambda/L_M > (\rho_a/a)^2$. If there is no gap or if $E$ lays above it, the quasilinear relaxation rebuilds $f_a$ in a shorter time $\tau_h \approx (L_M/\lambda)\tau_{QL}$ than the time $\omega_s^{-1}$ which an interchange perturbation takes to increase substantially, $\tau_h \ll \omega_s^{-1}$. A general tendency of any relaxational process is to cancel or, at least, to diminish any sort of absorption by minority ions and, as a sequence, to diminish $K_I$ too. Thus it stabilizes the resonant-particle instability. To avoid possible misunderstanding we should note that in other schemes of ICRF heating $\tau_{QL}$ can have another explicit expression that differs from (31). Therefore the gap determined by the conditions analogous to (30) and (32) is also changed in dimension; as a rule it becomes much wider.

In the remainder of the paper, we shall study the effects of

PMF for the case where the resonant-particle instability is absent or suppressed. This situation takes place if $n_a \ll n_a^*$, or $n_a \gg n_a^*$, or $\omega$ is not very close to $\Omega_a$, or if $n_a \approx n_a^*$ and $\omega = \Omega_a$ but $\tau_h \ll \omega_s^{-1}$. For simplicity we also suppose for RF spectrum to be centered at such $k_\parallel$ that $N_\parallel^2 \ll \varepsilon$. The latter condition is not very restrictive one; it allows us to use simplified expression (21) for $E_{q\varphi}$ instead of exact solution (20). By setting $\mathrm{Im}\chi^{j}_{\cdot j}=0$ we omit the term $iK_I$ which leads to the resonant-particle instability. As to the real RF term $K_R$ in (26), it can be calculated with the cold plasma susceptibility tensor $\vec{\chi}$ used since the final expression does not contain any marks of the $(\omega-\Omega_a)^{-1}$ singularity presented in the expression (12) for $f_{rf}$:

$$K_R = \int \frac{dz}{4\pi r^2 B^3} \frac{\omega_{pi}^2}{\Omega_i^2} \left[ -\frac{\partial}{a\,\partial r}|E_\varphi|^2 + \frac{1}{a^2}\left(1 - \frac{\omega^2}{\Omega_i^2}\right)|E_\varphi|^2 \right] \qquad (33)$$

where $a = -\left[\frac{\partial}{\partial r}\ln n\right]^{-1}$ and $E$ is defined in (21). Typical density profile $n(r)$ is peaked at the plasma axis so that $a>0$. It follows from (33) that for $a>0$ the PMF exerts a stabilizing effect on an interchange small-scale mode in the outmost part of a plasma cylinder where

$$\frac{r}{a} > \frac{\Omega_i^2}{\Omega_i^2 - \omega^2} \qquad (34)$$

provided tnat $\omega < \Omega_i$. A small-scale interchange activity in the inner part have no disastrous effect on plasma confinement. Whether the stabilization of plasma edge by the RF field is actually achieved depends on level of the RF power applied. The condition for the stabilization does occur is $K+K_R>0$. It seems to be quite achievable inspite of the fact that excitation of an eigen plasma oscillation can produce RF fields of much higher



amplitude than in the case of the antenna near field that we have considered.

## 5. Conclusion

We have considered the effect of the ponderomotive force induced by a RF field at ICRF heating on stability of small-scale interchange modes in an axisymmetric mirror device. RF field was assumed to be localized near a full-turn-loop antenna and have the frequency below the eigen frequency spectrum of the plasma column. We have found that dissipation of RF power (resulting in plasma heating) drives strong resonant-particle instability if minority ions scheme of heating is used. However at a sufficiently high level of RF power, quasilinear relaxation of minority ions distribution suppresses this instability. Assuming it to be actually suppressed, we have obtained a simple criterion for the ponderomotive force to have favourable effect on plasma stability. We have found that the plasma edge can be stabilized at $\omega < \Omega_i$ while an inner part of plasma column is accessible for a small-scale interchange instability. Fortunately, MHD activity inside the plasma column have no disastrous effect on plasma confinement.

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