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ABSTRACT

Dynamics of a complicated quantum system interacting with open decay channels is treated by means of the discretized effective non-hermitean Hamiltonian. The main subject is segregation of collective short-lived resonances, which are similar to the coherent Dicke states in optics, from the background of equilibrium compound states. The analysis is carried out with the use of two complementary representations (internal and doorway). The phase transition between regimes of weak and strong continuum coupling is considered. The effects of the channel thresholds as well as the problem of ergodicity (equivalence of energy and ensemble averages) are discussed. Two simple models illustrate results, show the relationship to the similar solid state problems, and display new collective phenomena for doorways strongly coupled to the background.

1. INTRODUCTION

Recently, a number of interesting phenomena was discovered in the dynamics of resonances embedded into continuum. New collective effects and their coexistence with the irregular structure of compound states are of special interest. The goal of the paper is to give an uniform treatment of such effects, to classify them according to relative values of relevant physical parameters and to discuss various manifestations of the collective dynamics in the continuum.

It is well known [1, 2] that the behavior of a quantum system with N internal states $|n\rangle$, $n = 1, \ldots, N$, decaying into k open channels c, $c = 1, \ldots, k$, can be described by means of the effective Hamiltonian $\mathcal{H}$ which acts within the intrinsic N-dimensional space only but acquires, due to the elimination of continuum variables, an antihermitean part,

$$\mathcal{H} = H - (i/2)W . \qquad (1)$$

Here H and W are both hermitean; H is the internal Hamiltonian with a discrete spectrum whereas W originates from on-shell self-energy contributions corresponding to open channels. Due to the discretization of the dynamical problems in the continuum, we have now at our disposal powerful matrix methods similar to those in physics of bound states.

In consequence of the unitarity of the scattering matrix, W has a specific factorized form

$$W = AA \Rightarrow W_{12} = \sum_{c(\text{open})} A_1^c A_2^c . \qquad (2)$$

The off-shell contributions generated by the open as well as by closed decay channels are assumed to be incorporated into H. The decay amplitudes A_1^c can be considered as real (for a T-invariant system) and energy-independent quantities. The latter assumption being valid within the limited energy interval implies, in particular, the decay thresholds to be remote from this interval.

In practice, we are interested in the region of the high level density $\rho = N/2a = D^{-1}$, $N \gg 1$, where D is the mean level spacing, and we denote our energy interval as $(-a, a)$. For many applications, it is possible to go to the limit $N \to \infty$ keeping a constant. As for the channel number k, two situations were analyzed: small number of explicitly considered channels (usually $k \leq 4$) and large k making up a finite fraction of N.

Various physical questions can be addressed with the use of the effective phenomenological Hamiltonian (1).

A. This approach allows one to study the dynamics in the continuum using the language of quasistationary states with exponential time dependence $\propto \exp(-i\mathcal{E}t)$. Those are eigenstates of the Hamiltonian (1) with complex energies $\mathcal{E} = E - (i/2)\Gamma$. The structure (2) of the matrix W ensures that $\Gamma \geq 0$ for all eigenvalues. In many respects such a consideration is similar to that in the discrete spectrum.

Friedrich and Wintgen [3] observed that in a two-level system with a common decay channel the width of one eigenstate can vanish at some parameter values. Moldauer [4] and Kleinwächter and Rotter [5] in their realistic numerical calculations noticed that, in the case of intrinsic levels strongly coupled to the continuum, the broad states are formed absorbing the significant part of the summarized level width and increasing the lifetime of the rest of states. The number of the short-lived states is correlated with the number of the open channels. The transition from the nonoverlapping levels with uniformly distributed widths to the regime with the striking difference of broad and narrow states is rather sharp. It occurs at $\langle\Gamma\rangle \approx D$ where $\langle\Gamma\rangle$ is the mean level width.

In Refs. [6 - 8] (see also [9, 10]) we have shown that the existence of two dynamical regimes with the distinct phase transition between them follows straightforwardly from the formalism using the effective Hamiltonian (1). If the characteristic parameter

$$\kappa = \langle\Gamma\rangle / D , \qquad (3)$$

is small, $\kappa \ll 1$, the antihermitean part W is a weak perturbation providing the stationary eigenstates of H with the small widths; off-diagonal elements of W are then of minor importance. As a result, we have well separated narrow resonances with the smooth width distribution. This is the case for the neutron resonances at low energies. In the opposite case $\kappa \gg 1$, the off-diagonal elements of W play the significant role. They generate the strong level coupling through the common decay channels resulting in the drastic redistribution of widths. For k open channels, k collective states are created accumulating the lion's share of the total width $w = \text{Tr}\, W = N\langle\Gamma\rangle$. The remaining N-k eigenstates become very narrow. The same effect was rediscovered for the simplest case of the degenerate levels and one decay channel in ref. [11] and then for more general cases in Refs. [12].

The detailed theory was developed in ref. [13] where the collectivization of widths was interpreted as the clear separation of direct and equilibrium processes and the similarity to the Dicke coherent state [14] in optics, created by the synchronization of the individual radiators through the common radiation field, was pointed out. As has been shown analytically and numerically [15], the transition between the two regimes is very distinctive in the limit $N \Rightarrow$

∞, $k \Rightarrow \infty$, $m = k/N$ = const. The counterplay between the conventional ("internal") collective effects in the discrete spectrum and the "external" collectivization via the continuum was analyzed in Ref. [16]; the implications for the giant resonances were discussed with the conclusion that the precursor of the main peak should exist in the isovector case near the unperturbed shell model energy.

B. Another aspect of the problem is connected with the complicated structure of the intrinsic states corresponding to some kind of chaotic motion. The irregularity of the internal wave functions does not destroy the collectivity created by the common decay channels [6, 8]. In the limit of the extreme chaoticity one can assume that the internal dynamics can be described by the Gaussian orthogonal ensemble (GOE) and the decay amplitudes A_1^c are normally distributed random variables. The joint distribution function of complex eigenvalues for this ensemble of nonhermitean Hamiltonians was obtained in [6, 8] (the details of the derivation can be found in [13]) for the one channel case. Even for the weak overlap, $\kappa \ll 1$, the Wigner level repulsion on the real energy axis vanishes at spacings less than $\langle\Gamma\rangle$. The repulsion of the complex energies is proportional to the cube of the distance in the complex plane if these points are not too near to the real axis. At $\kappa \geq 1$, the broad state stands out against a background of remaining resonances which return to the weak overlap regime (very narrow widths with the Porter-Thomas distribution). For the case of many uncorrelated equivalent channels some results concerning the eigenvalue density in the complex plane were obtained in [15] manifesting again the phase transition with the segregation of k broad states.

The interaction of collective states with the statistical background gives rise, in the limit of $N \to \infty$, to the spreading width of a collective resonance. The applications of the formalism under study to isobaric analog resonances and multipole giant resonances were considered in Refs. [17] and [16] respectively. Here the scaling properties of the random matrix elements H_{12} and decay amplitudes A_1^c ($\propto N^{-1/2}$) are crucial.

C. The whole approach can be of some use for the analysis and parameterization of data concerning resonance reactions. Actually it goes back to Ref. [18] where the problem of the neutral kaon decays was treated in similar way. Some general statements on the level repulsion for unbound states were formulated in [19]. It would be highly desirable to try to extend the standard analysis of the neutron resonances, which gives the Wigner nearest level spacing distribution and the Porter-Thomas width distribution, from the near-threshold region to higher neutron energies where the resonances become to overlap.

In what follows, we try to set forth the general scheme for the consideration of the dynamics governed by the effective Hamiltonian (1) and to systematize the typical situations which differ in the relationship between available physical parameters. Some domains in the parameter space apparently have not been analyzed up to now.

Sect. 2 contains the main equations written down in the basis of eigenstates of the real internal Hamiltonian H ("internal" representation, IR). Various methods of treatment and qualitative features of the phase transition to the regime of strong continuum coupling for the single- and many-channel cases are discussed. In particular, the scaling of the critical value of the overlap parameter κ (3) with the open channel number is derived.

Sect. 3 touches upon a question of ergodic properties of description of an open system where the internal states have complicated wave functions which manifest itself in the fine structure of cross sections. The preliminary answer we suggest at the current initial stage of research is that the conventional energy average and the ensemble average over the random intrinsic dynamics give different results in the many-channel case.

In Sect. 4 we transform the formalism to the basis of eigenstates of the decay Hamiltonian W ("doorway" representation, DR). Being strictly equivalent in the mathematical sense, as has been explained rather long ago, for the single-channel case, by Jeukenne and Mahaux [20], the two representations are complementary with respect to the domains of the physical applicability.

Sect. 5 gives the generalization taking into account the energy dependence of the decay amplitudes due to the proximity of the channel threshold.

Two simple models clearly illustrating the whole formalism and the variety of physical patterns are considered in Sect. 6. The model **A** establishes the analogy of the IR to the delocalized Bloch wave basis in solids whereas the DR corresponds to the localized surface states. The new situation with the strong coupling of the doorway state to the background is considered with the help of the model **B** of Sect. 6. Rather unexpectedly, it results in the splitting of the doorway resonance into components repelling each other as far as possible. Such an example was found in the model calculations by P.von Brentano [21]. The underlying physics is rather simple: the strong coupling to the doorway forms another collective state from the background.

In what follows, we prefer, as a rule, to use a language of models being as simple as possible, to make clear main features of arising physical pictures.

2. INTERNAL REPRESENTATION

Given the effective nonhermitean Hamiltonian $\mathcal{H}$, one can introduce the Green function

$$\mathcal{G}(\mathcal{E}) = (\mathcal{E} - \mathcal{H})^{-1} , \qquad (4)$$

describing the propagation through the set of the unstable states coupled to the continuum. The bare Green function for a stable system with the hermitean Hamiltonian H is

$$G(\mathcal{E}) = (\mathcal{E} - H)^{-1} , \qquad (5)$$

so that, due to the separable structure of W, Eq. (2), the two propagators are interconnected as follows (here and

below the hat marks $k \times k$ matrices in the channel space):

$$\mathcal{G}(\mathcal{E}) = G(\mathcal{E}) - (i/2)G(\mathcal{E})A\{1 + (i/2)\hat{K}(\mathcal{E})\}^{-1}A^{T}G(\mathcal{E}) , \qquad (6)$$

where **A** is the $k \times N$ matrix of the amplitudes A_1^c, and

$$\hat{K}(\mathcal{E}) = A\ G^{T}(\mathcal{E})A . \qquad (7)$$

Neglecting for simplicity the potential scattering one can, using eqs. (6) and (7), express the amplitude T^{ab} of the reaction $b \to a$ as a matrix element of the $k \times k$ matrix

$$\hat{T}(\mathcal{E}) = A^{T}\mathcal{G}(\mathcal{E})A = \hat{K}(\mathcal{E})\{1 + (i/2)\hat{K}(\mathcal{E})\}^{-1} , \qquad (8)$$

taken at the real reaction energy $\mathcal{E} \Rightarrow E + i0$. The full complete scattering matrix is

$$\hat{S}(\mathcal{E}) = 1 - i\hat{T}(\mathcal{E}) = [1 - (i/2)\hat{K}(\mathcal{E})][1 + (i/2)\hat{K}(\mathcal{E})]^{-1} . \qquad (9)$$

The amplitudes $T^{ab}(\mathcal{E})$ are the meromorphic functions in the complex energy plane having the poles in the points $\mathcal{E}_j = E_j - (i/2)\Gamma_j$ of the eigenvalues of the effective Hamiltonian $\mathcal{H}$. Eq. (8) shows immediately that these eigenvalues can be calculated as the roots of the secular equation

$$\mathcal{D}(\mathcal{E}) \equiv \det\{1 + (i/2)\hat{K}(\mathcal{E})\} = 0 , \qquad (10)$$

where det stands for the determinant in the k-dimensional channel space. Note the relationship between the determinants (Det) in the N-dimensional level space:

$$\mathrm{Det}(\mathcal{E} - \mathcal{H}) = \mathrm{Det}(\mathcal{E} - H)\mathcal{D}(\mathcal{E}) ; \qquad (11)$$

the poles of $\hat{K}(\mathcal{E})$ at real eigenvalues of H are canceled in the r.h.s. of Eq. (11) by the zeros of $\mathrm{Det}G^{-1}(\mathcal{E})$.

Since the two parts of $\mathcal{H}$ in general do not commute with each other they can not be diagonalized simultaneously with an orthogonal transformation. Instead one should deal with the complicated biorthogonal set of the eigenstates $|\Phi_j>$ of $\mathcal{H}$. As a rule, it is convenient to analyze the physical pattern in terms of an appropriate orthogonal complete basis. One of the natural options is to use the basis of the stable eigenstates $|n>$ of the hermitean part H ("internal" representation, IR). The matrix (7) can be written down in the IR as

$$K^{ab}(\mathcal{E}) = \sum_n A^a_n A^b_n (\mathcal{E} - \varepsilon_n)^{-1} , \qquad (12)$$

where ε_n are the energies of the stable states $|n>$ and the same notations A^c are maintained for the transformed real decay amplitudes.

The decay matrix W, Eq. (2), has the rank k (we assume $k<N$) which is equal to the number of open channels. k nontrivial eigenvalues of W coincide with those of the $k\times k$ matrix

$$\hat{X} \Rightarrow X^{ab} = \sum_n A^a_n A^b_n , \qquad (13)$$

so that, introducing partial widths $\gamma^c_n = (A^c_n)^2$, one has

$$w = \mathrm{TrW} = \mathrm{tr}\hat{X} = \sum_{c,n} (A^c_n)^2 = \sum_{c,n} \gamma^c_n = \sum_n \gamma_n = \sum_c \gamma^c = \sum_j \Gamma_j , \qquad (14)$$

where $\gamma_n = \Sigma_c \gamma^c_n$, $\gamma^c = \Sigma_n \gamma^c_n$ and Γ_j is the width of the j-th exact eigenstate $|\Phi_j>$ of the total Hamiltonian $\mathcal{H}$; Tr and tr stand for traces in the intrinsic and channel spaces respectively. The last equality in (13) follows from the invariance of TrW = - 2ImTr$\mathcal{H}$ with respect to the complex transformation diagonalyzing $\mathcal{H}$.

The IR is especially convenient in the limit of weak

overlap when the coupling to the continuum can be treated as perturbation. Here and below we assume that all internal states are of the same degree of complexity so that we can use for estimates the average parameters $<\Gamma>=<\gamma_n>=w/N=k<\gamma^c_n>$.
In the weak overlap case, by the definition (3), $\kappa = <\Gamma>/D = w/2a \ll 1$. Then the secular equation (10) shows that the access to the continuum results in small widths of isolated resonances,

$$\mathcal{E}_n \approx \varepsilon_n - (i/2)\gamma_n . \qquad (15)$$

Eq. (15) is valid for almost all roots except when the energy ε is just at the edge of the interval, $|\varepsilon \pm a|/a \simeq \exp(-1/\kappa)$. But, in any case, the sharp cut-off at the edge is the model artifact. The natural truncation is determined by the decay threshold, Sect. 5. Up to the second order corrections, the original distribution of energies is unchanged whereas the distribution of widths is fixed by that of the amplitudes A^c_n. The normally distributed uncorrelated A^c_n imply the χ^2 width distribution for k degrees of freedom (the Porter-Thomas distribution in the one-channel case).

In the opposite case of the strong coupling to the continuum ($\kappa > 1$) the antihermitean part with its specific structure (2) dominates the dynamics. One can still use the IR but the resulting pattern is unlike that of the uniformly spread overlapped resonances. For the single decay channel eq. (10) reads

$$1 + (i/2)\sum_n \gamma_n (\mathcal{E} - \varepsilon_n)^{-1} = 0 , \qquad (16)$$

which gives one broad state $\mathcal{E}_1 = E_1 - (i/2)\Gamma_1$ and N-1 very narrow states $\mathcal{E}_j$, j = 2, ..., N. At $\kappa \gg 1$, denoting $<\varepsilon> = \Sigma_n \varepsilon_n \gamma_n / w$, we get

$$E_1 = \langle\varepsilon\rangle + 4\sum_n \varepsilon_n(\langle\varepsilon\rangle-\varepsilon_n)^2\gamma_n/w^3 = \langle\varepsilon\rangle[1 + O(\kappa^{-2})] , \qquad (17)$$

$$\Gamma_1 = w - 4\sum_n \gamma_n(\varepsilon_n-E_1)^2/w^2 = w[1 - O(\kappa^{-2})] . \qquad (18)$$

Thus, the collective ("Dicke") resonance is located near the middle of the energy interval; it accumulates the major part of the total summed width w reducing the remaining $N - 1$ widths to very small values $\Gamma_j \cong w/\kappa^2N \ll \langle\Gamma\rangle = w/N$. These states $|\Phi_j\rangle$ correspond again to isolated resonances, their overlap parameter being $\langle\kappa_j\rangle = \langle\Gamma_j\rangle/D \approx \kappa^{-1} \ll 1$, and can be considered in terms of the statistical equilibrium since their lifetime $\tau_j=\hbar/\langle\Gamma_j\rangle\approx(\hbar/D)\kappa$ is much longer than the recurrence time $\hbar/D$. The collective state vector $|\Phi_1\rangle$ represented as a vector in the N-dimensional space is almost aligned along the decay amplitude vector $\mathbf{A} = \{A_n\}$; its "transverse" components are of the order of $a/\Gamma_1 \approx \kappa^{-1}$. On the contrary, compound states are situated mainly in the orthogonal subspace. The typical energy dependence for the scattering cross-section was discussed in Ref. [13] in terms of the separation of the long-scale (compound) and short-scale (direct) processes.

If one is not interested in the fine structure details they can be averaged out. Neglecting, in accordance with the picture of many intrinsic states of similar complicated structure, possible correlations between the level energies ε_n and their decay amplitudes, and introducing the intrinsic level density $\rho(\varepsilon)$ we come to the averaged version of Eq. (16):

$$1+(i/2)\int_{-a}^{a} d\varepsilon\gamma(\varepsilon)\rho(\varepsilon)(\mathcal{E}-\varepsilon)^{-1}\approx 1-(i/2)\langle\gamma\rho\rangle\ln[(\mathcal{E}-a)/(\mathcal{E}+a)]=0. \qquad (19)$$

Since, in the same manner, $\sum_n \gamma_n = w \Rightarrow \langle\gamma\rho\rangle\ 2a$, the mean value $\langle\gamma\rho\rangle$ is nothing but our overlap parameter, $\langle\gamma\rho\rangle = w/2a = \kappa$. Then Eq. (18) gives the collective root in conformity with Eq. (18),

$$E_1 = 0, \qquad \Gamma_1 = w\kappa^{-1}\cot\kappa^{-1} \approx w[1-(1/3\kappa^2)] . \qquad (20)$$

In the continuous approximation (19) the collective width vanishes at $\kappa = \kappa_c = 2/\pi$. Of course, then the approximation itself becomes invalid. But, as can be seen from (20), the collectivity condition $\Gamma_1 \gg D$ is fulfilled very near to the critical value of κ, namely, at $(\kappa - \kappa_c) \gg 4/N\pi^2$. Thus, for $N \gg 1$, the reorganization of the spectra occurs rather sharply showing up the phase transition .

Another way of averaging offers a possibility to take into account the specific features of the internal level density. For such a goal, we can right down the secular equation (16) as

$$1 + (i/2)w\langle g(\mathcal{E})\rangle = 0 , \qquad (21)$$

where $g(\mathcal{E})$ is defined for the stable system (5),

$$g(\mathcal{E}) = N^{-1}\mathrm{Tr}\{G(\mathcal{E})\} . \qquad (22)$$

For example, let us assume that the internal dynamics is chaotic and it can be described by the GOE [22]. The random matrix elements of H are Gaussian variables with the correlation function

$$\langle H_{12}H_{34}\rangle = (a^2/4N)(\delta_{14}\delta_{23} + \delta_{13}\delta_{24}) . \qquad (23)$$

Here we use the standard scaling factor N^{-1} [23-26] which leads, in the limit of N>>1, to the Tr $H^2 \propto N$. It is well known that this factor appears naturally in the estimates of matrix elements of interaction between typical complicated states if they have N simple (shell-model-like) components superimposed with the weights $\approx N^{-1/2}$; a is the typical magnitude of the interaction matrix elements between the simple states. Such a scaling is essential to explain qualitatively many aspects of statistical reactions; as examples the dynamical enhancement of the parity non-conservation in the fission process [23, 24] and the suppressed variations of the spreading widths of isobaric analog resonances [27, 17] can be mentioned.

In the GOE case (23) the average Green function (22) is

$$\langle g(\mathcal{E})\rangle = (2/a^2)\{\mathcal{E} - (\mathcal{E}^2-a^2)^{1/2}\} , \qquad (24)$$

leading to the semicircle level density distribution confined within the interval $(-a, a)$,

$$\rho(E) = (2N/\pi a^2)(a^2-E^2)^{1/2}, \quad |E| \le a . \qquad (25)$$

Substituting (24) in (21) we again get for the Dicke state

$$E_1 = 0, \quad \Gamma_1 = w(1-a^2/w^2) = w[1-(4\kappa^2)^{-1}] . \qquad (26)$$

in agreement with Eq. (18). Evidently, the arguments used in the end of the paragraph after Eq. (20) could be repeated here.

The diversity of possible patterns grows with the number k of the open channels. Apart from the parameters a and w, the resulting picture is determined now by the partial widths γ^c for the specific channels c (the likeness

of the intrinsic states is still assumed) and by $k(k - 1)/2$ angles ϑ^{ab} between the decay amplitude N-component vectors $A^c = \{A^c_n\}$. Thus, in the regime of the strong coupling to the continuum, κ>>1, for two decay channels a and b we get two broad states $|\Phi_\pm\rangle$ and N-2 long-lived states which share altogether the width of the order $w\kappa^{-2}$. Quite similar to Eqs. (16 - 18), one can find the complex energies of the broad resonances $\mathcal{E}_\pm = E_\pm - (i/2)\Gamma_\pm$ dependent on the bare widths $\gamma^a = (A^a)^2$ and $\gamma^b = (A^b)^2$ as well as on ϑ, $(A^aA^b) = (\gamma^a\gamma^b)^{1/2}\cos\vartheta$,

$$E_\pm = \frac{\sum_n \varepsilon_n[(\Gamma_\pm-\gamma^a)(A^a_n)^2 + (\Gamma_\pm-\gamma^b)(A^b_n)^2 + 2(A^aA^b)A^a_nA^b_n]}{\Gamma_\pm w - 2\gamma^a\gamma^b\sin^2\vartheta} , \qquad (27)$$

$$\Gamma_\pm = (1/2)\{w \pm [w^2 - 4\gamma^a\gamma^b\sin^2\vartheta]^{1/2}\} . \qquad (28)$$

In the "perpendicular" situation, $\vartheta = \pi/2$, the channels are almost decoupled (decoupling becomes perfect for the degenerate internal levels ε_n) and the broad resonances are matched to different channels. In the Hilbert space they are aligned along the orthogonal vectors A^a and A^b and their widths are unperturbed ones, γ^a and γ^b respectively. When the angle ϑ decreases the channel mixing increases. At small ϑ, one of the broad resonances disappears joining the quasicontinuum of the compound states. For this state the components along A^a and A^b interfere destructively so that their ratio equals $(-\gamma^b/\gamma^a)$ and the width goes to zero as $\Gamma_- = (\gamma^a\gamma^b/w)\vartheta^2$. The second state with the constructive interference (the ratio of the component is one) absorbs in the "parallel" situation, $\vartheta \to 0$, the total decay width, $\Gamma_+ = w = \gamma^a + \gamma^b$. The rest of the states is concentrated mainly in the subspace orthogonal to that spanned by the vectors A^a and A^b.

Again, for the broad states, the average description can be achieved with the use of the average Green function (22) and the decay matrix $\hat{X}$ (13). Let $\tilde{\gamma}_r$ be the eigenvalues of $\hat{X}$, $r = 1,\dots,k$, $\Sigma_r \tilde{\gamma}_r = w$ (the so called eigenchannel representation); for the two-channel case expressions for these eigenvalues coincide with $\Gamma_\pm$, Eq. (28). Then the secular equation (10) falls apart into the product of the equations analogous to that for the single channel case (21):

$$\prod_r [1+ (i/2)\tilde{\gamma}_r <g(\mathcal{E})>] = 0 \ . \qquad (29)$$

For each eigenchannel, the solution of (29) coincides with that of Eq. (26) with $w \Rightarrow \tilde{\gamma}_r$. However, one 0should have in mind that the average description (29) is valid under more strict condition than (26). Here the overlap condition $w>a$ is replaced by the similar constraint for each eigen-channel, $\tilde{\gamma} \approx w/k > a$. Therefore, the range of parameter values exists at large number of open channels, $k >> 1$, where $w/k < a < w$, so that the levels are overlapped (w/N>D) but the collectivization of widths does not occur yet: roughly speaking, one needs the strong coupling to the specific common decay channel to create the coherent state.

The mechanism deferring the onset of collectivization in the case of many "nonparallel" channels was discussed briefly in [10]. The matrix elements (2) of W are sums of k independent products of the decay amplitudes. Therefore one can expect that the off-diagonal matrix elements are proportional to $k^{1/2}$ at large k whereas the diagonal ones grow linearly with k. It can be seen easily if we introduce, similar to ϑ^{ab}, the angles χ_{mn} between the N k-dimensional

vectors $\mathbf{A} = \{A^c\}$,

$$W_{mn} = \mathbf{A}_m \mathbf{A}_n = (\gamma_m \gamma_n)^{1/2} \cos\chi_{mn} \ . \qquad (30)$$

At natural statistical assumptions, these angles are uncorrelated with γ_n and the mean value of $\cos\chi$ is $k^{-1/2}$ which implies the smallness of W_{mn}, $m \neq n$, in comparison with W_{nn}. Evaluating the second order correction to the eigen-values induced by these off-diagonal terms, we find that it ceases to be small at $w/Nk \sim$ D, i.e. at $\kappa = <\Gamma>/D \sim k$. It defines the border between the two dynamical regimes. This k-scaling can be seen in the realistic nuclear calculations [5] for the dipole resonances in ^{16}O where proton and neutron escape channels populating the ground and the first excited states in ^{15}N were taken into account, k=1, 2, or 4.

In the asymptotic limit of the large channel number,

$$N \to \infty, \ k \to \infty, \ m = k/N = \text{const} \ , \qquad (31)$$

the two regimes are separated with the sharp phase transition at $\kappa \approx k$ as was demonstrated very distinctly in the numerical simulation made in Ref. [15] for random matrices of type (1) (N = 200, k = 50) with the same statistical properties as in [13], namely the GOE for the hermitean part H and the Gaussian distribution for the amplitudes A^c_n with the uncorrelated equiprobable channels,

$$<A^a_n A^b_m> = (\gamma/N)\delta^{ab}\delta_{nm} \ . \qquad (32)$$

In the phase transition point ($\gamma \approx 2a$) the cloud depicting the eigenvalue distribution in the complex energy plane dissociates into two subclouds one of them (compound states) being flattened against the real axis and the second one (coherent short-lived states) stretched along the imaginary axis and contained the fraction k/N of the total number of states. The phase transition looks like as an appearance of the finite gap separating the two domains.

This scenario of the width collectivization when increasing the mean overlap parameter κ imposes the restraint on the standard concept of the Ericson fluctuations [28]. As has been pointed out in Ref. [13] the collectivization mechanism suppresses those fluctuations at small number of the open channels. At $\kappa<1$ the levels are not overlapped whereas at $\kappa > 1$ the majority of the quasistationary states again returns effectively into the same regime of isolated resonances. The room for the Ericson fluctuations appears in the case of $k>\kappa>1$ when there exist a large number of strongly overlapped states uncorrelated with each other. In the overcritical region of $\kappa>k$, one can expect attenuation of fluctuations. The whole question deserves to be elaborated in detail.

3. ENERGY AVERAGE AND ENSEMBLE AVERAGE

In this section, we discuss briefly the problem of ergodicity as concerns the theory of reactions in the region of overlapped resonances. In the conventional practice, the experimental cross-section is averaged over some energy interval I covering a number of internal levels, $I>>D$, but still keeping track of the intermediate structure energy behavior, $I < a$. For functions F(E) of energy, which have analytical properties similar to those of the S-matrix (9), the energy average can be taken explicitly using the Lorentzian weight function. It results in the simple recipe: $F(E) \Rightarrow \bar{F}(E) = F(E + iI)$. Such a procedure when applied to the scattering matrix (9) leads to

$$\bar{\hat{S}}(E) = [1-(i/2)\bar{\hat{K}}(E)][1+(i/2)\bar{\hat{K}}(E)]^{-1} . \qquad (33)$$

It implies the independent averaging in the numerator and in

the denominator of the original amplitude (8).

If the intrinsic levels are of equally complicated structure one would expect that, at N large enough, the level ensemble shows up the self-averaging properties and the ensemble average coincides with the energy average (33) with the possible exception of the outer edges of the energy interval. Indeed, it can be shown to be true for the small number of open channels, $m = k/N \to 0$ at $N\to\infty$. In this case, the terms taking into account the correlations between the K-matrices in the numerator and in the denominator of Eqs. (8) or (9), have the lesser number of traces in the internal space and therefore they are of higher order in N^{-1} than the main term originating from the independent averaging of the two K-matrices, so that

$$\langle\hat{S}(E)\rangle = [1-(i/2)\langle\hat{K}(E)\rangle][1+(i/2)\langle\hat{K}(E)\rangle]^{-1} . \qquad (34)$$

Here $\langle\ldots\rangle$ stands for the ensemble average. Note that the similarity of Eqs. (33) and (34) takes place independently of the assumptions concerning the decay amplitudes: one can treat them either as dynamic quantities [26] or as random variables [8, 13] like in Eq. (32).

The situation changes for the large channel number, i.e. in the asymptotic limit (32). The Green function (4) for the total Hamiltonian (1) can be averaged out with respect to the internal dynamics to give

$$\langle\mathcal{G}(z)\rangle = z^{-1}\{1 + \langle H\mathcal{G}(z)\rangle - (i/2)W\langle\mathcal{G}(z)\rangle\} . \qquad (35)$$

For the GOE case, calculating $\langle H\mathcal{G}\rangle$ by the formal expansion and resummation with the use of (23) in each term, we get

$$[z + (i/2)W]\langle\mathcal{G}(z)\rangle = 1 + (a^2/4)\langle g(z)\mathcal{G}(z)- N^{-1}(d\mathcal{G}/dz)\rangle , \qquad (36)$$

where the trace g(z) was defined in (22). In the limit of $N\to\infty$, the average product $\langle g\mathcal{G}\rangle = \langle g\rangle\langle\mathcal{G}\rangle + O(N^{-1})$ so that

$$\langle \mathcal{G}(z)\rangle = [z - (a^2/4)\langle g(z)\rangle + (i/2)W]^{-1} . \qquad (37)$$

Of course, at $W \to 0$ eq.(37) reproduces the result (24) for the stable system. Actually the same formula, with $[-(i/2)W]$ substituted by H_0, is valid for any deterministic operator H_0 added to the GOE Hamiltonian H. The average analog of Eq. (6) is

$$\langle \mathcal{G}(z)\rangle = \mathcal{G}_0(z)\{1 - (i/2)A[\mathcal{G}_0^{-1}(z) + (i/2)\hat{X}]^{-1}A^T\} , \qquad (38)$$

where now, instead of G(z), the function

$$\mathcal{G}_0(z) = [z - (a^2/4)\langle g(z)\rangle]^{-1} , \qquad (39)$$

appears, containing the trace g(z) of the full Green function including the decay effects. Eq. (38) immediately determines the average reaction matrix (8) and the scattering matrix (9),

$$\langle \hat{T}(z)\rangle = A^T\langle \mathcal{G}(z)\rangle A = \hat{X}[\mathcal{G}_0^{-1}(z) + (i/2)\hat{X}]^{-1} , \qquad (40)$$

$$\langle \hat{S}(z)\rangle = [1 - (i/2)\mathcal{G}_0(z)\hat{X}][1 + (i/2)\mathcal{G}_0(z)\hat{X}]^{-1} . \qquad (41)$$

This expression, being the exact result of the GOE for the internal interaction, differs from the "ergodic" formula (34) by the replacement

$$\langle \hat{K}(z)\rangle = \langle G(z)\rangle\hat{X} \;\Rightarrow\; \hat{K}(z) = \mathcal{G}_0(z)\hat{X} . \qquad (42)$$

The function $\mathcal{G}_0(z)$ would coincide with the pure GOE result $\langle G(z)\rangle$ (equal to $\langle g(z)\rangle$ given by Eq. (24)) for $m = k/N \to 0$. However, if in the asymptotic limit $m \to$ const, the distortion of $\langle g(z)\rangle$ and of the state density is not negligible. The trace of the full average Green function (38) is easily found to be

$$\langle g(z)\rangle = \mathcal{G}_0(z)[1 - (i/2)mt(z)] , \qquad (43)$$

where t(z) is the trace of the reaction matrix (40) averaged with respect to the channels,

$$t(z) = k^{-1}\mathrm{tr}\langle \hat{T}(z)\rangle . \qquad (44)$$

Hence, for finite m, the similarity between the energy average (33) and the ensemble average (34) is violated. Formally speaking, the small factor $\sim N^{-1}$ associated with the contractions of the random variables taken from the numerator and the denominator of the S-matrix (9), is compensated by the large number k of decay channels. Using the more physical language, we can estimate under which conditions the extra term $mt(z)$ in Eq. (43) ceases to be a small correction. In the resonance region, $|z|<a$, it happens, for the large channel number, $m \gtrsim 1$, when the typical matrix elements $\gamma \approx w/k$ of the matrix X reach the order of $|\mathcal{G}_0^{-1}(z)| \sim a$, see Eq. (40). This condition coincides with that of the width collectivization for the large channel number, $\kappa = w/a \approx k$. It is clear that on the border of the two dynamical regimes the formation of the coherently decaying states destroys the homogeneity along the energy axis which implies the violation of ergodicity.

The breakdown of ergodicity in the regime of the strong coupling to the continuum is of rather general nature which persists, as can be shown, even if the decay amplitudes are treated as random variables and the additional averaging is carried out, for example as in (32).

4. DOORWAY REPRESENTATION

The eigenbasis of the IR utilized in Sect. 2 becomes remote far away from the eigenstates of the effective Hamiltonian $\mathcal{H}$ in the regime of the strong coupling via continuum. For the general system with nondegenerate internal levels it turns out to be good practice to perform a transformation to the alternative basis [13] which can be called the doorway representation (DR).

Since the antihermitean part W of the total Hamiltonian $\mathcal{H}$ dominates now the dynamics, it is natural to start with the diagonalization of W. For the case of k open channels, $k<N$, we get k states $|d\rangle$, $d=1,\ldots,k$, with non-negative eigenvalues $\tilde{\gamma}_d$ which exhaust the summed width, $\sum_d \tilde{\gamma}_d = w$, and the orthogonal $(N-k)$-dimensional subspace where all eigenvalues of W are zero. Using this degeneracy we can in addition diagonalize the $(N-k)\times(N-k)$ submatrix of the hermitean part H in this subspace to obtain the new "energies" $\tilde{\varepsilon}_t$, $t = k+1,\ldots,N$.

The two consecutive orthogonal transformations bring the matrix $\mathcal{H}$ into the form where first k states $|d\rangle$ have the direct access to the continuum (doorway states) whereas the remaining states can decay only through the doorway ones. Note that the DR looks very different from the IR where the characteristic pattern is that of many uniformly distributed levels with the comparable decay amplitudes. Nevertheless, the two representations are strictly equivalent. The IR is more practical in the case of weak coupling to continuum. Meanwhile, at $\kappa>1$ the advantage of the DR is that here the coupling of the "trapped" states $|t\rangle$, $t = k + 1, \ldots, N$, to the doorways turns out to be weak.

The k-dimensional doorway subspace is spanned by the k unit eigenvectors $c^d=\{c^d_n\}$ of W corresponding to the nonzero eigenvalues $\tilde{\gamma}_d$, $d = 1,\ldots, k$ (we assume that the amplitude vectors A^a are linearly independent; otherwise the dimension

of the doorway space is less than k). The vectors A^a lie entirely in this subspace and their components $p^{ad} = A^a c^d = \sum_n A^a_n c^d_n$ along the new axes c^d can be treated as the components $(p^d)^a$ of the eigenvectors p^d diagonalyzing the matrix X (13),

$$\sum_b X^{ab} p^{bd} = \tilde{\gamma}_d p^{ad} , \tag{45}$$

the eigenvalues $\tilde{\gamma}_d$ coincide with those of W. For $t \geq k+1$, vectors c^t of the trapped states are orthogonal to A^a.

The analysis of the general properties of the DR can be done as follows. Let $c^t = \{c^t_n\}$ are the eigenvectors of the trapped states. The diagonalization of H within this subspace leads to

$$\sum_{mn} c^t_m c^{t'}_n H_{mn} = \tilde{\varepsilon}_t \delta_{tt'} . \tag{46}$$

The coupling matrix elements between the two subspaces are

$$H^{dt} = \sum_{mn} c^d_m c^t_n H_{mn} . \tag{47}$$

Using eqs. (46), (47) and the definition (5) of the Green operator G(z) for the internal Hamiltonian H in the total space, together with the closure condition,

$$\sum_d c^d_m c^d_n + \sum_t c^t_m c^t_n = \delta_{mn} , \tag{48}$$

the amplitudes c^t_m can be expressed in terms of H^{dt}:

$$c^t_m = - \sum_{mn} G_{mn}(\tilde{\varepsilon}_t)\, c^d_n H^{dt} . \tag{49}$$

Substituting (49) into (47) one gets the set of linear equations for the coupling matrix elements H^{dt},

$$H^{dt} = -\sum_{d'mn} c^d_m (HG(\tilde{\varepsilon}_t))_{mn} c^{d'}_n H^{d't} , \qquad (50)$$

and, after simple algebra using the orthonormalization properties $\Sigma c^d_n c^{d'}_n = \delta^{dd'}$, we come to the secular equation for

$$\tilde{\varepsilon}_t, \ \det\hat{Q}(\tilde{\varepsilon}) = 0 , \qquad (51)$$

$\tilde{\varepsilon}_t$ where the $k \times k$ matrix $\hat{Q}(z)$ is similar to $\hat{K}(z)$, Eq. (7), but, instead of the decay into the open channels, describes the exit to the doorway subspace,

$$\hat{Q}: \quad Q^{dd'}(z) = \sum_{mn} c^d_m G_{mn}(z) c^{d'}_n . \qquad (52)$$

It can be shown that Eq. (51) has $N-k$ roots $\tilde{\varepsilon}_t$ each of them being isolated within one of the $N - 1$ intervals $[\varepsilon_n, \varepsilon_{n+1}]$ between two next eigenvalues of H (that basis was used in the IR); $k - 1$ of those intervals contain no $\tilde{\varepsilon}_t$.

Below we show that, in the regime of the strong coupling via continuum, the interaction of d- and t - states becomes weak. It means that the energies $\tilde{\varepsilon}_t$ are close to the real parts E_t of the exact eigenvalues of the total Hamiltonian $\mathcal{H}$ (1). Then the above mentioned arrangement of ε_t implies that the repulsion of the exact E_t is, on the average, stronger than that of the energies ε_n of the stable system (a very close contact of ε_n and ε_{n+1} has no counterpart for E_t). This fact observed also numerically in [30]

reflects the randomizing influence of coupling through the common decay channels which can mix the intrinsic states of quite different character.

To estimate the typical coupling strength between d - and t - subspaces, we make use of the normalization condition for c^t_m,

$$\sum_{mn} f^t_m (G^2(\tilde{\varepsilon}_t))_{mn} f^t_n = 1, \qquad f^t_n \equiv \sum_d c^d_n H^{dt} . \qquad (53)$$

This condition being orthogonally invariant is especially simple in the IR basis where G is diagonal (for the transformed f^t_n we keep the same notations assuming as in the previous sections that the average properties of the amplitudes c^d_n do not change under orthogonal transformations of the internal basis):

$$\sum_n (\tilde{\varepsilon}_t - \varepsilon_n)^{-2} (f^t_n)^2 = 1 . \qquad (54)$$

In the sum of Eq. (54) the main contribution comes from the intrinsic levels in the immediate vicinity of $\tilde{\varepsilon}_t$. Therefore, $\langle f^2 \rangle \sim D^2 \sim a^2/N^2$. Since $\langle c^d_n c^{d'}_n \rangle \sim \delta^{dd'} N^{-1}$, it gives

$$\langle (H^{dt})^2 \rangle \sim (N/k) \langle f^2 \rangle \sim a^2/Nk . \qquad (55)$$

At large κ when one expects the DR to be useful, the widths Γ_t acquired by the trapped states due to the coupling H^{dt} are small and they can be evaluated by the perturbation theory. Indeed, one gets in such a way

$$\Gamma_t = \sum_d (H^{dt})^2 \tilde{\gamma}_d [(\varepsilon_d - \tilde{\varepsilon}_t)^2 + \tilde{\gamma}_d^2/4]^{-1} \le 4 \sum_d (H^{dt})^2 / \tilde{\gamma}_d . \qquad (56)$$

According to Eq. (55), the upper boundary of Γ_t can be estimated by the order of magnitude as $k(a^2/Nk)/\langle\tilde{\gamma}\rangle \sim w (k/N)\kappa^{-2}$. The estimate will be self-consistent if the widths Γ_t do not overlap the adjacent trapped states so that $D \sim w/N\kappa > wk/N\kappa^2$, or $\kappa > k$. In the previous sections we have already discussed that this inequality marks the border between the two regimes.

As for the subspace of the doorway states, the hermitean part H mixes the eigenvectors $|d\rangle$ of W by means of the matrix elements $H^{dd'}$ which can be estimated as $\langle (H^{dd'})\rangle^2 \sim a^2/N$. If the differences $|\tilde{\gamma}_d - \tilde{\gamma}_{d'}|$ are of the same order as the widths themselves, the simple perturbative estimate shows that the corrections to the widths $\delta\tilde{\gamma}_d$ are relatively small, $\delta\tilde{\gamma}_d/\tilde{\gamma}_d \sim mk^2/\kappa^2 < 1$ at $\kappa > k$. The case of many equiprobable channels with random decay amplitudes (32) gives different results since here the average density of eigenvalues $\tilde{\gamma}$ occupies the interval $[\gamma_-, \gamma_+]$ where $\gamma_\pm = \gamma(1\pm\sqrt{m})^2$ so that the mean spacing of the widths is $4\gamma\sqrt{m}/k \ll \gamma$. In this case the doorway states are strongly mixed.

To illustrate the general results, let us consider the simplest single channel model (d=1, t=2,..., N). Here the only doorway state absorbs the full width, $\tilde{\gamma}_1 = w$, and the Hamiltonian $\mathcal{H}$ in the DR has the nonzero matrix elements

$$\mathcal{H}_{11} = h - (i/2)w, \quad \mathcal{H}_{1t} = \mathcal{H}_{t1} = h_t, \quad \mathcal{H}_{tt'} = \tilde{\varepsilon}_t \delta_{tt'} . \tag{57}$$

The exact secular equation for the complex energies $\mathcal{E} = E - (i/2)\Gamma$ can be written down as

$$\Omega(\mathcal{E}) \equiv \mathcal{E} - h + (i/2)w - \mathcal{R}(\mathcal{E}) = 0 , \tag{58}$$

$$\mathcal{R}(\mathcal{E}) = \sum_t h_t^2 (\mathcal{E} - \tilde{\varepsilon}_t)^{-1} . \tag{59}$$

Earlier we have justified the applicability of the perturbation theory in h_t. Therefore one obtains for the widths, in agreement with what was found in the IR (18),

$$\Gamma_1 = w\{1 - \sum_t h_t^2[(h - \tilde{\varepsilon}_t)^2 + w^2/4]^{-1}\} = w\,[1 - O(\kappa^{-2})] , \tag{60}$$

$$\Gamma_t = w\, h_t^2\, [(h - \tilde{\varepsilon}_t)^2 + w^2/4]^{-1} \sim w/N\kappa^2 . \tag{61}$$

The scattering amplitude (8) takes very simple form when using the DR. The K-matrix (7) is equal to

$$K(\mathcal{E}) = w\, G_{11}(\mathcal{E}) = w\,[\mathcal{E} - h - \mathcal{R}(\mathcal{E})]^{-1} , \tag{62}$$

so that ($\Omega(\mathcal{E})$ is defined by Eq. (58))

$$T(\mathcal{E}) = w\, \Omega^{-1}(\mathcal{E}) . \tag{63}$$

As it should be, the poles of $T(\mathcal{E})$ coincide with the complex eigenvalues of $\mathcal{H}$ (roots of Eq. (58)) whereas the amplitude (63) and, hence, the cross section, has zeros at real energies $E = \tilde{\varepsilon}_t$ corresponding to the trapped states (poles of $\mathcal{R}(E)$, Eq. (59)).

In the extreme limit $\kappa \gg 1$, the coupling h_t is very weak and eq. (63) displays the scattering via the single Dicke resonance,

$$T(E) \approx w\,[E - h + (i/2)w]^{-1} . \tag{64}$$

The intermediate coupling, $\kappa \gtrsim 1$, reveals the fine structure of the cross sections, namely N-1 narrow resonances (their shape is not the simple Breit-Wigner one) due to the long-lived states involved in $\mathcal{R}(E)$. Outside this energy region, the amplitude (63) has the Breit-Wigner wings corresponding to the doorway state with the width w. If the fine structure is not resolved the sum (59) over large number of intermediate states can be substituted by the integral with the appropriate level density $\rho(\tilde{\varepsilon})$, see for example (25). It gives rise to the imaginary part of $\mathcal{R}(E)$ which adds the spreading width [29] of the doorway state

$$\Gamma^{\downarrow}(E) = -2\mathrm{Im}\mathcal{R}(E) = 2\pi\rho(E)\langle h_t^2\rangle . \qquad (65)$$

Originating from the interaction of the doorway mode with the background of the trapped states, the spreading width is of the same order of magnitude as the energy region a occupied by the internal states coupled to the given channel: from (55) one gets $\Gamma^{\downarrow} \sim (N/a)(a^2/N) \sim a$.

Using the results (60) and (61) for the strong coupling regime, the amplitude (63) can be presented [10] in the form

$$T(E)=-2\sin\delta_1\exp(i\delta_1) + \sum_t \exp(2i\delta_1+i\alpha_t)\Gamma_t[E-E_t+(i/2)\Gamma_t]^{-1}, \qquad (66)$$

$$\mathrm{tg}\delta_1(E) = -(1/2)\,\Gamma_1[E-E_1]^{-1} , \qquad (67)$$

where phases of residues for the trapped states are small, $\alpha_t \sim \kappa^{-1}$. Thus, the Dicke resonance simulates potential scattering as a fast direct process, $\tau_d \sim \hbar/\Gamma_1 \sim \hbar/w$, which is short as compared to the reactions proceeding via the compound states (the sum in Eq. (66)), $\tau_{CN} \sim \hbar/\Gamma_t$. According to Eq. (61), $\tau_{CN} \sim \hbar\kappa/D$ guarantees the equilibrium character of the compound process exceeding by the factor κ the recurrence time $\tau_R \sim \hbar/D$.

5. THRESHOLD PROXIMITY

In the previous account, we neglected completely effects originated from the possible energy dependence of the matrix elements of the Hamiltonian (1). The whole approach is based on the elimination of the channel variables and, in order to be able to work with the effective dynamics within the internal space, one have to pay by the nonstandard features of the Hamiltonian $\mathcal{H}$: it is nonhermitean and energy depen- dent. The imaginary part W arises from the δ-functional contribution to the self-energy operator [2] corresponding to the open channels. Therefore the amplitudes A_n^c are to be taken at running energy.

The main energy dependence of the decay amplitudes comes from the proximity of thresholds. If the thresholds for the essential channels are far away from the energy domain under study, the energy dependence is generally accepted to be smooth and, hence, to be of minor importance in the small region with the high level density. The principal value contributions to the self-energy operator do not discriminate open and closed channels. Therefore, as earlier we neglect the energy dependence of the hermitean part H.

To get the impression of what we should expect due to the threshold behavior, we consider the simplest model where the decay amplitudes are taken to be of the s-wave form,

$$A_n^c(\mathcal{E}) = p^c(\mathcal{E})u_n^c \equiv (\mathcal{E} - E_o^c)^{1/2}u_n^c . \qquad (68)$$

Here E_o^c is the threshold energy, $p^c(\mathcal{E})$ is the relative motion momentum, and u_n^c are reduced energy-independent amplitudes; Coulomb effects being essential near the threshold for the emission of a charged particle are not taken into account in Eq. (68). The generalization of Eq. (68) for higher partial waves is straightforward.

We start with the one-channel case when the IR secular equation (16) can be written down as

$$1 + (i/2)(\mathcal{E} - E_o) \sum_n \eta_n (\mathcal{E} - \varepsilon_n)^{-1} = 0 , \qquad (69)$$

where we have omitted the channel superscript and introduced the reduced widths $\eta_n = (u_n)^2$. As earlier, we assume that the internal levels are distributed more or less uniformly within the interval $(-a, a)$. We specify the threshold energy $E_o < -a$ and denote $w = -E_o \sum_n \eta_n$ to ensure the correct limiting transition to the previous results for the remote threshold ($E_o \to -\infty$, $E >> E_o$). Note that now, due to the additional threshold singularities, the analytical properties of the scattering amplitude are more complicated and w is not bound to be equal to the sum of imaginary parts Γ_j of the roots $\mathcal{E}_j$ of the secular equation. In the time evolution language, it means that the decay of the unstable intermediate states located near a threshold is not exponential [31]. However, the structure (2) of W still guarantees the unitarity of the S-matrix as well as the inequality $\Gamma_j \geq 0$.

The solution of (69) is obvious in the extreme limit of very strong coupling via continuum: w, $E_o >> a$. Then the internal levels are concentrated near the origin, $\varepsilon_n \to 0$, and Eq. (69) gives the collective Dicke state $\mathcal{E}_1 = E_1 - (i/2)\Gamma_1$, which can be expressed in terms of the new parameter $\xi = w/2|E_o| = \sum_n \eta_n/2$,

$$\Gamma_1 = 2|E_o|\xi/(1+\xi^2), \qquad E_1 = E_o \xi^2/(1+\xi^2) , \qquad (70)$$

and N-1 stable states $\mathcal{E}_n = 0$, $n \geq 2$. We see that the threshold attracts the resonance ($E_o < 0$). For the remote threshold, $\xi << 1$, $\Gamma_1 \approx w$ and the displacement of E_1 to the left is small. But in the opposite case of the close threshold, $\xi >> 1$, the resonance gets on the threshold, $E_1 \approx E_o$, and $\Gamma_1 \approx w/\xi^2$ is much less than the "natural" width w. Of course, we always assume to stay in the region where the amplitudes still obey the threshold law (68).

The scattering cross section being, in our units, equal to

$$\sigma^{ba}(E) = |T^{ba}(E)|^2 / 4[p^a(E)]^2 , \qquad (71)$$

can be easily calculated with the aid of Eqs. (8) and (68):

$$\sigma(E) = \frac{\xi^2}{\xi^2 + 1} (E - E_o) [(E - E_1)^2 + \Gamma_1^2/4]^{-1} . \qquad (72)$$

Proximity of the threshold distorts drastically the Breit-Wigner resonance shape. The cross-section starts linearly from the threshold with the slope $d\sigma/dE = \xi^2/E_o^2$ and reaches the maximum in the point

$$E_{max} = E_o + [(E_1 - E_o)^2 + \Gamma_1^2/4]^{1/2} = E_o [1-(1+\xi^2)^{-1/2}] , \qquad (73)$$

where it equals

$$\sigma(E_{max}) = [1 + (1+\xi^2)^{1/2}]/2|E_o| . \qquad (74)$$

At $\xi << 1$ the Breit-Wigner peak centered in the origin is restored and $\sigma(E_{max}) \to |E_o|^{-1}$ which corresponds, in our units, to the absolute maximum of $|T(E)|$, Eq. (8), attainable, for the remote threshold, in the poles of K(E), $|T(E)|^2 = 4$. For $\xi >> 1$ the cross-section grows very steeply and the pattern resembles that of the cusp anomaly.

The corrections associated with the finite extent 2*a* of the energy interval can be calculated in the way similar to (17) and (18). Introducing

$$\langle\varepsilon\rangle = \Sigma\eta_n\varepsilon_n/\Sigma\eta_n = -E_o\Sigma\eta_n\varepsilon_n/w; \quad b^2 = -E_o\Sigma\eta_n(\varepsilon_n - \langle\varepsilon\rangle)^2/w \ , \tag{75}$$

where $b \sim a$ is assumed to be small, $b \ll \min(|E_o|, w)$, one gets

$$E_1 = [\langle\varepsilon\rangle + E_o\xi^2](1+\xi^2)^{-1} \ , \tag{76}$$

$$\Gamma_1 = 2|E_o|\xi[1-\langle\varepsilon\rangle/E_o](1+\xi^2)^{-1} - 2b^2/\xi[\langle\varepsilon\rangle - E_o] \ . \tag{77}$$

These results agree with Eqs. (17) and (18) as well as with Eq. (70) in the limits $E_o \to -\infty$ and $a \to 0$ respectively.

The methods of Sect. 2 for taking into account the average influence of the background of the internal levels also can be generalized to include the threshold effects. Thus, in the strong overlap region, $\kappa = w/2a \gtrsim 1$, we obtain instead of (19):

$$1 - (i/2)(\xi/a)(\mathcal{E}-E_o)\ln[(\mathcal{E}-a)/(\mathcal{E}+a)] = 0 \ . \tag{78}$$

As can be checked, at $\xi \to 0$ and w fixed, we return to eq.(20). This approximation is valid if $\xi < \kappa$ i.e. if the threshold is still outside the level interval. Within the accuracy of $(\xi/\kappa)^4$, the collective energy E_1 given by (78) coincides with that of Eq. (70) whereas the width combines both correction due to the finite κ, Eq. (20), and due to the threshold proximity (70),

$$\Gamma_1 = w\,[(1+\xi^2)^{-1} - (3\kappa^2)^{-1}] \ . \tag{79}$$

The whole consideration can be extended to the many-channel case. The most intriguing situation (and the most important practically), arising when the new decay channels become open one by one within the region under study, should be treated separately. Since the interaction of intrinsic states via continuum shifts the real parts of energies, their disposition with respect to the thresholds is to be computed self-consistently. Here small contributions from the principal value of the original self-energy operator can be essential so that it might be necessary to go back to the total hermitean Hamiltonian including the channel variables explicitly.

We conclude this section noting that, in the case of rather remote threshold, the value of the parameter ξ can be found immediately from the universal Eq. (76),

$$\xi^2 = (\langle\varepsilon\rangle - E_1)\,/\,(E_1 - E_o) \ , \tag{80}$$

using the experimentally known shift $\delta E = \langle\varepsilon\rangle - E_1$ of the peak from the bare position towards the threshold and distance $\Delta E = E_1 - E_o$ of the peak to the threshold. Eq. (80) defines the bare total width $w = 2\xi|E_o|$. Similarly, for the particle escape in the partial wave with the orbital angular momentum l, one gets

$$\xi^2 = (2l+1)^{-1}(\delta E/|E_o|)(\Delta E/|E_o|)^{-(4l+1)} \ . \tag{81}$$

These rough estimates can be applied to the long standing problem of the Δ-resonance in nuclei. Assuming that pionic and nonmesonic channels are approximately orthogonal in the sense of Eq. (32), we can use (81) for the *p*-wave pions to get from the $^{12}C(^3He,t)$ data (see, for example, the review talk [32]) the value $\xi^2 \approx .63$ which gives for the summed width in pionic channels $w \approx 250$ MeV. Then the coherent resonance width can be estimated, similar to (81), as $\Gamma_1 \approx w(\Delta E/|E_o|)^{2l+1} \approx 80$ **MeV**. The rest of the observed resonance width can be attributed to nonmesonic decay channels of the Δ-hole (and more complicated) states. Let us note the universal character of the displacement of the collective peak towards the threshold. The same is valid for internal states with the energy center <ε> under the threshold: the cross section in this case looks as if this cluster of states is shifted up, again to the threshold.

6. TWO SPECIFIC EXAMPLES

In this section we discuss two simple models which stress various noticeable aspects of dynamics of open systems. The first example describes a typical system with the exit being possible from the surface only, namely a finite chain of potential wells where the outer wells are coupled to the exterior. The second example concerns the model with the strong coupling of doorway states to internal ones.

A. Many-well potential.

Let us consider N potential wells coupled to each other by the tunneling amplitudes $v_{n,n-1}$ and $v_{n,n+1}$; let the outer wells have access to the continuum. Keeping the most generic features of the dynamical problem we assume, for simplicity, all wells to be identical so that the effective Hamiltonian (1) takes the form

$$\mathcal{H}_{nm} = \varepsilon\delta_{nm} + v(\delta_{m,n+1}+\delta_{m,n-1}) - (i/2)(\gamma^L\delta_{n1}\delta_{m1} + \gamma^R\delta_{nN}\delta_{mN}) , \qquad (82)$$

where ε stands for the level energy in each well (neglecting the jumping probability); $\gamma^L = (A_{n=1})^2$ and $\gamma^R = (A_{n=N})^2$ are the decay probabilities to the left and to the right respectively from the edge cells of the chain (in this "two-channel" formulation the LL and RR processes correspond to reflections from two sides of the system whereas LR and

RL processes describe transmission).

The basis $|n>$ of localized stable states utilized in (82) can be transformed into that of the DR, Sect. 4, by means of the diagonalization of the hermitean part in the (N-2)-dimensional space spanned by the interior states $|n>$, n=2, ..., N-1. To have the continuous transition to the closed chain, we construct first the IR.

The coupling v between wells splits the N-fold degenerate level ε (below we put ε = 0) into the band of crystal standing waves with the integer wave vector q=1, ..., N and the corresponding energies ω_q, $|\omega_q| < 2v$,

$$\omega_q = 2v\cos\varphi_q , \qquad \varphi_q = \pi q/(N+1) . \qquad (83)$$

The level density at N>>1 has maxima on the band edges,

$$\rho(\omega) = [N/2\pi v\sin\varphi_q]_{\omega=\omega_q} = (N/\pi)(4v^2-\omega^2)^{-1/2} . \qquad (84)$$

The basis $|q> = \sum_m |m><m|q>$ of these delocalized states is the one used in the IR. The components of the eigenvectors $|q>$ are

$$<m|q> = [2/(N+1)]^{1/2}\sin m\varphi_q , \qquad (85)$$

leading to the matrix elements of the antihermitean part

$$W_{qq'} = [2/(N+1)]\sin\varphi_q \sin\varphi_{q'}[\gamma^L + (-)^{q+q'}\gamma^R] . \qquad (86)$$

Finally, one has to diagonalize the total Hamiltonian matrix $\mathcal{H}_{qq'} = \omega_q\delta_{qq'} - (i/2)\, W_{qq'}$. The secular equation for the complex energies $\mathcal{E}$ takes the form

$$\Omega(\mathcal{E}) \equiv 1 + i(\gamma^L+\gamma^R)P_+(\mathcal{E}) + \gamma^L\gamma^R[P_-^2(\mathcal{E}) - P_+^2(\mathcal{E})] = 0 , \qquad (87)$$

$$P_{\pm}(\mathcal{E}) = (N+1)^{-1}\sum_{q} (\pm 1)^{q}\sin^{2}\varphi_{q}(\mathcal{E} - \omega_{q})^{-1} . \qquad (88)$$

The results for the limiting cases follow immediately. At weak coupling to the continuum, we neglect the last term of (87) and get small imaginary corrections to the band energies (83):

$$\mathcal{E}_{q} = \omega_{q} - (i/2)\Gamma_{q}, \quad \Gamma_{q} = W_{qq} = (\gamma^{L}+\gamma^{R})[2/(N+1)]\sin^{2}\varphi_{q} . \qquad (89)$$

The width being written down as

$$\Gamma_{q} = \gamma^{L}|\langle 1|q\rangle|^{2} + \gamma^{R}|\langle N|q\rangle|^{2} , \qquad (90)$$

agrees with the intuitive argument that the decay executes the decomposition of the quasistationary state isolating the components matched to the specific decay channels. Such an argument is used in the conventional derivation [22] of the width distribution for the compound states (the Porter-Thomas distribution for the neutron resonances). As has been discussed in [8, 13], simple identification of the widths with the particular components squared of an internal wave function is not justified in the case of the strong coupling via continuum. In our problem the width distribution (89) replicates that of the group velocity squared reaching the maximum in the middle of the band. The weak coupling approximation (89) is valid, for $N >> 1$, if $\gamma^{L,R} << v/|\sin\varphi_{q}|$, i.e. $\gamma^{L,R} << v$ for the states $|q\rangle$ well within the band and $\gamma^{L,R} << Nv$ near the band edges (q =1 or q = N).

The opposite limiting case of the strong overlap, $\gamma^{L,R} > v$, can be treated as in Eq. (16). According to the general results of Sect. 2, to the main order in κ^{-1}, the two collective short-lived states can be found from (88) discarding ω_{q} in the denominators of $P_{\pm}$. Then $P_{+} \to (2\mathcal{E})^{-1}$

whereas P_{-} vanishes. Thus, for $N>>1$ the collective states have pure imaginary energies

$$\mathcal{E}_{L,R} = -(i/2)\gamma^{L,R} . \qquad (91)$$

Corrections of the order κ^{-2} can be computed by the help of the various methods developed in Sect. 2.

In the middle of the band we have two Dicke states superimposed on the background of (N-2) long-lived states. The Dicke states accumulating the total width for the escape to the left (γ^{L}) or to the right (γ^{R}) can be labeled as $|L\rangle$ and $|R\rangle$ respectively since, in this limit, they become localized on the chain edges, $|L\rangle=|m=1\rangle$ and $|R\rangle=|m=N\rangle$: for such states the irreversible decay outside occurs much faster than the hopping to the next well. (Note that the limit $P_{-} \to 0$ found in the previous paragraph is the simple consequence of the orthogonality of the states $|L\rangle$ and $|R\rangle$). Thus, in the problem under study, the phase transition between the two dynamical regimes manifests itself as the segregation of the short-lived surface-localized states from the band of the delocalized Bloch waves.

In the regime of the strong continuum coupling, it is useful to go to the DR of Sect.4. Here the doorways $|d\rangle$ are the surface states $|L\rangle$ and $|R\rangle$ of the previous paragraph. The subspace of the trapped states $|t\rangle$, t=1, ..., N-2, is spanned by the interior states $|m\rangle$, m = 2, ..., N-1; similar to (83) and (85), the energies (46) and wave functions (49) are, respectively,

$$\tilde{\omega}_{t} = 2v\cos\tilde{\varphi}_{t} \equiv 2v\cos[\pi t/(N-1)] , \qquad (92)$$

$$c^{t}_{m} = \langle m|t\rangle = [2/(N-1)]^{1/2}\sin(m-1)\tilde{\varphi}_{t} . \qquad (93)$$

The dispersion law (92) for the trapped states satisfies the secular equation (51) which takes, using the $|q\rangle$-basis of Eq. (83), the form

$$P_+^2(\tilde{\omega}) - P_-^2(\tilde{\omega}) = 0 \qquad (94)$$

where the sums $P_\pm(z)$ have been defined in (88). As it should be, the N-2 roots $\tilde{\omega}_t$ coincide with the real solutions $\mathcal{E}_t$ of the full secular equation (87) in the limit of very strong continuum coupling. On the other hand, (94) represents an example of the nontrivial mathematical identity including the two sets of trigonometric functions (83) and (92) with intermittent frequencies (for N odd there is a coincidence of $\tilde{\omega}_t = 0$, $t = (N - 1)/2$, with $\omega_q = 0$, $q = (N + 1)/2$).

The matrix elements (47) which couple in the DR the doorways to the trapped states can be readily found to be

$$H^{Lt} = (-1)^{t+1} H^{Rt} = v[2/(N - 1)]^{1/2} \sin \tilde{\varphi}_t , \qquad (95)$$

which means, in conformity with (55), that the admixtures of the doorways are weak, $\propto v^2/N$, and distributed over the trapped states in the same way (89) as the widths in the limit of isolated resonances. The amplitudes (53) are equal to $f_q^t = \langle 1|q\rangle H^{Lt} [1+(-)^{q+t}]$, and Eq. (54) becomes an another identity:

$$2\sin^2\tilde{\varphi}_t \sum_n [1+(-)^{q+t}] \sin^2\varphi_q [\cos\tilde{\varphi}_t - \cos\varphi_q]^{-2} = N^2 - 1 . \qquad (96)$$

The final diagonalization of interaction (95) in the DR basis containing $|L\rangle$, $|R\rangle$ and $|t\rangle$ states gives, analogously to the one-channel case (58), (59), the secular equation for the complex eigenvalues $\mathcal{E}$ of the total Hamiltonian (82)

$$[\mathcal{E} + (i/2)\gamma^L - \mathcal{R}_+(\mathcal{E})][\mathcal{E} + (i/2)\gamma^R - \mathcal{R}_+(\mathcal{E})] - \mathcal{R}_-^2(\mathcal{E}) = 0 , \qquad (97)$$

$$\mathcal{R}_\pm(\mathcal{E}) = \sum_t (\pm)^t (H^{Lt})^2 [\mathcal{E} - \tilde{\omega}_t]^{-1} . \qquad (98)$$

In the case of $\gamma^{L,R} \gg v$ which is the most favourable for the application of the DR, the perturbation theory is valid (see Sect. 4) so that one gets small widths of the trapped states

$$\Gamma_t = [2/(N-1)] (v^2/\gamma) \sin^2\tilde{\varphi}_t , \qquad (99)$$

$$\gamma = (1/4)[(1/\gamma^L)+(1/\gamma^R)]^{-1} , \qquad (100)$$

obtained at the cost of the depletion of the doorway widths,

$$\Gamma_{L,R} = \gamma^{L,R} - 4v^2/\gamma^{L,R} ; \qquad (101)$$

it is easy to check that

$$\Gamma_L + \Gamma_R + \sum_t \Gamma_t = \gamma^L + \gamma^R . \qquad (102)$$

Of course, one can analyze (97) more in detail.

For the conclusion, we write down the expressions for the scattering amplitude (8). Calculating the K-matrix in the IR, we get, in terms of $\Omega(E)$, Eq. (87), and $P_\pm(E)$, Eq. (88),

$$T^{LL,RR}(E) = \Omega^{-1}(E)\, 2\gamma^{L,R}\{P_+(E) + i\gamma^{R,L}[P_+^2(E) - P_-^2(E)]\} , \qquad (103)$$

$$T^{LR}(E) = T^{RL}(E) = \Omega^{-1}(E)(-2)(\gamma^L\gamma^R)^{1/2} P_-(E) . \qquad (104)$$

In the limit of the weak continuum coupling, the scattering pattern is that of isolated Breit-Wigner resonances corresponding to the individual Bloch waves; the pole contribution at the complex energy $\mathcal{E}_q$ (89) is

$$T_q^{LL,RR,LR}(E) =$$

$$= [\gamma^L,\ \gamma^R,\ (-)^{q+1}(\gamma^L\gamma^R)^{1/2}](\gamma^L + \gamma^R)^{-1}\Gamma_q/(E - \mathcal{E}_q). \qquad (105)$$

In the opposite situation, one sees at $|E|>2v$ the wings of the broad Dicke resonances (91) in the reflection channels,

$$T^{LL,RR}(E) \approx \gamma^{L,R}[E + (i/2)\gamma^{L,R}]^{-1} , \qquad (106)$$

and the very small transmission (percolation) amplitude

$$T^{LR}(E)\approx-(\gamma^L\gamma^R)^{1/2}(v/E)^{N-1}E\{[E+(i/2)\gamma^L][E+(i/2)\gamma^R]\}^{-1} . \qquad (107)$$

Evidently, in this case the whole chain acts for passage as an extended barrier with the penetrability $\propto v^{N-1}$.

B. Doorways strongly coupled to background

Here we discuss very briefly a little bit different case where a number of doorways $|d\rangle$, $\mathcal{E}_d = \varepsilon_d - (i/2)\gamma_d$, are coupled to the set of internal states $|n\rangle$, with real energies ε_n, by means of the strong hermitean interaction V_{dn}.

As we have seen in Sect. 4, if the doorway states are generated through the Dicke mechanism from the same intrinsic reservoir, the interaction V is bound to be weak so that the doorways are close to the exact eigenstates of the total nonhermitean Hamiltonian. It was illustrated explicitly by the simple model A of the previous subsection. The strong coupling V can arise for the intruder states $|d\rangle$ of foreign

origin falling within an internal energy interval. To imitate such a situation, one can, for instance, modify the model A assuming that the surface states have the tunneling amplitudes V to the interior larger than the amplitudes v between internal wells.

The effect to which we would like to pay attention occurs already in the single doorway case. It was observed in the numerical simulation of neutron resonances by P.von Brentano [21]. The complex eigenvalues of the problem are determined by the secular equation ($V_{dn} \equiv V_n$)

$$\mathcal{E} = \mathcal{E}_d + \sum_n V_n^2\,(\mathcal{E} - \varepsilon_n)^{-1} . \qquad (108)$$

The full analysis can be carried out with the use of methods of Sect. 1. Here we give the limiting result only for the case when the energy interval $\sim 2a$ of internal levels is small as compared with the doorway width γ_d as well with the summed interaction strength $V = |V| = (\sum_n V_n^2)^{1/2}$. Putting the origin of the real energy scale into the point $\langle\varepsilon\rangle = \sum_n \varepsilon_n V_n^2/V^2$ we obtain, to the second order in a/V or a/γ_d, two collective solutions

$$\mathcal{E}_\pm = E_\pm - (i/2)\Gamma_\pm = [\mathcal{E}_d \pm (\mathcal{E}_d^2 + 4V^2)^{1/2}]/2 . \qquad (109)$$

For example, if the doorway is centered near the origin, $\varepsilon_d \approx \langle\varepsilon\rangle$,

$$E_\pm = \pm(V^2 - \gamma_d^2/16)^{1/2}\ \theta(V - \gamma_d/4) , \qquad (110)$$

$$\Gamma_\pm = [\gamma_d \pm (\gamma_d^2 - 16V^2)^{1/2}\ \theta(\gamma_d/4 - V)]/2 . \qquad (111)$$

Here a peculiar phase transition occurs at the critical coupling strength $V = \gamma_d/4$. If the interaction is under-critical both solutions stay at the center, $E_{\pm} \approx 0$, and one is able to discriminate the doorway state by the large width $\Gamma_{+} \approx \gamma_{d}$. The corresponding wave function gets the main contribution from the doorway state (the amplitude $c_{d}^{(+)} \approx 1$) whereas the contributions $c_{n}^{(+)}$ of the internal states are small, $|c_{n}^{(+)}|^{2}/|c_{d}^{(+)}|^{2} \sim V^{2}/N\gamma_{d}^{2}$. The second solution is the collectivized representative of the background states which at increasing V takes over gradually the decay width from the doorway state. The corresponding wave function is built up mainly from the internal states so that $|c_{n}^{(-)}|^{2}/|c_{d}^{(-)}|^{2} \sim \gamma_{d}^{2}/NV^{2}$.

In the overcritical regime, the total width γ_{d} is divided equally between the two resonances repelling each other along the real axis to become located at $E_{\pm} = \pm V$ for $V >> \gamma_{d}/4$. In this limit, the collective states can be shown to be coherent superpositions of the internal states hybhidized, constructively or destructively, with the door way, $c_{n}^{(\pm)} \approx \pm c_{d}^{(\pm)} V_{n}/V$.

This behavior resembles that of the giant resonance considered [16] from the viewpoint of competition of two collectivizing interactions (standard separable forces of multipole-multipole type and coupling (2) via the common decay channels). Typically, one obtains two collective peaks, the conventional giant resonance and the Dicke satellite, sharing the total decay width as well as the multipole strength. The variety of arising patterns is created due to the additional parameter available, namely the angle (similar to that in Eqs. (27) and (28)) between the N-dimensional vectors **A** of decay amplitudes and **D** of the multipole moment. However, in the giant resonance problem the limit of very strong coupling V seems to be rather

meaningless.

The many-channel version of the model under study can be analyzed along the same line. For the two-channel case we start from the two doorways with complex energies $\mathcal{E}_{1,2}$ and vectors $\mathbf{V}_{1,2}$ of coupling with internal states. As a new parameter, we again get the angle ϑ between these vectors. The eigenvalue equation obviously takes the form similar to (97)

$$[\mathcal{E} - \mathcal{E}_{1} - R_{11}(\mathcal{E})][\mathcal{E} - \mathcal{E}_{2} - R_{22}(\mathcal{E})] - R_{12}^{2}(\mathcal{E}) = 0 \,, \qquad (112)$$

$$R_{dd'}(\mathcal{E}) = \sum_{n} V_{dn} V_{d'n} \, (\mathcal{E} - \varepsilon_{n})^{-1} \,. \qquad (113)$$

Omitting all particular results we just mention that in the limit of very strong coupling V_{d} the number of collective states is twice the number of doorways; in the orthogonal case, $\cos\vartheta = 0$, they are located at $\pm V_{d}$ whereas in the parallel situation, $\cos\vartheta = 1$, two states are still at the origin and two peaks are shifted to $\pm(V_{1}^{2} + V_{2}^{2})^{1/2}$.

7. CONCLUSION

Let us try to summarize the content of the paper.

(i) The effective phenomenological non-hermitean Hamiltonian (1) is shown to be an adequate tool for the analysis of the dynamics for an open quantum system in the limited energy interval.

(ii) The ways of the practical treatment in various physical approximation schemes are developed. Two mathematically equivalent representations, IR and DR, turn out to be useful in complementary conditions of weak and strong continuum coupling, respectively. In both representations, the dynamical solutions can be combined with the statistical consideration of irregular properties associated with the chaotic nature of the complicated internal states.

(iii) Collective dynamic phenomena generated by the interaction of the intrinsic states through the common decay channels were investigated. The generality of the Dicke mechanism generating the coherent short-lived states in the strong overlap limit was stressed. We have analyzed the specific features of the resulting patterns under various relationships of relevant parameters. The phase transition between the regimes of weak and strong continuum coupling is proved to be rather sharp as a function of the overlap parameter κ (3). The critical value of this quantity scales roughly proportional to the number k of the open channels. This is related to the problem of the Ericson fluctuations which can show up at $k >> 1$.

(iv) The preliminary analysis of the ergodic properties of the dynamics shows that the two averaging procedures, namely that over the energy interval including many fine structure levels and that with respect to the ensemble of random internal wave functions, are not equivalent in the region of many open channels. The Dicke collectivization destroys the uniformity of the energy scale and violates the ergodicity.

(v) In the vicinity of the channel thresholds the energy dependence of the decay amplitudes should be taken into account. It modifies significantly the reaction cross sections attracting the resonance energy to the threshold.

(vi) The similarity of problems under consideration to the simple solid state models was noticed. The simple recipe for the translation from the nuclear physics language to that of the solid state reads: the IR corresponds to the delocalized Bloch wave basis whereas the DR describes the localized surface states. The model was considered where the strong hermitean interaction of the doorway state with the background forms the new collective resonance repelling the original state.

The future applications of the formalism may include the statistical analysis of the available body of data on nuclear reactions, especially induced by neutrons and photons, in the broad energy range where the transition between the two dynamical regimes could be traced. Another related fields of interest are the structure of giant resonances [16] and its temperature dependence; propagation of an unstable particle through the nuclear medium (the example of the Δ-isobar was mentioned in Sect. 5); the influence of switching on new decay channels with the thresholds within the energy domain under study; description of the relaxation processes competing with the decay into the open channels, and so on. There are also diverse possibilities of application to the solid state physics.

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