38

ИНСТИТУТ ЯДЕРНОЙ ФИЗИКИ
им. Г.И. Будкера СО РАН

V.M. Popik and N.A.Vinokurov

THE SIMPLE MODEL OF THE SUPERMODES IN FEL WITH A INTRACAVITY ETALON

BUDKERINP 93-72

НОВОСИБИРСК

The Simple Model of the Supermodes in FEL With a Intracavity Etalon

V.M. Popik and N.A. Vinokurov.

Budker Institute of Nuclear Physics

11 Lavrentyev Ave., Novosibirsk, 630090, Russia
Tel: 7-(3832)-359977 Fax: 7-(3832)-352163
Email: popik@inp.nsk.su

Abstract

The influence of intracavity glass plate with parallel planes on characteristics of FEL is theoretically investigated. For the given thickness of the plate the minimum of lasing linewidth is calculated. The decrease of FEL gain with a intracavity etalon is estimated.

The technique of linewidth narrowing using the intracavity Fabry-Perot etalon had been demonstrated previously on our FEL [1]. To conserve the longitudinal modes synchronization we used the etalon with small thickness and reflectivity, actually it was simple 1.2 mm thick glass plate. In this paper we improve the longitunal supermodes [1,2] for this particular case. Let's consider the evolution of the wave packet

$$\mathcal{E}(z,t)=\mathrm{Re}\left[a(z-ct)e^{i(kz-\omega t)}\right], \quad (1)$$

travelling inside the FEL optical resonator (see Fig.1). To find the supermodes we are to write down the round-trip transformation of wave packet (1):

$$a(z)=\int_z^{\infty}\mathcal{K}(z,z')a(z')dz'. \qquad (2)$$

Eigenfunctions of this transformation are the supermodes. Assuming $a(z)$ is slow enough and $\mathcal{K}(z,z')\neq0$ only at small $z'-z$ we may expand (2):

$$a \approx Aa + Ba' + Ca'' \qquad (3)$$

with

$$A=\int_z^{\infty}K(z,z+\xi)d\xi,\quad B=\int_z^{\infty}K(z,z+\xi)\xi d\xi \text{ and, } \quad C=\int_z^{\infty}K(z,z+\xi)\frac{\xi^2}{2}d\xi.$$

Let the electron bunches enter the FEL with period T, and the bunch duration is much less than T. Then choosing ω /near maximum of amplification of the FEL we have the transformation of amplitude:

$$a_f(z)\approx(1+\frac{1}{2}G(z))a_i, \qquad (4)$$

where $G(z)$ - gain per pass. For simplicity we suppose $G \ll 1$. After that light is reflected by the mirrors and passes through the glass plate (the simplest Fabry-Perot etalon). The last one transforms the wave as follows:

$$\mathcal{E}_f(z)=\mathcal{E}_i(z)(1-R)+R\mathcal{E}_i(z+D), \qquad (5)$$

where $R=\left(\frac{n-1}{n+1}\right)^2 \ll 1$, n - the glass refraction index, $D=2nd_p$, d_p - the plate thickness. To introduce slow amplitude it's better to generalize (1) as:

$$\mathcal{E}(z)=\mathrm{Re}[a(z)f(z)], \qquad (6)$$

$f(z+D)=f(z)$ - a periodical function. Then from (5) we obtain:

$$a_f(z)\approx a_i(z) + RDa_i' + \frac{RD^2}{2}a_i''. \qquad (7)$$

If the optical length between mirrors is less than the resonant one on value δ the full round-trip transformation over the period T is:

$$a_f=a_i\left[1+\frac{G(z)}{2}-\frac{\gamma}{2}\right] + a_i'(2RD-2\delta) + RD^2a_i'', \qquad (8)$$

where γ - losses on the mirrors. Putting $a_f=\lambda a_i$ at (8) we obtain equation for the eigenmodes:

$$a'' + 2\frac{RD-\delta}{RD^2}a' + \frac{a}{RD^2}\left[1-\frac{\gamma}{2}-\lambda+\frac{G(z)}{2}\right]=0. \qquad (9)$$

Substitution $a=be^{-(1-\frac{\delta}{RD})\frac{z}{D}}$ gives:

$$b'' + \left[1-\frac{\gamma}{2} - \lambda - R\left(1-\frac{\delta}{RD}\right)^2 + \frac{G(z)}{2}\right]\frac{1}{RD^2}\, b = 0. \qquad (10)$$

Choosing

$$G(z) = \frac{G_0}{ch^2 \alpha z}, \qquad (11)$$

we can obtain eigenvalues [3,4]

$$\lambda = 1 - \frac{\gamma}{2} - R\left(1-\frac{\delta}{D}\right)^2 + RD^2\frac{\alpha^2}{4}\left[-(2n+1)+\sqrt{1+\frac{2G_0}{RD^2\alpha^2}}\right]^2. \qquad (12)$$

Note, that quantity inside the square brackets is to be positive. Therefore the maximal eigenvalue corresponds to n=0 and

$$b = (ch\ \alpha z)^{-s}, \qquad (13)$$

$$s = \frac{1}{2}\sqrt{1+\frac{2G_0}{RD^2\alpha^2}} - \frac{1}{2}. \qquad (14)$$

The remarkable feature of this situation is the finite number of eigenmodes: $0 \le n \le s$. In particular case $s<1$ there is only one eigenmode (13). It's useful to rewrite (12):

$$\lambda = 1 - \frac{\gamma}{2} + \frac{G_0}{2} - R\left(1-\frac{\delta}{D}\right)^2 - RD^2\alpha^2\left(n+\frac{1}{2}\right)\left(\sqrt{1+\frac{2G_0}{RD^2\alpha^2}} - 1 - n\right) \qquad (15)$$

Last term explicits the reduction of increment due to glass plate.

As above the threshold λ is to be more than one, G_0 is to be more than $\gamma + 2R\left(1-\frac{\delta}{RD}\right)^2$.

Then for the thin plate

$$D \ll \sqrt{\frac{2}{R\alpha^2}\left[\gamma+2R\left(1-\frac{\delta}{RD}\right)^2\right]} \qquad (16)$$

the reduction of increment for n=0 is

$$\delta\lambda \approx -\alpha D\sqrt{\frac{G_0}{2R}}. \qquad (17)$$

If we are interesting in the "maximal" characteristical thickness we may put $\delta\lambda = -G_0/4$ and obtain:

$$D_{max} = \alpha^{-1}\sqrt{\frac{G_0 R}{8}} \qquad (18)$$

and

$$d_{max} = \frac{\sigma}{4n}\cdot\frac{n-1}{n+1}\sqrt{G_0}, \qquad (19)$$

where $\sigma = \frac{1}{\sqrt{2}\,\alpha}$ - the electron bunchlength. In this case $S \approx \frac{2}{R}$ and according to (13) the light pulse is $\sqrt{2/R} = \frac{n+1}{n-1}\sqrt{2}$ times shorter (by amplitude) than the electron bunch and the "minimal" linewidth is:

$$\left(\frac{\sigma_\omega}{\omega}\right)_{min} = \frac{1}{\sigma k} \cdot \frac{n+1}{n-1} . \qquad (20)$$

Here we are to remind that we do not specify the periodical function f(z) in (6) yet. Therefore spectrum might consist of many peaks with relatively narrow linewidth (20).

REFERENCES

[1] M.-E. Couprie *et al.*, Nucl. Instr. and Meth. A308(1991) 39-44.

[2] V.N. Litvinenko and N.A. Vinokurov, Nucl.Instr. and Meth. A304(1991) 66-71.

[3] L.D. Landau and E.M. Livshitz. Quantum Mechanics.

[4] S. Flugge. Practical Quantum Mechanics I. Springer-Verlag, 1971.

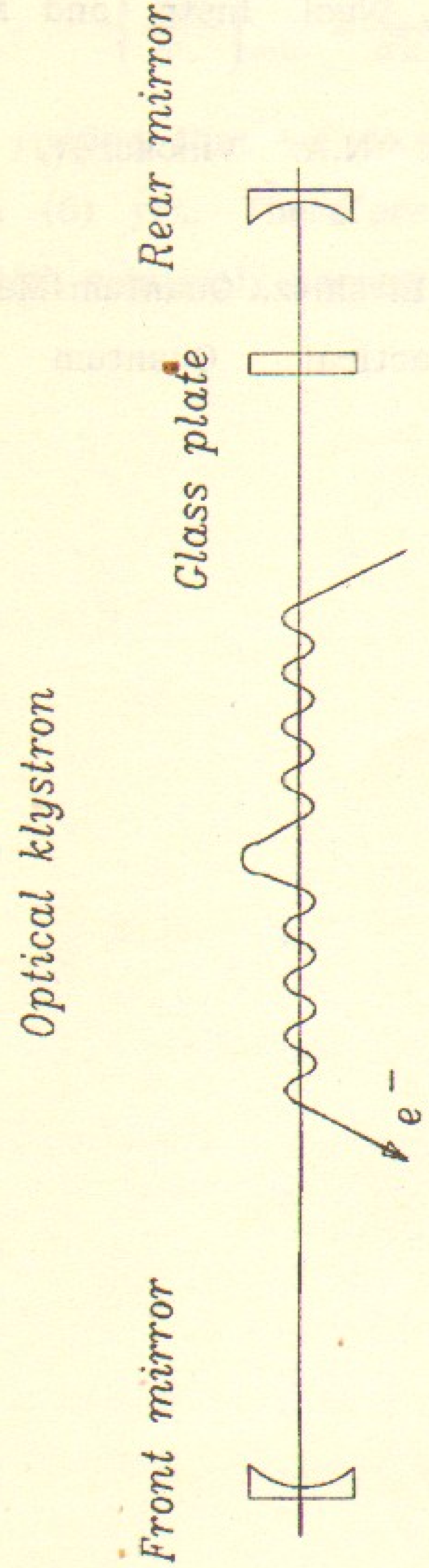

Fig. 1. Optical cavity with Fabry-Perot etalon.

V.M. Popik and N.A. Vinokurov

The Simple Model of the Supermodes
in FEL with a Intracavity Etalon

В.М. Попик, Р.А. Винокуров

Простая модель супермод в ЛЭС
с внутрирезонаторным эталоном

Подписано в печать 8.09.1993 г.
Формат бумаги 60×90 1/16 Объем 0,8 печ.л., 0,7 уч.-изд.л.
Тираж 200 экз. Бесплатно. Заказ N° 72

Обработано на IBM PC и отпечатано на
ротапринте ИЯФ им. Г.И. Будкера СО РАН,
Новосибирск, 630090, пр. академика Лаврентьева, 11.