

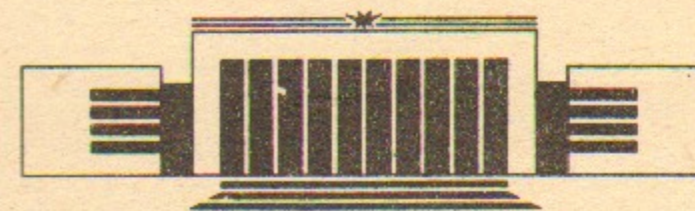


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ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ  
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ONCE MORE  
ABOUT THE  $\omega \rightarrow 3\pi$  CONTACT TERM

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НОВОСИБИРСК

# ONCE MORE ABOUT THE $\omega \rightarrow 3\pi$ CONTACT TERM

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## ABSTRACT

The manifestations of the  $\omega \rightarrow 3\pi$  contact term and its unitary partners are investigated in the framework of the chiral effective lagrangian theory with vector mesons. We conclude that nowadays the existence and magnitude of the contact term can't be extracted neither from theory, nor experiment. The theoretical uncertainty is caused by the one-loop corrections. Some speculations about them lead to the generalized KSRF relation.

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## 1. Introduction

The experimental study of the  $e^-e^+ \rightarrow 3\pi$  reaction [1] has confirmed the Gell-Mann, Sharp, Wagner suggestion [2] that the  $\omega \rightarrow 3\pi$  transition is dominated by the  $\omega\rho\pi$  pole diagram, though the experimental accuracy is not sufficient at present to exclude completely the existence of the possible contact term. This four-point contact term was discussed on quite general grounds [3,4], inspired by dispersion theory and current-algebra. No reason was found to neglect it, but its magnitude remained undefined until Rudaz had remarked [5] that one needs quite definite contact term to satisfy simultaneously the KSRF relation [6] and the low energy theorem [7] concerning  $\pi^0 \rightarrow 2\gamma$  and  $\gamma \rightarrow 3\pi$  amplitudes.

Meantime, Witten's topological reinterpretation [8] of the Wess-Zumino [9] chiral anomaly [10] stimulated a renewal of interest in effective chiral lagrangian theories [11]. A plenty of models were suggested (see f.e. [12 - 16]), especially for including vector (and axial-vector) degrees of freedom, with attempts [17] to derive the corresponding effective (non-renormalizable) lagrangians directly from QCD.

Although it is commonly believed nowadays that a chiral perturbation theory [18] gives a suitable and phenomenologically successful framework for the low energy meson physics, some specific suppositions about vector mesons [12, 14] is also interesting, because they reduce the number of phenomenological constants in the theory, so raising its predictability.

Namely, in [12] Kaymakcalan, Rajeev and Schechter introduced vector and axial-vector mesons as gauge bosons of local  $SU_L(3) \times SU_R(3)$  symmetry, the idea which can be traced back to Sakurai [19]. The contact term and the  $\omega\rho\pi$  coupling is fixed in their model by demanding Bardeen's form for the chiral anomaly, but it had been noticed soon [5] that the magnitude of the contact term was insufficient to ensure the validity of the Terentiev's et al. low energy theorem [7], which had been experimentally confirmed [20].

The situation was clarified by Brihaye, Pak and Rossi [21], who showed an elegant way how to construct counter-terms [22] needed for vector mesons not to break the low energy theorems of current-algebra. Actually, in their formalism it is not obligatory for vector mesons to be gauge bosons and throughout this paper we will use just such "minimal" realization [23] without self-interactions of the vector particles.

A new stage of experiments at Novosibirsk VEPP-2M storage ring is under way now with two modern detectors [24,25]. A few percent accuracy is expected can be reached for many processes in the energy range  $\sim 1$  GeV. So a simple phenomenological vector meson dominance [19,24] picture, used earlier, becomes insufficient and it is interesting if effective chiral lagrangian models and chiral perturbation theory can take up this challenge.

In this article we analyze how the model [21,23] can confront some experimental tests, with special emphasize of the effect of the rather large  $V \rightarrow 3P$  contact term. The phenomenological consequences of current-algebra based and effective chiral Lagrangian models were thoroughly investigated [27-29]. Therefore we omit some technical details which can be found in the cited literature.

## 2. $\Gamma(\omega \rightarrow 3\pi)$ and $e^-e^+ \rightarrow 3\pi$ .

The one of the successful predictions of [12] was a correct  $\omega \rightarrow 3\pi$  decay width, especially compared with last experimental result [30,31]. Adjusting [21] the  $\omega \rightarrow 3\pi$  contact term for low energy theorem [7] to be valid, we end

with four times larger magnitude for it and, as a result too small  $\Gamma(\omega \rightarrow 3\pi)$ , as will be shown below.

Defining the  $\omega_\mu(Q) \rightarrow \pi^+(q_+) \pi^-(q_-) \pi^0(q_0)$  amplitude as

$$M_\mu = i F(s_{12}, s_{13}, s_{23}) \epsilon_{\mu\nu\lambda\sigma} q_+^\mu q_-^\nu q_0^\sigma, \quad (1)$$

a standard calculation gives the formula for the decay width

$$\Gamma(\omega \rightarrow 3\pi) = \frac{M_\omega}{768\pi^3} \int_{x_{\min}}^{x_{\max}} dx \int_{y_{\min}}^{y_{\max}} dy G(x,y) |M_\omega^3 F(x,y)|^2, \quad (2)$$

where  $m=m(\pi^\pm)$ ,  $m_0=m(\pi^0)$ ,  $x = \frac{E_+}{M_\omega}$ ,  $y = \frac{E_-}{M_\omega}$ ,

$$G(x,y) = 4 \left( x^2 - \frac{m^2}{M_\omega^2} \right) \left( y^2 - \frac{m^2}{M_\omega^2} \right) - \left( 1 - 2x - 2y + 2xy + \frac{2m^2 - m_0^2}{M_\omega^2} \right)^2 \quad (3)$$

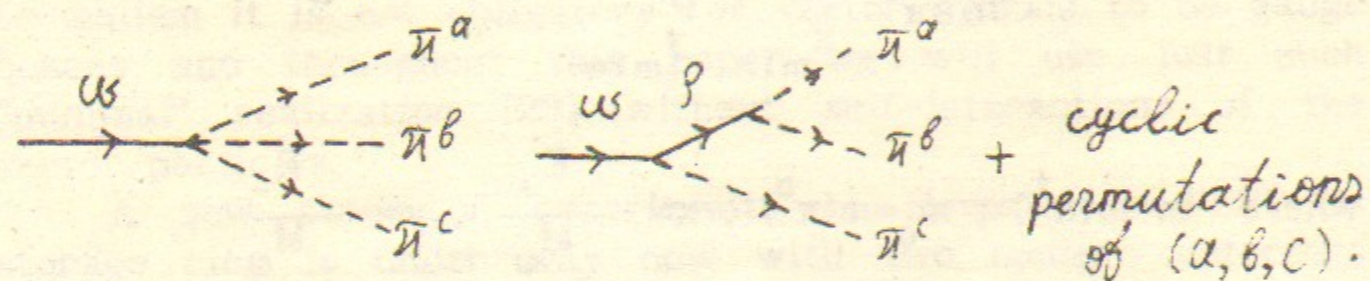
and

$$x_{\min} = \frac{m}{M_\omega}, \quad x_{\max} = \frac{1}{2} \left( 1 - \frac{m_0(2m+m_0)}{M_\omega^2} \right),$$

$$y_{\min}^{\max} = \frac{1}{2 \left( 1 - 2x + \frac{m^2}{M_\omega^2} \right)} \left\{ (1-x) \left( 1 - 2x + \frac{2m^2 - m_0^2}{M_\omega^2} \right) \mp \right. \quad (4)$$

$$\left. \left[ \left( x^2 - \frac{m^2}{M_\omega^2} \right) \left( 1 - 2x + \frac{m_0(2m - m_0)}{M_\omega^2} \right) \left( 1 - 2x - \frac{m_0(2m + m_0)}{M_\omega^2} \right) \right]^{\frac{1}{2}} \right\}.$$

The expression for the invariant amplitude  $F$  can be obtained from the diagrams



and looks like

$$\begin{aligned} |M_\omega^3 F(x,y)|^2 &= \\ &= \left( \frac{3}{4\pi^2} \right)^2 \left( \frac{M_\omega}{f_\pi} \right)^6 \left( \frac{M_\rho}{f_\pi} \right)^2 \alpha_K \cdot |1 - 3\alpha_K - \alpha_K H(x,y)|^2, \end{aligned}$$

where

$$H(x,y) = R_\rho(Q_0^2) + R_\rho(Q_+^2) + R_\rho(Q_-^2) \quad (5)$$

is defined through the  $\rho$ -meson Breit-Wigner propagators

$$R_V(Q^2) = \left[ \frac{Q^2}{M_V^2} - 1 + i \frac{\Gamma_V(Q^2)}{M_V} \right]^{-1} \quad (6)$$

and

$$Q_0^2 = M_\omega^2 (2x - 2y - 1) + m_0^2,$$

$$Q_+^2 = M_\omega^2 (1 - 2y) + m^2, \quad Q_-^2 = M_\omega^2 (1 - 2x) + m^2. \quad (7)$$

As for  $\alpha_K$ , it is defined as

$$\alpha_K = \left( \frac{f_\pi g_{\rho\pi\pi}}{M_\rho} \right)^2, \quad (8)$$

$\alpha_K = \frac{1}{2}$  being the KSRF relation [6].

For off-mass-shell resonance widths we assume that they are proportional to the main decay channel phase space. For example:

$$\Gamma_\rho(Q^2) = \Gamma_\rho \frac{M_\rho^2}{Q^2} \left( \frac{Q^2 - 4m_\pi^2}{M_\rho^2 - 4m_\pi^2} \right)^{3/2}, \quad (9)$$

where  $\Gamma_\rho \approx 151$  MeV.

Taking for the other parameters

$$f_\pi \approx 93 \text{ MeV}, \quad m_0 \approx m \approx 140 \text{ MeV},$$

$$M_\omega \approx 782 \text{ MeV}, \quad M_\rho \approx 768 \text{ MeV},$$

and  $\alpha_K \approx 0.55$  (which corresponds to  $\frac{g_{\rho\pi\pi}^2}{4\pi} \approx 3$ ), we get  $\Gamma(\omega \rightarrow 3\pi) \approx 4.9$  MeV. The experimental value is [32]  $\Gamma_{\text{exp}}(\omega \rightarrow 3\pi) = (7.49 \pm 0.14)$  MeV. If we take four times smaller contact term from [12], we get almost experimental width:  $\Gamma_{[12]}(\omega \rightarrow 3\pi) \approx 7.3$  MeV, and if we drop the contact term altogether, the width increases up to 8.4 MeV.

Taking into account a small deviation  $\epsilon \approx 3.4^\circ$  [33] from the ideal  $\omega$ - $\phi$  mixing, the following predictions for the  $\phi \rightarrow 3\pi$  decay width can be get also :

model	$\Gamma(\varphi \rightarrow 3\pi)$ MeV
[21, 23]	0.67
[12]	0.79
no contact term	0.84
experiment [31]	$0.63 \pm 0.04$

Of course, these values depend on the details of the unitary symmetry breaking [34] (for some new ideas about the  $\omega$ - $\varphi$  mixing problem see [35]), and so are not a clear test for the chiral effective theories.

Somewhat small  $\Gamma(\omega \rightarrow 3\pi)$  for the correct (from the low energy theorem's point of view) contact term, maybe indicate the importance of the one-loop and radial excitations corrections. Their magnitude can be estimated according to [36] by dual model inspired change

$$R_\rho(x, y; Q^2) \longrightarrow R_\rho(x, y; Q^2) \left( \frac{F_V(Q^2)}{F_V(0)} \right)^2,$$

where

$$F_V(Q^2) = \Gamma(\beta-1) \frac{\Gamma(1-\alpha'(Q^2-m_\rho^2))}{\Gamma(\beta-1-\alpha'(Q^2-m_\rho^2))}, \quad \alpha' = \frac{1}{2m_\rho^2}, \quad \beta \approx 2.33.$$

This increases  $\Gamma(\omega \rightarrow 3\pi)$  from 4.9 MeV up to 6.7 MeV in the Brihaye, Pak, Rossi model [21,23].

Closely related to the  $\omega \rightarrow 3\pi$  transition is the  $e^-e^+ \rightarrow 3\pi$  process [37]. Its cross-section is

$$\begin{aligned} \sigma(e^-e^+ \rightarrow 3\pi) &= \\ &= \frac{\alpha}{192\pi^2 s} \int_{x_{\min}}^{x_{\max}} dx \int_{y_{\min}}^{y_{\max}} dy G(x, y) |(2E)^3 F_{3\pi}(x, y)|^2, \end{aligned} \quad (10)$$

where  $s=(2E)^2$ ,  $x = \frac{E_+}{2E}$ ,  $y = \frac{E_-}{2E}$ , and  $x_{\min}^{\max}$ ,  $y_{\min}^{\max}$ ,  $G(x, y)$  are given by (3), (4) with change  $M_\omega \rightarrow 2E$ ,  $E$  being beam energy.  $F_{3\pi}$  formfactor has the following form (here and later the coupling constants from [23] is assumed, if not otherwise stated)

$$M_\mu(\gamma \rightarrow \pi^+ \pi^- \pi^0) = -i\epsilon_{\mu\nu\sigma\tau} q_+^\nu q_-^\sigma q_0^\tau F_{3\pi}(s_{12}, s_{13}, s_{23}),$$

$$|(2E)^3 F_{3\pi}(x, y)|^2 = \frac{3\alpha}{4\pi^3} \left( \frac{2E}{f_\pi} \right)^6 |\sin\theta \cos\epsilon R_\omega(s)$$

$$-\cos\theta \sin\epsilon R_\varphi(s)|^2 |1-3\alpha_K - \alpha_K H(x, y)|^2. \quad (11)$$

Recall  $M_\omega \rightarrow 2E$  substitution in the definition of  $H(x, y)$  from (5) and (7).  $\theta$  is  $\omega$ - $\varphi$  mixing angle:

$$\omega = \cos\theta \omega_{(1)} + \sin\theta \omega_{(8)}, \quad \varphi = \cos\theta \omega_{(8)} - \sin\theta \omega_{(1)},$$

and  $\epsilon = \theta - \arcsin(1/\sqrt{3}) \approx 3.4^\circ$  —its departure from the "ideal" mixing.

Numerical calculations give the following resonance cross-sections (when  $s=M_V^2$ ):

model	[21,23]	[12]	no contact term	experiment
$\sigma_\omega$ $\mu\text{b}$	0.99	1.47	1.71	$1.54 \pm 0.03 \pm 0.12$ [39]
$\sigma_\varphi$ $\mu\text{b}$	0.58	0.69	0.75	$0.66 \pm 0.04$ [40]

As expected, the situation is much the same as for  $\Gamma(\omega \rightarrow 3\pi)$ . Fig.1 shows calculated cross-sections between  $\omega$  and  $\varphi$  re-

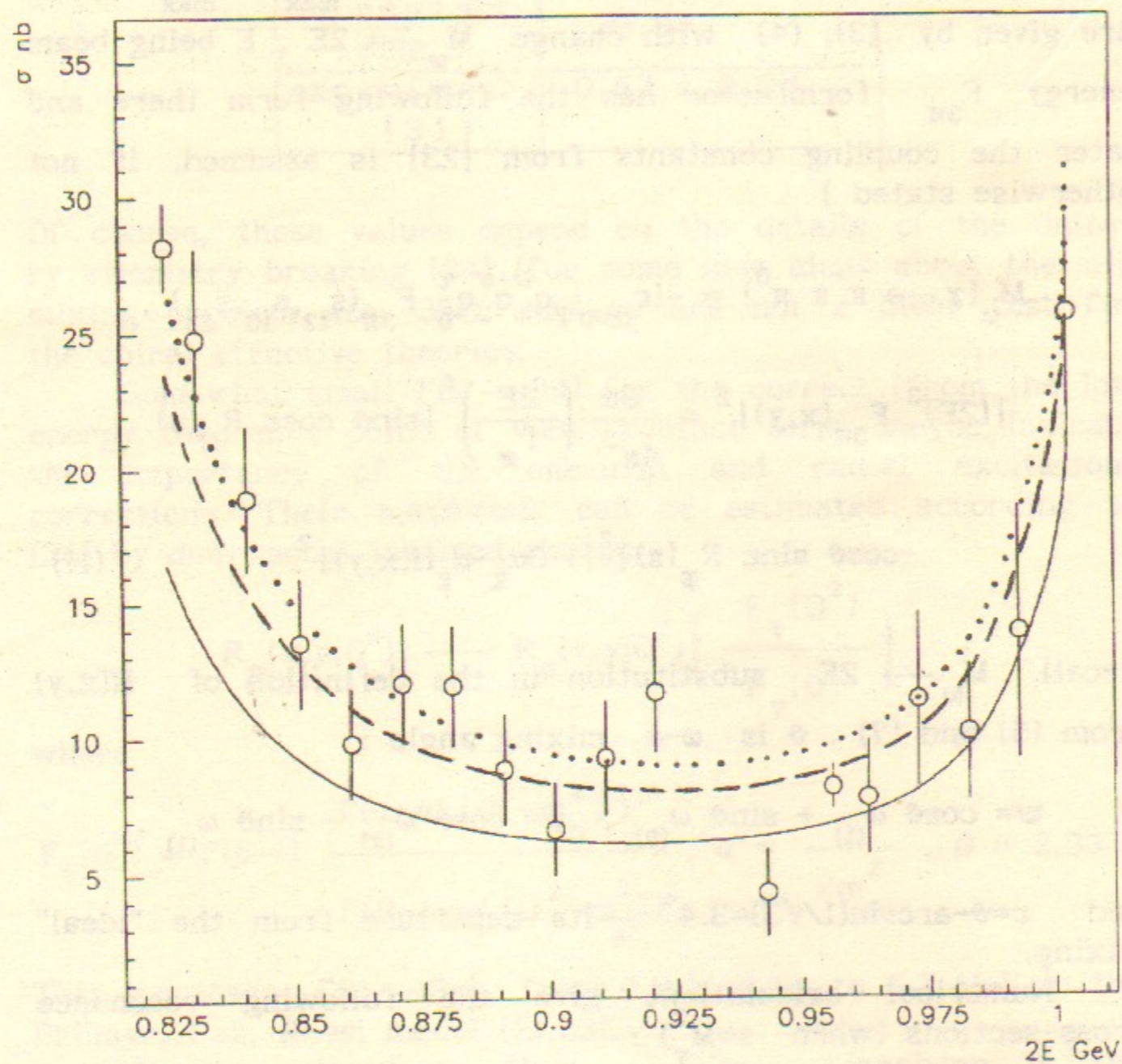


Fig. 1.  $\sigma(e^+e^- \rightarrow 3\pi)$  between  $\omega$  and  $\phi$ . Dashed line [12], dotted line—no contact term and solid [23]. Experimental points are from [31].

sonances (again [12] fits experiment better, than [21,23]), and Fig.2 — after  $\phi$ -meson.

As we see, above  $\phi$ -meson chiral-model predictions disagree significantly with experiment. Of course, the assumption that resonance widths, being normalized at their physical values, increase as the main decay channel phase space, is not the best thing to be done when we are so far from the  $\omega$ -meson peak [40]. But even disregarding completely a  $Q^2$ -dependence of  $\Gamma_V$  in the Breit-Wigner propagators, we get only a factor about 3, not enough to remove discrepancy.

Maybe this experimental result can't be explained without radial excitations ( $\rho(1450)$  for example). In any case, we see that above 1 GeV predictions of chiral effective theory must be dealt with caution. Nevertheless, below we consider some unitary partners of  $\omega \rightarrow 3\pi$  and  $e^+e^- \rightarrow 3\pi$ , such as  $K^* \rightarrow K\pi\pi$  [27,28],  $e^+e^- \rightarrow K\bar{K}\pi$  [41],  $\eta \rightarrow \pi\pi\gamma$  [27, 28,42] and  $e^+e^- \rightarrow \eta\pi\pi$  [43].

### 3. $\Gamma(K^* \rightarrow K\pi\pi)$

Because of isospin and charge conjugation invariance, only  $K^{*+}$  decay modes can be considered. Needed formulas are the same as for  $\Gamma(\omega \rightarrow 3\pi)$  with obvious changes  $M_\omega \rightarrow M_{K^*}$ ,  $m_0 \rightarrow m(K)$ . The contributing diagrams are



and corresponding formfactors look like (for the model [12] change  $1-3\alpha_K$  to  $1-3\alpha_K + \frac{3}{2}\alpha_K^2$ )

$$F(K^* \rightarrow K^0 \pi^0 \pi^+) = -\frac{g_{\rho\pi\pi}}{2^{3/2} \pi^2 f_\pi^3} \left[ 1-3\alpha_K - \frac{3}{4} \alpha_K \left\{ 2R_\rho(Q_0^2) + \right. \right.$$

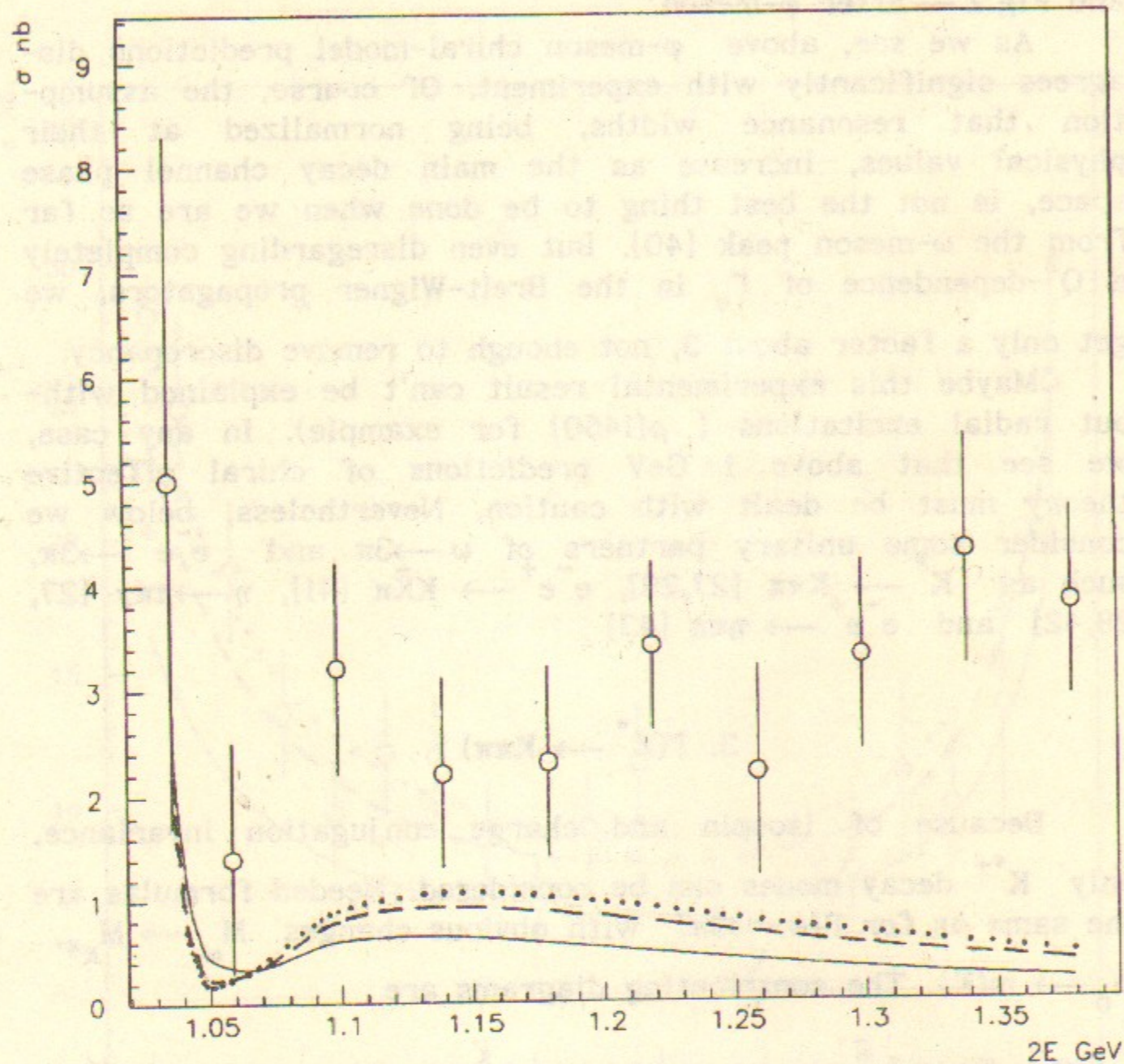


Fig. 2.  $\sigma(e^-e^+ \rightarrow 3\pi)$  after  $\varphi$ . Designations are the same as for Fig. 1.

$$+ \frac{m_\rho^2}{m_{K^*}^2} [R_{K^*}(Q_+^2) + R_{K^*}(Q_-^2)] \Bigg\} ,$$

$$F(K^* \rightarrow K^+ \pi^- \pi^+) =$$

$$= \frac{g_{\rho\pi\pi}}{4\pi^2 f_\pi^3} \left[ 1 - 3\alpha_K - \frac{3}{2} \alpha_K \left\{ R_\rho(Q_0^2) + \frac{m_\rho^2}{m_{K^*}^2} R_{K^*}(Q_+^2) \right\} \right]. \quad (12)$$

The third formfactor satisfies relation

$$F(K^* \rightarrow K^+ \pi^0 \pi^0) = F(K^* \rightarrow K^+ \pi^- \pi^+) + \frac{1}{2^{1/2}} F(K^* \rightarrow K^0 \pi^0 \pi^+), \quad (13)$$

expected from the isospin invariance.

The numerical results are collected below (we have taken  $m_0 = m(K) \approx 500$  MeV,  $M_{K^*} \approx 890$  MeV,  $\Gamma_{K^*} \approx 50$  MeV)

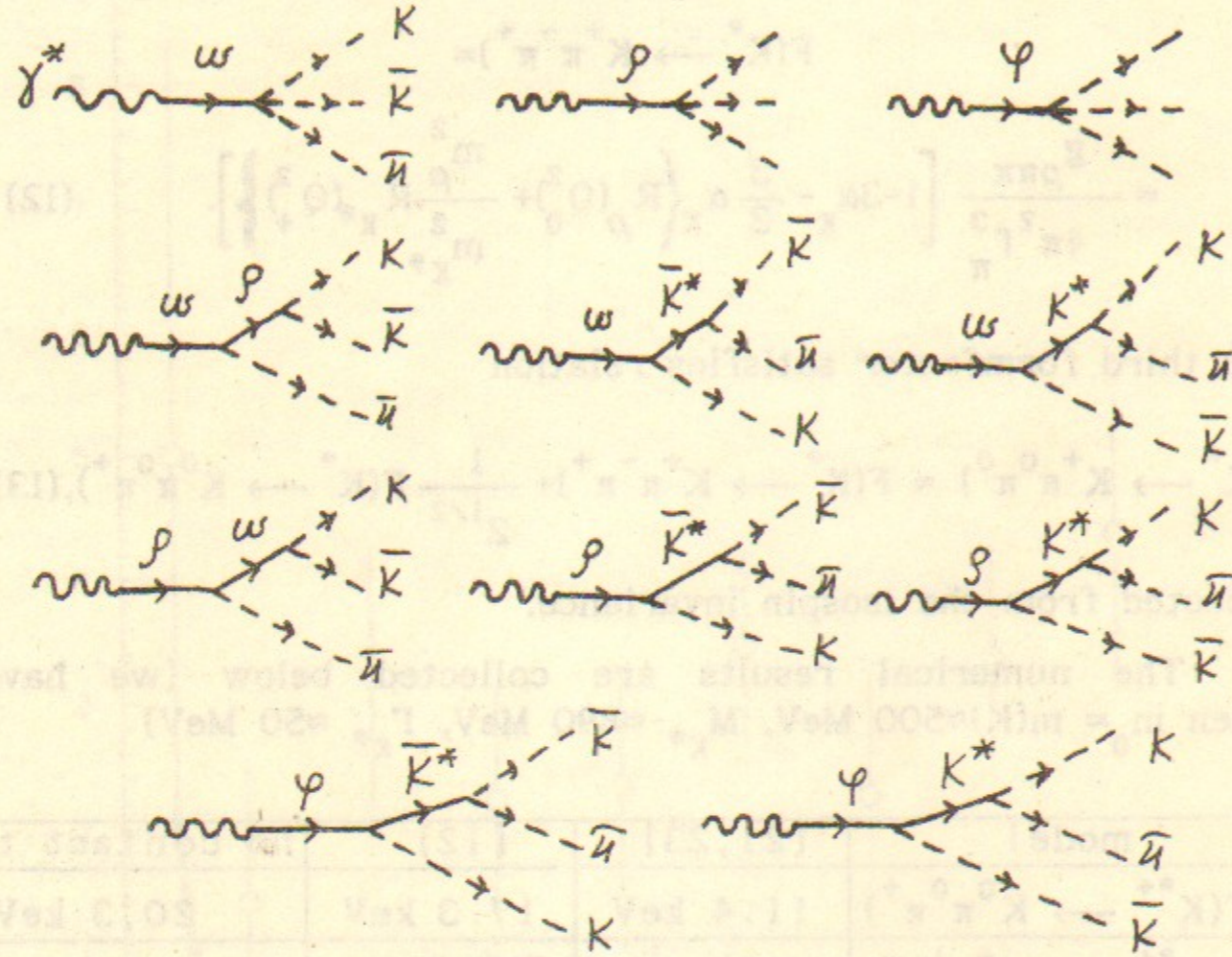
model	[21,23]	[12]	no contact term
$\Gamma(K^{*+} \rightarrow K^0 \pi^0 \pi^+)$	11.4 keV	17.3 keV	20.3 keV
$\Gamma(K^{*+} \rightarrow K^+ \pi^+ \pi^-)$	5.7 keV	8.7 keV	10.2 keV
$\Gamma(K^{*+} \rightarrow K^+ \pi^0 \pi^0)$	0.03 keV	0.03 keV	0.03 keV

Any choice of the contact term is compatible with the current experimental bound [32] on the sum of all three modes  $\Gamma(K^* \rightarrow K\pi\pi) < 35$  keV.

#### 4. $e^-e^+ \rightarrow K\bar{K}\pi$ near the threshold

This reaction was not yet observed for the energies  $s \sim (1 \text{ GeV})^2$ . It is interesting if the new VEPP-2M experiments can see them. Our results show that the expected cross-sections are several picobarns, so their investigation is not a simple, though possible task for such a kind of storage ring as VEPP-2M.

There are many diagrams contributing in this process. For the ideal  $\omega$ - $\phi$  mixing, they are listed below:



The formfactors can be easily derived from them using coupling constants of [23] and have the following form:

$$F(e^-e^+ \rightarrow K^+ \bar{K}^0 \pi^0) = \frac{e}{12\pi^2 f_\pi^3} [ F_\omega + 3F_\rho - F_\phi ],$$

$$F(e^-e^+ \rightarrow K^0 \bar{K}^0 \pi^0) = \frac{e}{12\pi^2 f_\pi^3} [- F_\omega + 3F_\rho + F_\phi ], \quad (14)$$

$$F(e^-e^+ \rightarrow K^+ \bar{K}^0 \pi^-) = \frac{e\sqrt{2}}{12\pi^2 f_\pi^3} [ F_\omega + 3\tilde{F}_\rho - F_\phi ],$$

where

$$F_\omega = R_\omega(s) \left\{ 1 - 3\alpha_K - \frac{3}{2} \alpha_K R_\rho(Q_0^2) - \right.$$

$$\left. - \frac{3}{4} \alpha_K \frac{m_\rho^2}{m_{K^*}^2} [ R_{K^*}(Q_+^2) + R_{K^*}(Q_-^2) ] \right\},$$

$$F_\rho = R_\rho(s) \left\{ 1 - 3\alpha_K - \frac{3}{2} \alpha_K \frac{m_\rho^2}{m_\omega^2} R_\omega(Q_0^2) - \right.$$

$$\left. - \frac{3}{4} \alpha_K \frac{m_\rho^2}{m_{K^*}^2} [ R_{K^*}(Q_+^2) + R_{K^*}(Q_-^2) ] \right\},$$

$$F_\phi = R_\phi(s) \left\{ 1 - 3\alpha_K - \frac{3}{2} \alpha_K \frac{m_\rho^2}{m_{K^*}^2} [ R_{K^*}(Q_+^2) + R_{K^*}(Q_-^2) ] \right\}$$

and

$$\tilde{F}_\rho = \frac{3}{4} \alpha_K \frac{m_\rho^2}{m_{K^*}^2} R_\rho(s) [ R_{K^*}(Q_+^2) - R_{K^*}(Q_-^2) ].$$

$Q_0^2$ ,  $Q_+^2$  and  $Q_-^2$  are given by (7) with changes

$$M_\omega^2 \rightarrow s = (2E)^2, \quad m_0 \rightarrow m(\pi), \quad m \rightarrow m(K).$$

The formula for the cross-section is actually the same as for the  $e^-e^+ \rightarrow 3\pi$  and Fig.3-5 present the numerical results.



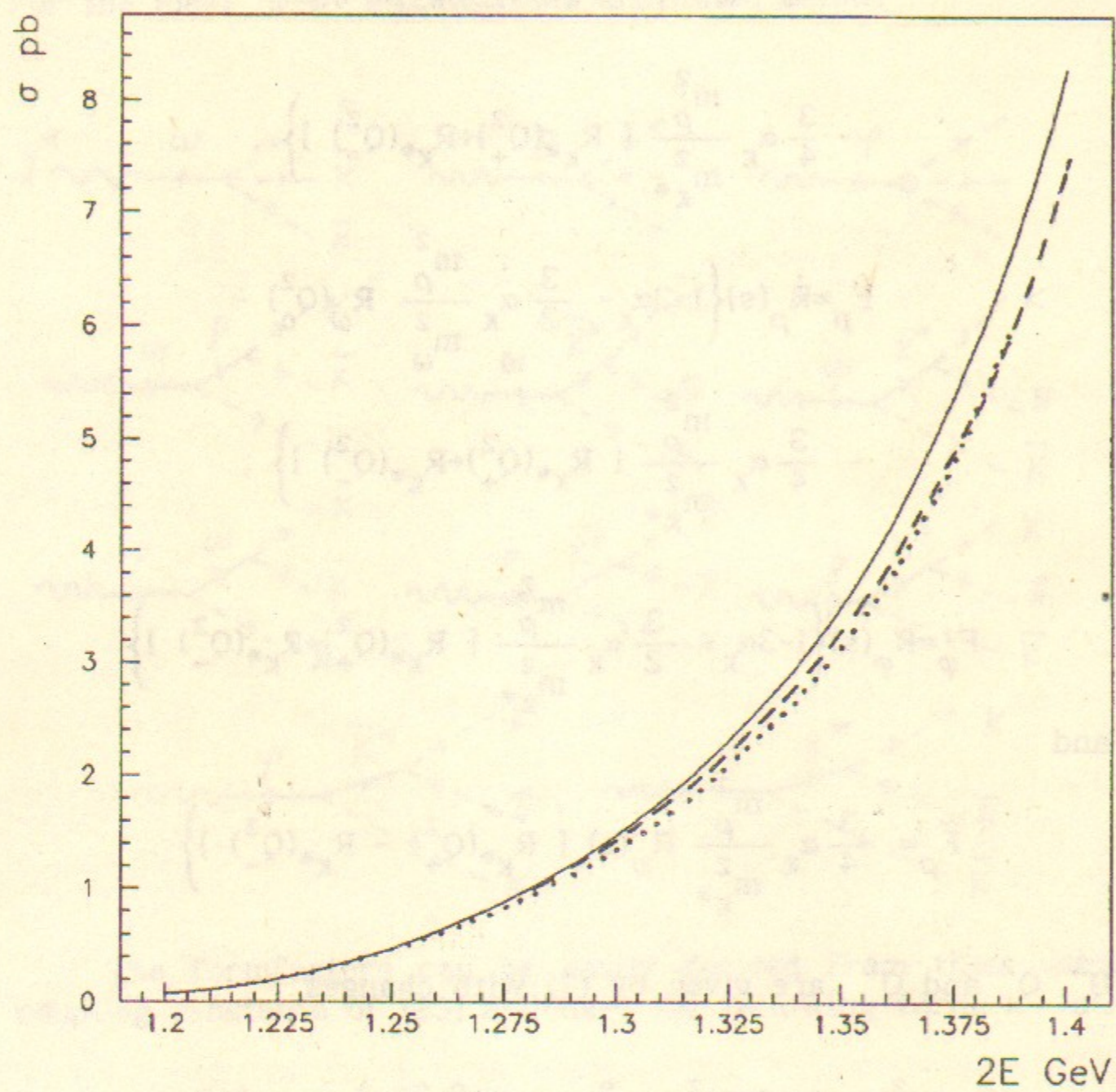


Fig. 3.  $\sigma(e^-e^+ \rightarrow K^-K^+\pi^0)$ . Dashed [12], dotted line—no contact term, solid line [23].

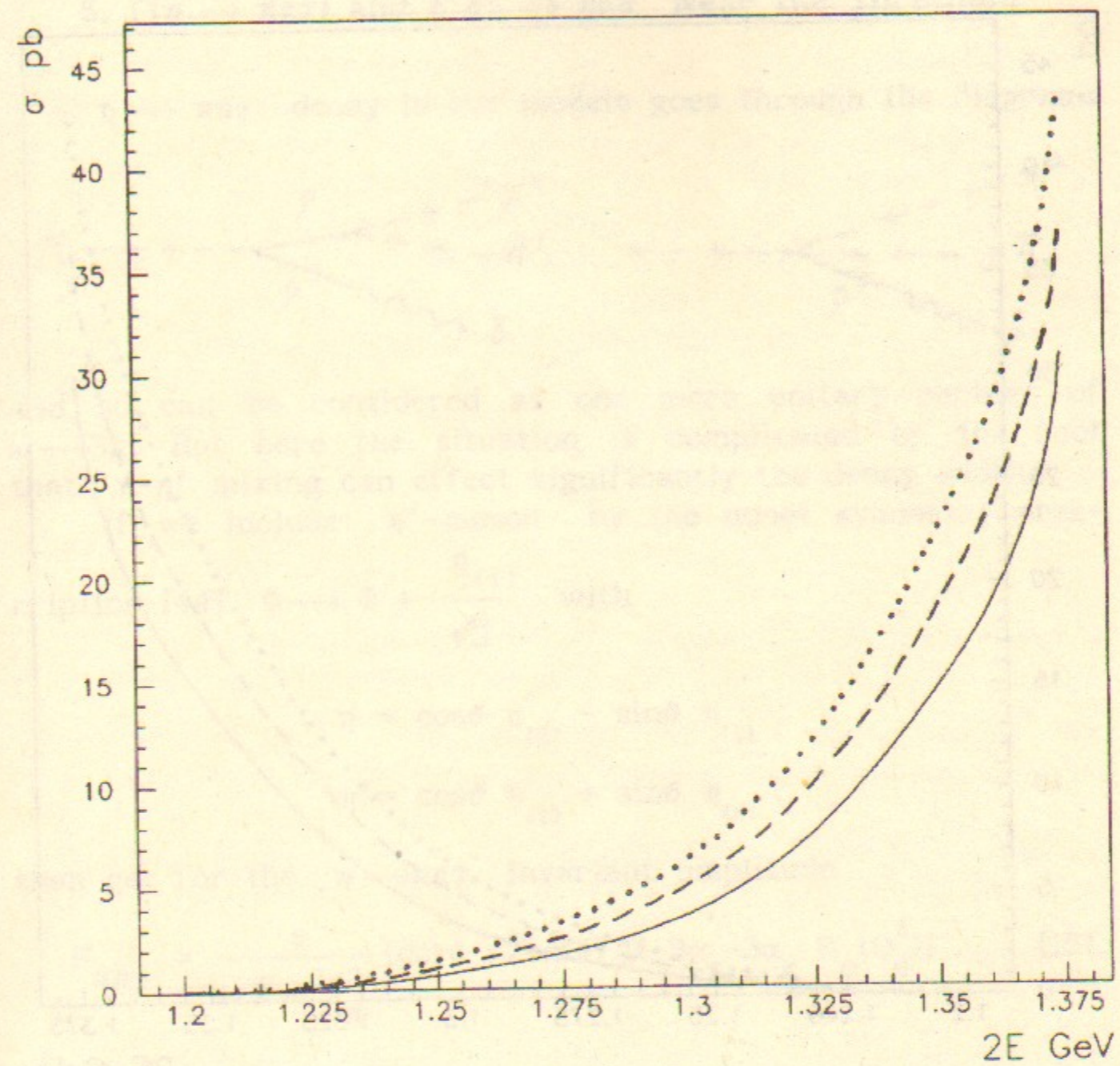


Fig. 4.  $\sigma(e^-e^+ \rightarrow K^0 \bar{K}^0 \pi^0)$ . Designations are the same as for Fig. 3.

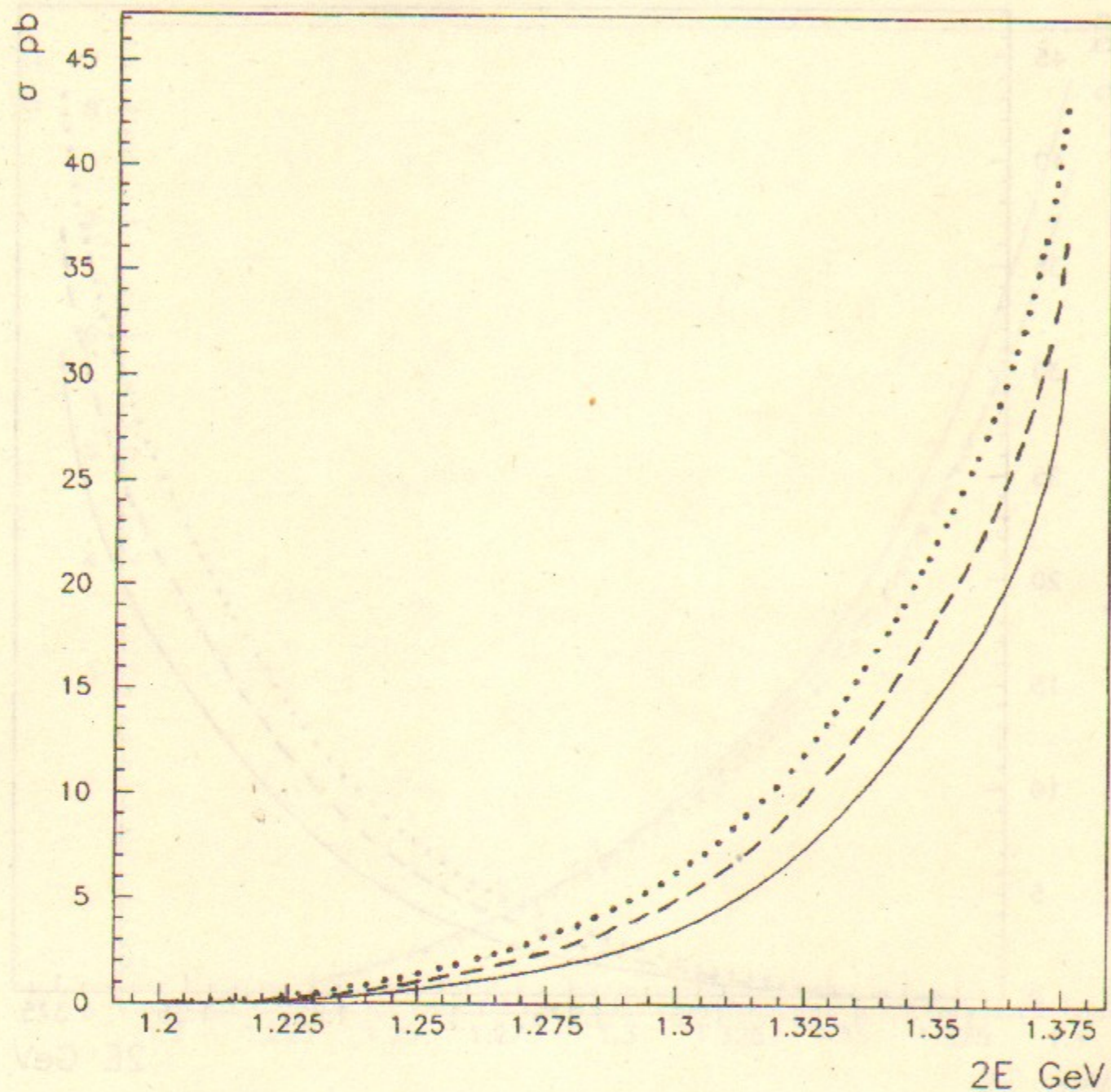
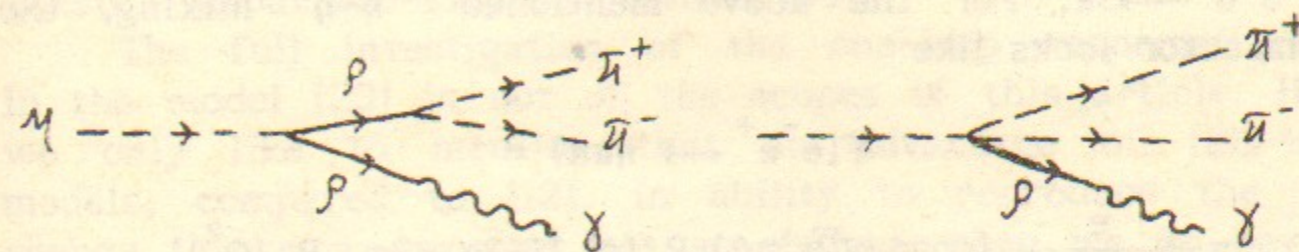


Fig. 5.  $\sigma(e^-e^+ \rightarrow K^+ \bar{K}^0 \pi^-)$ . Designations are the same as for Fig. 3.

### 5. $\Gamma(\eta \rightarrow \pi\pi\gamma)$ and $e^-e^+ \rightarrow \eta\pi\pi$ Near the Threshold

$\eta \rightarrow \pi\pi\gamma$  decay in our models goes through the diagrams



and so can be considered as one more unitary partner of  $\omega \rightarrow 3\pi$ . But here the situation is complicated by the fact that  $\eta$ - $\eta'$  mixing can effect significantly the decay width.

If we include  $\eta'$ -meson by the nonet symmetry prescription [44]  $\Phi \rightarrow \Phi + \frac{\eta_{(1)}}{\sqrt{3}}$  with

$$\eta = \cos\theta \eta_{(8)} - \sin\theta \eta_{(1)},$$

$$\eta' = \cos\theta \eta_{(1)} + \sin\theta \eta_{(8)},$$

then get for the  $\eta \rightarrow \pi\pi\gamma$  invariant amplitude

$$F_{\pi\pi\gamma} = \frac{e}{4\sqrt{3} \pi^2 f_\pi^3} (\cos\theta - \sqrt{2}\sin\theta) [1 - 3\alpha_K - 3\alpha_{\bar{K}} R_\rho(Q_0^2)]. \quad (15)$$

Assuming  $M_\omega \rightarrow m(\eta)$ ,  $m=m(\pi)$ ,  $m_0=0$  in (2),(3),(4) and  $\theta \approx -20^\circ$  [45], we can calculate the decay width and the results are :

model	[21,23]	[12]	no contact term
$\Gamma(\eta \rightarrow \pi\pi\gamma)$ eV	93	152	183

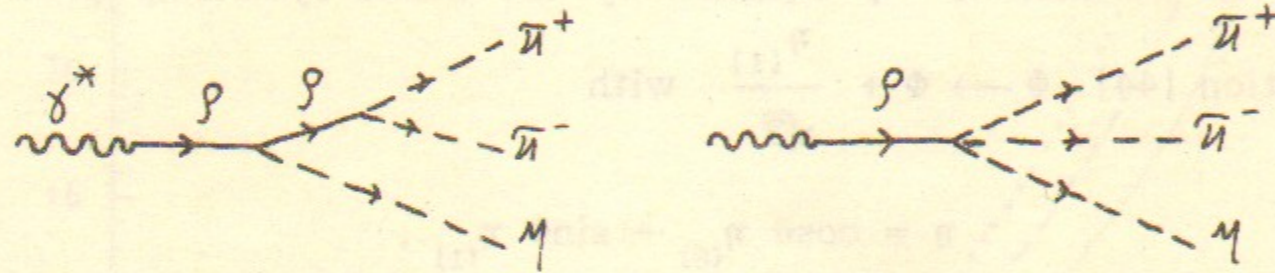
This evidently overestimates the experimental width [32]  $\Gamma_{\text{exp}} = (58 \pm 7)$  eV. But remember that  $\cos\theta - \sqrt{2}\sin\theta \approx \sqrt{2}$  factor in

(15) is due to the  $\eta$ - $\eta'$  mixing. So it is not clear if this discrepancy indicates the important one-loop corrections [46] or more refined  $\eta$ - $\eta'$  mixing scheme [47].

$e^-e^+ \rightarrow \eta\pi\pi$  reaction can be considered in the same way as  $e^-e^+ \rightarrow 3\pi$ . For the above mentioned  $\eta$ - $\eta'$  mixing, the formfactor looks like

$$F(e^-e^+ \rightarrow \eta\pi\pi) = \frac{e}{4\sqrt{3} \pi^2 f_\pi^3} (\cos\theta - \sqrt{2}\sin\theta) R_\rho(s) [1 - 3\alpha_K - 3\alpha_K R_\rho(Q_0^2)],$$

and corresponds to the diagrams



The predicted cross-sections are (in picobarns):

2E GeV	model			experiment [31]
	[21,23]	[12]	no contact term	
1.075	8	12	15	0±500
1.15	18	28	32	0±500
1.25	57	75	81	200±400
1.325	133	162	176	300±500

This process was considered earlier in [43] with similar results. It is premature to compare them to the existing experimental data because of a very big statistical errors.

For higher energies it is known [48] that the reaction goes through the  $\rho$ -meson radial excitations, so we don't expect that the predictions of our chiral effective lagrangians can be trusted far from the threshold.

## 6. Beyond the Trees

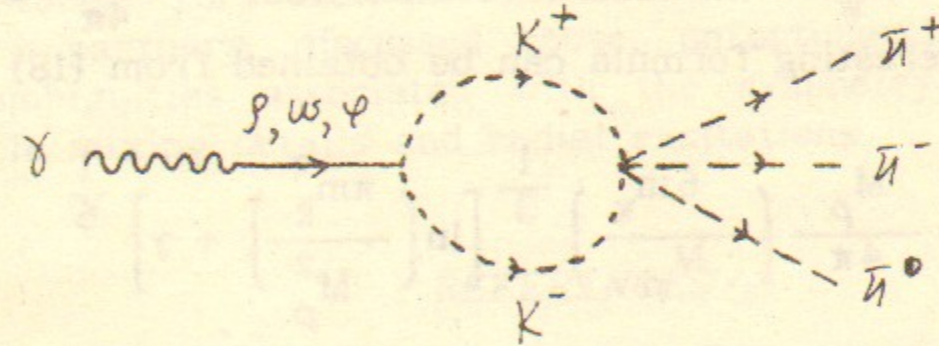
As we have seen, phenomenological consequences of chiral effective theory with correct  $\omega \rightarrow 3\pi$  contact term can be hardly considered as successful. This naturally raises a question about one-loop corrections [49].

The full investigation of the one-loop renormalization in the model [23] is out of the scopes of this article. Here we only like to mention that an advantage of [21]-type models, compared to [12], in ability to reproduce the low energy theorems, becomes not so obvious when we go beyond the tree level. In particular, let us note the curious observation that the purely pseudoscalar loops can restore in [12] the validity of the Terentiev's et al. low energy

$$\text{theorem } F_{3\pi} = \frac{F_\pi}{ef_\pi^2} \quad [7].$$

The one-loop renormalization of the  $\pi^0 \rightarrow 2\gamma$  amplitude has been already considered [50]. It was found that  $F_\pi$  (i.e. the  $\omega\rho\pi$  vertex) remains unrenormalized, as was expected from the Adler-Bardeen theorem about the non-renormalizability of the chiral anomaly [51].

The contributions from the pion-loop contained diagrams in the low energy  $\gamma \rightarrow 3\pi$  amplitude are proportional to  $m_\pi^2$  and can be neglected in the spirit of current-algebra. So it remains only the  $K\bar{K} \rightarrow 3\pi$  Wess-Zumino anomaly contribution:



Using dimensional regularization, we get the following expression for it (assuming  $\frac{1}{\epsilon}$  pole is absorbed by suitable counterterm)

$$F_{3\pi} = -\frac{3em_K^2}{4(2\pi)^4 f_\pi^5} \left[ \ln\left(\frac{\pi m_K^2}{\mu^2}\right) + \gamma \right], \quad (16)$$

$\gamma=0.5772$  being the Euler constant.

Together with the tree level contributions in  $F_\pi$  and  $F_{3\pi}$  [5], we get that the low energy theorem [7] is satisfied, if

$$-\frac{3m_K^2}{16\pi^2 f_\pi^2} \left[ \ln\left(\frac{\pi m_K^2}{\mu^2}\right) + \gamma \right] + 3\alpha_K + 1 - 3\alpha_K + \frac{3}{2} \alpha_K^2 = 1. \quad (17)$$

For the renormalization scale the natural choice is [52]  $\mu=M_\rho$ . So we get from (17) a generalized KSRF relation

$$\alpha_K = \frac{m_K}{2\sqrt{2}\pi f_\pi} \left[ \ln\left(\frac{\pi m_K^2}{M_\rho^2}\right) + \gamma \right] \frac{1}{2}. \quad (18)$$

For  $m_K=494$  MeV,  $M_\rho=768$  MeV and  $f_\pi=93$  MeV the r.h.s. gives just the experimental value 0.55.

Using  $SU_W(6)$  motivated relation [53]  $\frac{g_{\rho\pi\pi}^2}{4\pi} = \frac{2}{3}\sqrt{2}\pi$ , one more interesting formula can be obtained from (18):

$$f_\pi = \frac{M_\rho}{4\pi} \left( \frac{6m_K}{M_\rho} \right)^{\frac{1}{3}} \left[ \ln\left(\frac{\pi m_K^2}{M_\rho^2}\right) + \gamma \right]^{\frac{1}{6}}. \quad (19)$$

We don't know, if a very good accuracy by which (18) and (19) are fulfilled is a mere accident, or the consistency of the vector meson dominance, chiral loops and low energy theorems really requires such a kind of relations.

## 7. Conclusions

It seems to us that at present neither theory nor experiment points out to the existence and magnitude of the  $\omega \rightarrow 3\pi$  contact term. A relative size of the contact interactions, compared to the  $\rho$ -meson pole one, can be in principle extracted from a  $e^-e^+ \rightarrow 3\pi$  data investigating a  $\frac{d\sigma}{dx dy}$  distribution, where  $x = \frac{E_+}{2E}$  and  $y = \frac{E_-}{2E}$  are the charge pion energy fractions:

$$\frac{d\sigma}{dx dy} \sim |h_{\text{cont.}} - 3\alpha_K H(x,y)|^2.$$

For example,  $h_{\text{cont.}} = 1 - 3\alpha_K \approx -\frac{1}{2}$  for [21] and  $h_{\text{cont.}} = 1 - 3\alpha_K + \frac{3}{2}\alpha_K^2 \approx -\frac{1}{8}$  for [12].

The similar analysis was already performed [1] with the result that the Gell-Mann, Sharp, Wagner mechanism gives (85±15)% of the total cross-section near the  $\varphi$ -meson [31], indicating that the contact term, if present, is small.

As for the theory, the complete one-loop analysis of the chiral effective models with vector mesons is greatly desired, especially in context of the vector meson dominance and low energy theorems.

$\Gamma(\omega \rightarrow 3\pi)$  and  $\sigma(e^-e^+ \rightarrow 3\pi)$  are the only clear touchstones for the  $\omega \rightarrow 3\pi$  contact term, because their unitary partners, discussed above, unfortunately suffer from the ambiguities associated with the symmetry breaking and particle mixing details and radial excitations.

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