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V.L. Chernyak

CALCULATION OF THE D AND B MESON LIFETIMES

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V.L. Chernyak[1]

Budker Institute of Nuclear Physics
630090, Novosibirsk 90, Russia

ABSTRACT

Using the expansions of the heavy meson decay widths in the heavy quark mass and QCD sum rules for estimates of corresponding matrix elements, we calculate the $D^{\pm,o,s}$ decay widths and the $B^{\pm,o,s}$ lifetime differences. The results for D mesons are in agreement with the data, while it is predicted that $[\Gamma(B^o)-\Gamma(B^-)]/\Gamma_B \simeq 4\%$, and the lifetime difference of the B^o and B_s mesons is even smaller. The role of the weak annihilation and Pauli interference contributions to the lifetime differences are described in detail. In the course of self-consistent calculations the values of many parameters crucial for calculations with charmed and beauty mesons are found. In particular, the perturbative pole quark masses are: $M_c \simeq 1.65\,GeV$, $M_b \simeq 5.04\,GeV$, and the decay constants are: $f_D(M_c) \simeq 165\,MeV$, $f_B(M_b) \simeq 113\,MeV$. It is also shown that the nonfactorizable corrections to the $B^o - \bar{B}^o$ mixing are large, $B_B \simeq (1-18\%)$. The values of the unitarity triangle parameters are found which are consistent with these results and the data available (except for the NA31 result for the ϵ'/ϵ which is too large): $\lambda \simeq 0.22$, $A \simeq 0.825$, $\rho \simeq -0.4$, $\eta \simeq 0.2$.

[1] e-mail: CHERNYAK@INP.NSK.SU

1. Introduction

It is a long standing challenge for theory to calculate the D and B meson decay widths. On the qualitative side, two mechanisms were invoked to explain the pattern of the D meson lifetime differences: weak annihilation (WA) [1], [2], [3], and Pauli interference (PI) [4], [5], [6], [7]. As for WA, it was expected that because an admixture of the wave function component with an additional gluon or the emission of a perturbative gluon, both remove a suppression due to helicity conservation (which leads to $Br(\pi \to e\,\nu)/Br(\pi \to \mu\,\nu) \sim 10^{-4}$), the D^o meson decay width is enhanced. On the other hand, it was expected that the destructive Pauli interference of two d-quarks (the spectator and those from a final state) suppresses the $D^{\pm}$ meson decay width. As for WA, there were no reliable calculations at all. For PI, simple minded estimates (see sect.8) give too large an effect which results in a negative $D^{\pm}$ decay width.

For B mesons, it was clear qualitatively that all the above effects which are of a pre-asymptotic nature and die off at $M_Q \to \infty$, will be less important. However, because the patern of the D meson lifetime differences was not really explained and well understood, this prevented to obtain reliable estimates of the B meson lifetime differences, and only order of magnitude estimates are really available: $[\delta\Gamma(B)/\Gamma_B] : [\delta\Gamma(D)/\Gamma_D] \sim O(f_B^2 M_c^2/f_D^2 M_B^2) \sim O(10^{-1})$.

Moreover, as for WA contributions through perturbative gluon emission (which is formally a leading correction $\sim O(\Lambda_{QCD}/M_Q)$ to the deacay width and was expected before to be potentially the most important), it has been emphasized recently [8] that such contributions are of no help at all because, being large (at least formally at $M_Q \to \infty$) term by term, they cancel completely in the inclusive widths, both $O(1/M_Q)$ and $O(1/M_Q^2)$ terms [2]. It will be shown below (see sect.9) that, nevertheless, there are important WA contributions but on the nonperturbative level.

Considerable progress has been achieved recently in applications of the operator product expansion to the calculation of the heavy meson decay width. In particular, it was shown that there are no $O(1/M_Q)$ corrections to the Born term and first nonperturbative corrections $O(1/M_Q^2)$ were calculated explicitly [9], [10]. However, these contributions are all nonvalence (i.e. one and the same for all $D^{\pm,o,s}$ mesons), and so have nothing to do with lifetime differences. They are important however for the calculation of the absolute decay rates. Recently a number of papers appeared [11], [12], [13], [14], where these results were applied to determine the values of the quark masses, M_c and M_b, and $|V_{cb}|$.

The purpose of the present work was to calculate the D and B meson decay widths, with the main emphasis on the calculation of lifetime differences. It is shown that the $D^{\pm,o,s}$ lifetimes can be calculated with a reasonable accuracy, and concrete predictions for the $B^{\pm,o,s}$ meson lifetime differences are given also.

The scope of the paper is as follows:

1. Introduction.
2. Definition of the heavy quark mass.
3a. General formulae: c-quark.
3b. General formulae: b-quark.
4. $D \to e\nu + X$. Determination of M_c.
5. Mass formulae. Determination of M_b and $\bar{\Lambda}$.
6. $B \to e\nu + X$. Determination of $|V_{cb}|$.
7. Calculation of f_D, f_B.
8. Difficulties with naive estimates.
9. Nonfactorizable contributions: gluon condensates.
10. Nonfactorizable contributions: quark condensates.
11. Corrections to semileptonic widths.
12. λ and $\bar{\lambda}$.
13a. Calculation of $D^{\pm,o,s}$ decay widths.
13b. Calculation of $B^{\pm,o,s}$ lifetime differences.
14. $B^o - \bar{B}^o$ mixing.
15. The unitarity triangle.
16. Summary and conclusions.

[2] The separate perturbative WA contributions into the total width have the form: $\delta\Gamma/\Gamma_{Born} \sim f_D^2/\epsilon_o^2$, $f_D^2 \sim \mu_o^3/M_Q$, where ϵ_o is the binding energy of the spectator quark in the meson. So, they are highly infrared sensitive and singular at $\epsilon_o \to 0$. It seems clear that there can not be power infrared singularities in the inclusive decay width, so that it is not so surprising that all such terms cansel (i.e. the terms $\sim O(1/\epsilon_o^2)$ and $\sim O(1/\epsilon_o)$, but not $\log\epsilon_o$ which describes hybrid log.) .

2. Definition of the heavy quark mass

In what follows, expansions in powers of the heavy quark mass and some formulae of the HQET (Heavy Quark Effective Theory) (see, i.e. the reviews [15], [16], [17]) are used. In the standard approach, the perturbative pole mass, M_p, is chosen as the HQET expansion parameter. The perturbative pole mass is, clearly, a distinguished parameter because it is scheme and gauge independent. So, it is natural that all calculations of observable quantities are expressed usually through M_p.

It was pointed out recently [18], [19], [20] that the perturbative pole mass is an ill-defined quantity and contains an intrinsic uncertainty $\sim O(\Lambda_{QCD})$, because the perturbation theory series for it diverges due to renormalon effects.

Analogous renormalon divergences are well known and originate from the fact that there is an internal scale in the theory: Λ_{QCD}- the position of the coupling constant infrared pole. So, this scale reveals itself when perturbative loops are integrated up to $k_i \to 0$ (as in the case with the pole mass). But because Λ_{QCD} can not appear explicitly at any order of renormalized perturbation theory, the perturbative series diverges. These infrared singularities are cured usually by nonperturbative contributions, and it seems most people are not worrying especially about them.

To avoid from the beginning the infrared region contributions, it was proposed in [18] to use some infrared safe mass parameter, M_μ, instead of M_p. As for a concrete choice of M_μ, two variants are considered usually. The first one (in a spirit of the Wilson operator expansion) is to calculate all loops by cutting out the $k_i \le \mu$ region contributions. A deficiency of this variant is that it is practically impossible to perform such calculations. The second one is to use the current quark mass normalized far off mass shell which suppresses the infrared region contributions. Because this current mass differs from M_p already at the first loop level, this variant spoils finally all the usefullness of the standard HQET approach, replacing the standard decomposition in powers of $1/M_p$ by the usual perturbation theory decomposition in powers of $1/\log M_\mu$. Besides, because the current quark mass is renormalization scheme and gauge dependent, all this is highly inconvenient.

In connection with this, we describe below another redefinition of M_p and introduce the "hard pole mass", M_o, which is free of renormalon singularities by definition and differs from M_p by only the term $\sim \Lambda_{QCD}$ (not by $\sim \alpha_s M_p$). We emphasize that, as far as we do not calculate explicitly high order perturbation theory corrections to observable quantities, the whole problem of the renormalon is of abstract interest only and can be formally solved on a "hand waving level", i.e. by some redefinitions only, without making any real changes in all the formulae available.

The above "hard pole mass", M_o, can be connected with a definition of the matrix element of the local operator. Although it is clear beforehand, let us demonstrate explicitly, using the example considered in [18], that the relevant operator to use for a redefinition of M_p is H_{light}, i.e. the light degrees of freedom part of the Hamiltonian. With this purpose, let us consider the Hamiltonian and take its average over the heavy meson state, $|M_H>$:

$$< M_H|\, H_{tot}\, |M_H >= M_\mu < M_H|\, (\bar{Q}\,Q)_\mu\, |M_H > +$$

$$< M_H|\, \frac{1}{2}(\vec{E}^2 + \vec{H}^2)_\mu |M_H > + < H_q > + O(\mu^2/M)\,, \qquad (1)$$

where μ serves as an upper cut off in the matrix elements of operators and as lower cut off in M_μ, H_q is the light quark Hamiltonian, $< M_H|(\bar{Q}Q)_\mu|M_H >= 1 + O(\mu^2/M^2)$.

In the example considered in [18] (in the static limit):

$$M_\mu = M_p - \delta M, \quad \delta M = -i\frac{16\pi}{3}\int\frac{d^4k}{(2\pi)^4}\,\alpha_s\,\phi(k)\,,$$

$$\phi(k) = \frac{1}{k_o + i\epsilon}\,\frac{1}{k^2 + i\epsilon}\,\frac{\mu^2}{\mu^2 - k^2 - i\epsilon}\,, \qquad (2)$$

and when α_s in Eq.(2) is substituted by running $\alpha_s(k^2)$ in the form:

$$\alpha_s(\vec{k}^2) = \alpha_s(\mu^2)\sum_{n=0}^{\infty}\left(\frac{b_o\,\alpha_s(\mu^2)}{4\pi}\log\frac{\mu^2}{\vec{k}^2}\right)^n \qquad (3)$$

this leads to a divergent perturbative series:

$$\delta M = \mu\sum_n^{\infty}\alpha_s{}^n\,C_n\,, \quad C_n \sim (\frac{b_o}{2\pi})^n\, n!\,. \qquad (4)$$

On the other hand, it is not difficult to see that the insertion of the vertex $\vec{E}^2/2$ into the same diagram which gives δM, adds the contribution:

$$< M_H|\,\frac{1}{2}\vec{E}^2\,|M_H >= -i\frac{16\pi}{3}\int\frac{d^4k}{(2\pi)^4}\,\alpha_s\,\phi(k)\left(\frac{-\vec{k}^2}{k^2 + i\epsilon}\right) = \delta M\,, \qquad (5)$$

which cancels the renormalon contribution in Eq.(1) and leaves the pole mass M_p instead of M_μ.[3]

Let us proceed now with a standard on mass shell renormalization scheme and rewrite Eq.(1) in the form:

$$< M_H|\, H_{tot}\, |M_H >= M_p < M_H|\, \bar{Q}\,Q\, |M_H > + < M_H|\, H_{light}|M_H > +$$

$$\frac{2\,\delta\tilde{m}^2}{M} + O(\Lambda^3_{QCD}/M^2), \qquad (6)$$

where [9] (see sect.5 for the explicit form of $\delta\tilde{m}^2$):

$$< M_H|\,\bar{Q}Q\,|M_H >= 1 - \delta\tilde{m}^2/M^2 + O(\Lambda^3_{QCD}/M^3), \quad \delta\tilde{m}^2 = O(\Lambda^2_{QCD}). \qquad (7)$$

It is not difficult to check that:

$$< M_H|\, H_{light}\, |M_H >= \Lambda_o\,(1 + O(\Lambda^2_{QCD}/M^2))\,, \qquad (8)$$

where Λ_o is a finite number $\sim \Lambda_{QCD}$, independent of M. Clearly, it can be understood as the light quark self energy plus the binding energy.

Now, let us redefine the quark pole mass, M_p, and introduce the "hard pole mass" M_o:

$$M_p = M_o\,(1 + \Delta_1 + \Delta_2 + \cdots), \qquad (9)$$

where Δ_1 is defined formally, for instance, as the leading asymptotic part of the divergent perturbative series for M_p:

$$\Delta_1 = \sum_{n=N_1}^{\infty}\alpha_s^n(M_o)\,C_n^{(1)} = A_1\frac{\Lambda_{QCD}}{M_o}\,, \quad C_n^{(1)} \sim \left(\frac{b_o}{2\pi}\right)^n\, n!\,, \qquad (10)$$

and $A_1 \sim (1 + O(\alpha_s))$, $N_1 \simeq 1/\alpha_s(M_o)$.[4] Δ_2 in Eq.(9) is defined analogously through a subleading part of the series, so that $\Delta_2 = A_2\,\Lambda^2_{QCD}/M_o^2$, etc. So, by definition, M_o in Eq.(9) is free of divergent parts of renormalon contributions. What is important, is that the "bad" quantity M_p and the "good" quantity M_o differ not by $\sim \alpha_s(M_o)M_o$ terms, but terms $\sim \Lambda_{QCD}$ only.

[3] This is natural because the whole answer has to be independent of μ, and considering formally the case when there are no nonperturbative contributions and the matrix element is taken over the on mass shell heavy quark state, the total answer is M_p.

[4] More precisely, if any meaning can be given to the divergent sum in Eq.(10) , it can also give higher order terms $\sim \Lambda^2_{QCD}/M_o^2 + \ldots$, but these can be included into a redifinition of Δ_2, etc.

Clearly, we can absorb the term Δ_1 into a redefinition of Λ_o, the term Δ_2 into a redefinition of $\delta\tilde{m}^2$, etc., so that Eq.(6) can be rewritten as:

$$< M_H|\, H_{tot}\, |M_H > = M_o < M_H|\, \bar{Q}\, Q\, |M_H > + \bar{\Lambda} + \frac{2\,\delta m^2}{M_o} + O(\Lambda^3_{QCD}/M_o^2)\,, \qquad (11)$$

$$\bar{\Lambda} = \Lambda_o + \Delta_1 M_o\,, \quad \delta m^2 = \delta\tilde{m}^2 + \Delta_2 M_o^2\,. \qquad (12)$$

In connection with the above, let us point out the following. Strictly speaking, because the real expansion parameter is Λ_{QCD}/M, taking account of higher order terms of the perturbative series in $\alpha_s(M)$ is not justified without simultaneously taking account of power corrections in Λ_{QCD}/M. So, even if we have a convergent series: $\sum \alpha_s^n(M) B_n$, the term $\alpha_s^{N_1}(M)\, B_{N_1}$ becomes $O(\Lambda_{QCD}/M)$ starting with N_1 (depending on the behaviour of B_n at large n); starting with N_2 the term $\alpha_s^{N_2}(M)\, B_{N_2}$ becomes $O(\Lambda^2_{QCD}/M^2)$, etc. For instance, for $B_n \sim 1$ at large n: $N_1 \sim [\alpha_s(M)\, \log 1/\alpha_s(M)]^{-1}$, while for $B_n \sim n!$: $N_1 \sim [\alpha_s(M)]^{-1}$, which is only slightly larger than those for the convergent series. Therefore, in any case, the account of distant terms of the perturbative series has to be made only simultaneously with account of power corrections, and we can always transfer these distant perturbative terms into a redefinition of power corrections (remembering especially that we are unable to calculate directly these nonperturbative corrections at present).

Let us turn now to the heavy meson decay width which can be represented in the form [9]:

$$\Gamma \sim M_p^5 \left[F\Big(\alpha_s(M_p)\Big) + O(\Lambda^2_{QCD}/M_p^2) \right]\,, \qquad (13)$$

where $F(\alpha_s)$ represents a perturbative series, and power corrections in Eq.(13) are explicitly expressed through the matrix elements of higher dimension operators (see sect.3).

As was emphasized in [18], a new element in the example considered is that the renormalon effects lead to a contribution $\sim \Lambda_{QCD}$ in the pole mass, while there are no nonperturbative corrections to Γ at this level. Correspondingly, the perturbative series for $F(\alpha_s)$ is also divergent, and only a product of them in Eq.(13) contains no $\sim \Lambda_{QCD}/M$ renormalon contributions.

Now, let us reexpress the pole mass in Eq.(13) through the formally defined "hard pole mass", M_o. Then in the product: $(1 + 5\,\Delta_1 + O(\Delta_2))\, F(\alpha_s(M_o))$ (where Δ_1 is understood as the divergent series Eq.(10)), the leading divergences cancel, so that there is no Λ_{QCD}/M_o correction. As

for subleading divergencies which give corrections $\sim \Lambda^2_{QCD}/M_o^2$, they will be cured in the usual way by explicit nonperturbative matrix elements in Eq.(.13), etc.

The final result of all the above manipulations can be formulated as follows.

1. We can use the standard perturbative pole mass, M_p, in the HQET to obtain usual expansions in powers of $1/M_p$, putting no attention at the time that it is not a "good" quantity.

2. When dealing with formulae for directly observable quantities, we can reexpress M_p through the "hard pole mass", M_o, which is a "good" quantity. All renormalon divergencies will be either explicitly canceled in perturbative expansions, or absorbed by redefinitions of the original nonperturbative matrix elements. The net effect will be that all the original formulae of the $1/M$ expansions will formally stay intact after all the above redefinitions. Moreover, if we calculate explicitly only a few lowest order loop corrections, which is the case usually, we need not even explicit expressions for the divergent tails of perturbative expansions. It is sufficient to have in mind that all the divergent contributions are cured by redefinitions.

3. What is really important, is that (having in mind all the above redefinitions) we can use now our formulae with only a few first loop corrections explicitly calculated for a comparison with the data, not worrying much that there may be numerically large corrections from distant terms of the perturbative series.

3a. General formulae: c-quark

The weak Hamiltonian used in what follows has the form ($\Gamma_\mu = \gamma_\mu(1+\gamma_5)$):

$$H_W^c(\mu = M_c) = \frac{G_F}{\sqrt{2}}\, V_{cs}\, V_{ud}^*\, \{\, C_1(M_c)\, \bar{s}\,\Gamma_\mu c \cdot \bar{u}\,\Gamma_\mu d + C_2(M_c)\, \bar{u}\,\Gamma_\mu c \cdot \bar{s}\,\Gamma_\mu d \,\} + \cdots\,, \qquad (14)$$

where the dots denote corresponding Cabibbo-suppressed contributions. The coefficients $C_\pm = (C_1 \pm C_2)/2$ are determined by:

$$C_\pm(\mu^2) = C_\pm^L(\mu^2) \left\{ 1 + \frac{\alpha_s(\mu^2) - \alpha_s(M_W^2)}{\pi}\, \rho_\pm \right\}, \qquad (15)$$

$$C_-^L = \left(\frac{\alpha_s(\mu^2)}{\alpha_s(M_W^2)} \right)^{4/b_o}, \quad C_+^L(\mu^2) = 1/\sqrt{C_-^L(\mu^2)}\,, \qquad (16)$$

and the values of $\rho_\pm$ can be found in [21]. We use below in our calculations:

$$\frac{d\,\alpha_s(t)}{dt} = \beta(\alpha_s) = -\frac{b_o}{4\,\pi}\,\alpha_s^2\,(1 + \Delta\,\alpha_s)\,, \tag{17}$$

$$\Delta = \frac{1}{4\pi}\frac{b_1}{b_o},\ b_o = (11 - \frac{2}{3}n_f),\ b_1 = \frac{2}{3}(153 - 19n_f),\ \ t = \log(Q^2/\mu^2). \tag{18}$$

One obtains from Eq.(17) $(Z = 1/\alpha_s)$:

$$Z(\mu^2) = Z(Q^2) - \frac{b_o}{4\,\pi}\log(Q^2/\mu^2) - \Delta\,\log\left(\frac{Z(Q^2) + \Delta}{Z(\mu^2) + \Delta}\right). \tag{19}$$

We use as an input the value:

$$\alpha_s(M_W^2) = 0.118\,, \tag{20}$$

which corresponds to: $\Lambda^{(5)}_{\overline{MS}} \simeq 200\,MeV$, $\Lambda^{(4)}_{\overline{MS}} \simeq 300\,MeV$. One obtains then from Eqs.(19),(20) [5]:

$$\alpha_s(\mu = M_b = 5.04\,GeV) \simeq 0.204,\ \alpha_s(\mu = M_c = 1.65\,GeV) \simeq 0.310, \tag{21}$$

and the coefficients C_i are equal to:

$$C_-(M_c) \simeq 1.770,\quad C_+(M_c) \simeq 0.762,$$
$$C_1(M_c) \simeq 1.266,\quad C_2(M_c) \simeq -0.504. \tag{22}$$

The decay width of a hadron containing a heavy quark:

$$\Gamma = \frac{1}{2\,M_H}\int d\,x < M_H|\,H_W(x)\,H_W(0)\,|M_H >$$
$$\equiv \frac{1}{2M_H} < H|\,L_{eff}(0)\,|H >, \tag{23}$$

can be represented in the form of the operator expansion [9]:

$$L_{eff} = \sum C_i\,O_i(0)\,, \tag{24}$$

where the leading term is the dimension 3 operator $\bar{Q}Q$ which describes (at $M_Q \to \infty$) the free quark decay, fig.1. The next term is the dimension 5 operator $\bar{Q}\sigma GQ$ which describes an emission of a soft gluon, fig.2, etc.

[5] The quark masses are found below.

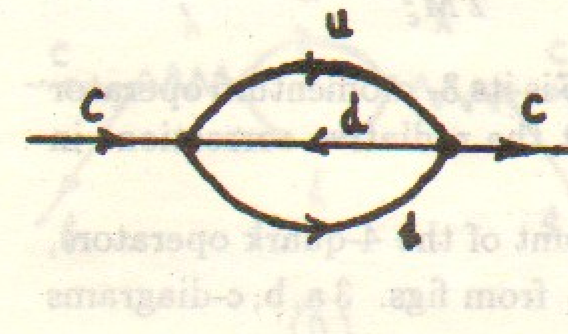


Fig. 1. The Born contribution.

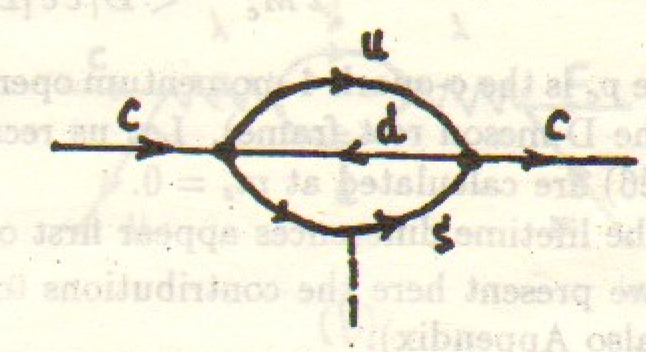


Fig. 2. The soft gluon emission giving rise the correction $O(1/M_Q^2)$.

If we confine ourselves temporarily to these terms only, then all $D^{o,\pm,s}$-mesons will have equal decay widths, and (on account of the radiative correction [21]) it can be represented in the form [9]:

$$\Gamma^o_{nl} \simeq \Gamma_o z_o \left(\frac{2\,C_+^2 + C_-^2}{3}\right)\frac{< D|\,\bar{c}\,c\,|D >}{2\,M_D}\left[I_{rad}\,\left(1 - 2\,\frac{z_1}{z_o}\Delta_G\right) + \right.$$
$$\left. 4\frac{z_2}{z_o}\left(\frac{C_-^2 - C_+^2}{2\,C_+^2 + C_-^2}\right)\Delta_G\right]. \tag{25}$$

Here:

$$I_{rad} = \left(\frac{2\,C_+^2}{2\,C_+^2 + C_-^2}\right)\left[1 - \frac{2}{3}\,\pi\,\alpha_s(M_c^2) + \frac{43}{12}\,\frac{\alpha_s(M_c^2)}{\pi}\right] +$$
$$\left(\frac{C_-^2}{2\,C_+^2 + C_-^2}\right)\left[1 - \frac{2}{3}\,\pi\,\alpha_s(M_c^2) + \frac{25}{3}\,\frac{\alpha_s(M_c^2)}{\pi}\right], \tag{26}$$

$$z_o = (1 - 8x + 8x^3 - x^4 - 12\,x^2\,\log x)\,,$$
$$z_1 = (1 - x)^4\,,\ z_2 = (1 - x)^3\,,\ x = \frac{m_s^2}{M_c^2}, \tag{27}$$

$$\Gamma_o = \frac{G_F^2\,M_c^5}{64\,\pi^3},\quad \frac{< D|\,\bar{c}\,c\,|D >}{2\,M_D} \simeq \left(1 + \frac{1}{2}\,\Delta_G - \Delta_K\right), \tag{28}$$

$$\Delta_G = \frac{1}{M_c^2}\frac{< D|\bar{c}(M_c^2 - p_c^2)c|D >}{< D|\bar{c}c|D >} = \frac{1}{M_c^2}\frac{< D|\bar{c}\frac{i}{2}g_s\sigma_{\mu\nu}G^a_{\mu\nu}\frac{\lambda^a}{2}c|D >}{< D|\bar{c}c|D >}, \tag{29}$$

$$\Delta_K = \frac{1}{2\,M_c^2}\,\frac{< D|\,\bar{c}\,(\vec{p}^2)\,c\,|D >}{< D|\,\bar{c}\,c\,|D >} \equiv \frac{<(\vec{p}^2)_c>}{2\,M_c^2}\,, \qquad (30)$$

where p_c is the c-quark 4-momentum operator, $\vec{p}$ is its 3- momentum operator (in the D-meson rest frame). Let us recall that the radiative corrections in Eq.(26) are calculated at $m_s = 0$.

The lifetime differences appear first on account of the 4-quark operators, and we present here the contributions to $\delta L^{(c)}_{eff}$ from figs. 3 a, b, c-diagrams (see also Appendix):

$$\delta L^c_{eff}(M_c) = \frac{G_F^2\,|V_{cs}|^2}{2\pi}\,\{g_{\mu\nu}\,\bar{\lambda}^2\,L^d_{\mu\nu} + T_{\mu\nu}(L^u_{\mu\nu} + L^s_{\mu\nu})\}_{\mu=M_c} + \cdots, \qquad (31)$$

$$L^d_{\mu\nu} = \Big\{\, S^o_d\,(\bar{c}\,\Gamma_\mu\,d)\,(\bar{d}\,\Gamma_\nu\,c) + O^o_d\,(\bar{c}\,\Gamma_\mu\frac{\lambda^a}{2}\,d)\,(\bar{d}\,\Gamma_\nu\frac{\lambda^a}{2}\,c)\Big\}\,, \qquad (32)$$

$$L^u_{\mu\nu} = \Big\{\, S^o_u\,(\bar{c}\,\Gamma_\mu\,u)\,(\bar{u}\,\Gamma_\nu\,c) + O^o_u\,(\bar{c}\,\Gamma_\mu\frac{\lambda^a}{2}\,u)\,(\bar{u}\,\Gamma_\nu\frac{\lambda^a}{2}\,c)\Big\}\,, \qquad (33)$$

$$L^s_{\mu\nu} = \Big\{\, S^o_s\,(\bar{c}\,\Gamma_\mu\,s)\,(\bar{s}\,\Gamma_\nu\,c) + O^o_s\,(\bar{c}\,\Gamma_\mu\frac{\lambda^a}{2}\,s)\,(\bar{s}\,\Gamma_\nu\frac{\lambda^a}{2}\,c)\Big\}\,. \qquad (34)$$

$$T_{\mu\nu} = \tfrac{1}{3}\,(\lambda_\mu\,\lambda_\nu - \lambda^2\,g_{\mu\nu})\,, \qquad (35)$$

where λ (or $\bar{\lambda}$) is the total 4-momentum of the integrated quark pair. It can be read off from each diagram and differ from p_c by the spectator quark momenta.

The coefficients S^o_i and O^o_i are:

$$S^o_d = \frac{1}{3}\,(C_1^2 + C_2^2) + 2\,C_1\,C_2\,, \quad O^o_d = 2\,(C_1^2 + C_2^2)\,, \qquad (36)$$

$$S^o_u = 3(C_2 + \frac{1}{3}C_1)^2,\ \ O^o_u = 2C_1^2,\ \ S^o_s = 3(C_1 + \frac{1}{3}C_2)^2,\ \ O^o_s = 2C_2^2, \qquad (37)$$

while C_1, C_2 are given in Eq.(22). Therefore:

$$\begin{pmatrix} S^o_d & O^o_d \\ S^o_u & O^o_u \\ S^o_s & O^o_s \end{pmatrix}_{\mu=M_c} \simeq \begin{pmatrix} -0.66 & 3.71 \\ 0.02 & 3.20 \\ 3.61 & 0.51 \end{pmatrix}$$

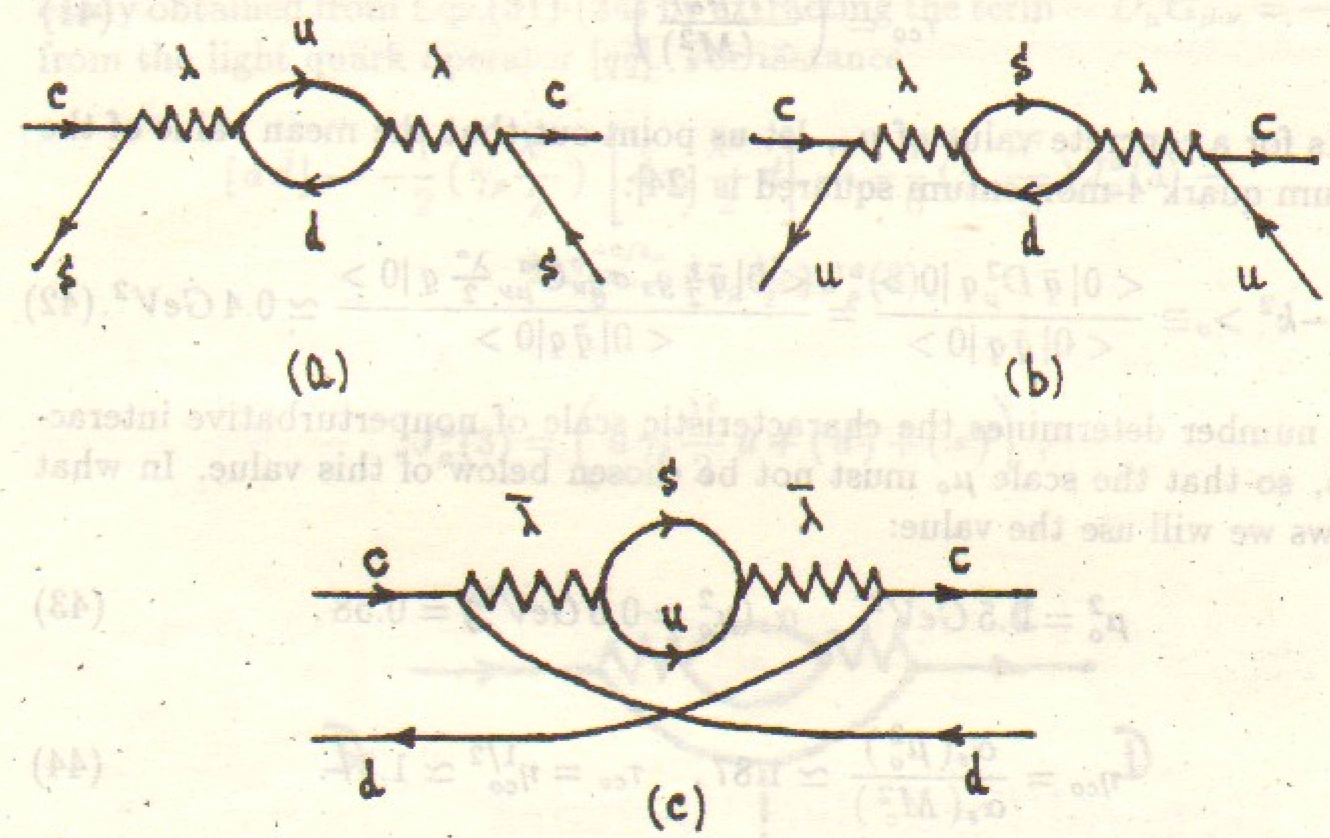


Fig. 3. The diagrams contributing into the 4-fermion operators.

The leading contribution at $M_c \to \infty (\lambda \simeq \bar{\lambda} \simeq p_c \simeq M_c)$ in Eqs.(31)-(34) coincides with those obtained before in [5], [6], [7]. As will be shown below, however, it is important for calculations with charmed mesons to account for nonzero momenta of the spectator quarks: $\lambda \neq \bar{\lambda} \neq M_c$.

The operators entering $\delta L^{(c)}_{eff}$ are normalized at the point $\mu = M_c$. So, it is reasonable to separate out explicitly the "hybrid" log effects [7],[22],[23] by renormalizing to the point $\mu = \mu_o$, before the calculation of matrix elements. The 4-quark operators in δL_{eff} are renormalized as [22] (we neglect possible changes in the renormalization formulae originating from the presense of the spectator quark momentum operators in $\bar{\lambda}$) :

$$(S_{\mu\nu})_{\mu=M_Q} = \Big[(\tau_{co} - \frac{\tau_{co}-1}{9})\,S_{\mu\nu} - \frac{2}{3}\,(\tau_{co}-1)\,O_{\mu\nu}\Big]_{\mu=\mu_o}\,, \qquad (38)$$

$$(O_{\mu\nu})_{\mu=M_Q} = \Big[(1 + \frac{\tau_{co}-1}{9})\,O_{\mu\nu} - \frac{4}{27}\,(\tau_{co}-1)\,S_{\mu\nu}\Big]_{\mu=\mu_o}\,, \qquad (39)$$

$$S^q_{\mu\nu} = (\bar{c}\,\Gamma_\mu\,q)\,(\bar{q}\,\Gamma_\nu\,c)\,, \quad O^q_{\mu\nu} = (\bar{c}\,\Gamma_\mu\frac{\lambda^a}{2}\,q)\,(\bar{q}\,\Gamma_\nu\frac{\lambda^a}{2}\,c)\,, \qquad (40)$$

$$\tau_{co} = \left(\frac{\alpha_s(\mu_o^2)}{\alpha_s(M_c^2)} \right)^{9/2b_o} . \tag{41}$$

As for a concrete value of μ_o, let us point out that the mean value of the vacuum quark 4-momentum squared is [24]:

$$< -k^2 >_o \equiv \frac{< 0|\, \bar{q}\, D_\mu^2 q\, |0 >}{< 0|\, \bar{q}\, q\, |0 >} = \frac{< 0|\, \bar{q} \frac{i}{2} g_s\, \sigma_{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2}\, q\, |0 >}{< 0|\, \bar{q}\, q |0 >} \simeq 0.4\, GeV^2 . \tag{42}$$

This number determines the characteristic scale of nonperturbative interactions, so that the scale μ_o must not be chosen below of this value. In what follows we will use the value:

$$\mu_o^2 = 0.5\, GeV^2, \quad \alpha_s(\mu_o^2 = 0.5\, GeV^2) = 0.58 . \tag{43}$$

$$\eta_{co} = \frac{\alpha_s(\,\mu_o^2\,)}{\alpha_s(\,M_c^2\,)} \simeq 1.87\,, \quad \tau_{co} = \eta_{co}^{1/2} \simeq 1.37 . \tag{44}$$

Now, after renormalization $\delta L_{eff}^{(c)}$ has the form:

$$\delta L_{eff}^{(c)}(\mu_o \equiv \Delta L(\mu_o) + L_{PNV}^{(c)}(\mu_o) , \tag{45}$$

$$\Delta L(\mu_o) = \frac{G_F^2}{2\,\pi} \left\{ T_{\mu\nu}\, L_{\mu\nu}^u + \Lambda^d + \Lambda^s \right\}_{\mu_o} , \tag{46}$$

where:

$$\Lambda^d = [\, |V_{ud}|^2\, g_{\mu\nu}\, \bar{\lambda}^2\, L_{\mu\nu}^d + |V_{cd}|^2\, T_{\mu\nu}\, (\, S_s\, S_{\mu\nu}^d + O_s\, O_{\mu\nu}^d\,)] , \tag{47}$$

$$\Lambda^s = [\, |V_{cs}|^2\, T_{\mu\nu}\, L_{\mu\nu}^s + |V_{us}|^2\, g_{\mu\nu}\, \bar{\lambda}^2\, (\, S_d\, S_{\mu\nu}^s + O_d\, O_{\mu\nu}^s\,)] , \tag{48}$$

and the coefficients are:

$$\begin{pmatrix} S_d & O_d \\ S_u & O_u \\ S_s & O_s \end{pmatrix}_{\mu=\mu_o} \simeq \tau_{co} \begin{pmatrix} -0.73 & 2.93 \\ -0.11 & 2.43 \\ 3.48 & -0.26 \end{pmatrix} \simeq \begin{pmatrix} -1.07 & 4.02 \\ -0.15 & 3.33 \\ 4.76 & -0.36 \end{pmatrix} .$$

The "penguin nonvalence" (PNV) term, $L_{PNV}^{(c)}$, originates from the contribution of the diagram in fig.4. With logarithmic accuracy L_{PNV} can be

easily obtained from Eqs.(31)-(34) by extracting the term $\sim D_\mu G_{\mu\nu} = -g_s\, J_\nu$ from the light quark operator $[q\bar{q}]$. For instance:

$$[\, d\, \bar{d}\,] \to -\frac{1}{2} (\, \gamma_\rho \frac{\lambda^a}{2}\,) \left[\bar{d}\, \gamma_\rho \frac{\lambda^a}{2}\, d \right] \to -\frac{1}{6} (\, \gamma_\rho \frac{\lambda^c}{2}\,)\, J_\rho^a(3) \to \frac{1-\eta_{co}^{-2/b_o}}{6} (\, \gamma_\rho \frac{\lambda^c}{2}\,)\, J_\rho^a(3) , \tag{49}$$

$$J_\rho^a(3) = \left(\bar{u}\, \gamma_\rho \frac{\lambda^c}{2}\, u + (\, d\,) + (\, s\,) \right) , \tag{50}$$

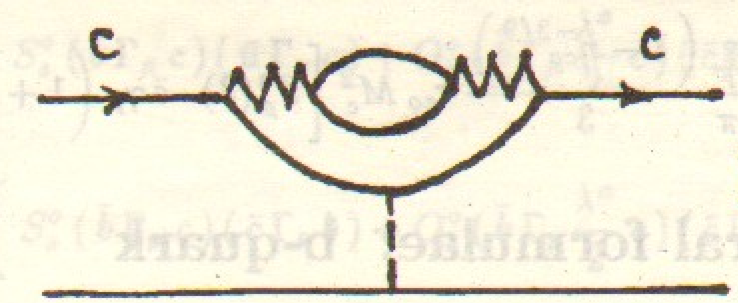


Fig. 4. The nonvalence factorizable penguin contribution (PNV).

where the last step in Eq.(49) describes the evolution of J_ρ^a from M_c to μ_o. Proceeding in this way (and using $\lambda \simeq \bar{\lambda} \simeq p_c$, $\lambda^2 \simeq \bar{\lambda}^2 \simeq M_c^2$, which is justified for these contributions, see fig.4) we obtain:

$$L_{PNV}^{(c)}(\mu_o) \simeq \frac{G_F^2}{2\,\pi} (1 - \eta_{co}^{-2/9})\, M_c^2\, J_\rho^a(3) \left[\bar{c}\, \gamma_\rho \frac{\lambda^a}{2} (\, N_v + N_a\, \gamma_5)\, c \right]_{\mu_o} , \tag{51}$$

$$N_v = \frac{1}{3} (\, A_u + A_s - 2\, A_d\,) \simeq 2.54\,, \tag{52}$$

$$N_a = \frac{1}{3} \left(\frac{1}{3}\, A_u + \frac{1}{3}\, A_s - 2\, A_d \right) \simeq 1.62\,, \tag{53}$$

$$A_i = (\, S_i - \frac{1}{6}\, O_i\,) . \tag{54}$$

For the semileptonic decays, δL_{eff}^{lept} can be easily obtained from Eqs.(45)-(48),(51) by corresponding replacements and has the form:

$$\delta L_{eff}^{lept} \equiv \Delta L^{lept} + L_{PNV}^{lept}\,, \tag{55}$$

$$\Delta L^{lept} \simeq$$
$$\frac{G_F^2}{2\pi} T_{\mu\nu} \left[|V_{cs}|^2 \left(1.327\, \bar c\,\Gamma_\mu s \cdot \bar s\,\Gamma_\nu c - 0.245\, \bar c\,\Gamma_\mu \tfrac{\lambda^a}{2} s \cdot \bar s\,\Gamma_\nu \tfrac{\lambda^a}{2} c \right) + |V_{cd}|^2 \left(1.327\, \bar c\,\Gamma_\mu d \cdot \bar d\,\Gamma_\nu c - 0.245\, \bar c\,\Gamma_\mu \frac{\lambda^a}{2} d \cdot \bar d\,\Gamma_\nu \frac{\lambda^a}{2} c \right) \right]_{\mu_o}, \tag{56}$$

$$L_{PNV}^{lept} \simeq \frac{G_F^2}{2\pi} \frac{\left(1 - \eta_{co}^{-2/9}\right)}{3} \tau_{co}\, M_c^2 \left[J_\rho^a(3) \cdot \bar c \gamma_\rho \left(1 + \frac{1}{3}\gamma_5\right) \frac{\lambda^a}{2} c \right]_{\mu_o}. \tag{57}$$

3b. General formulae: b-quark

The relevant part of the weak Hamiltonian has the form:

$$H_W^b(\mu = M_b) = \frac{G_F}{\sqrt2} V_{cb} V_{ud}^* \left\{ C_1(M_b)\, \bar c\,\Gamma_\mu b \cdot \bar d\,\Gamma_\mu u + C_2(M_b)\, \bar d\,\Gamma_\mu b \cdot \bar c\,\Gamma_\mu u \right\} + \cdots \tag{58}$$

$$\alpha_s(\mu = M_b = 5.04\,GeV) \simeq 0.204\,, \tag{59}$$

$$C_-(M_b) \simeq 1.385\,, \quad C_+(M_b) \simeq 0.855$$
$$C_1(M_b) \simeq 1.120\,, \quad C_2(M_b) \simeq -0.265\,. \tag{60}$$

For the beauty-hadrons one can obtain δL_{eff}^b from δL_{eff}^c by evident replacements:

$$\delta L_{eff}^b(M_b) = \frac{G_F^2 |V_{cb}|^2}{2\pi} \left\{ (\bar L_u + L_d + L_c) + (\bar L_c + L_s + \widetilde L_c) \right\}_{\mu = M_b} + \cdots \tag{61}$$

$$\bar L_u = g_{\mu\nu}\, \bar\lambda^2 \left(1 - \frac{M_c^2}{\bar\lambda^2}\right)^2 \left\{ S_d^o (\bar b\,\Gamma_\mu u)(\bar u\,\Gamma_\nu b) + O_d^o (\bar b\,\Gamma_\mu \tfrac{\lambda^a}{2} u)(\bar u\,\Gamma_\nu \tfrac{\lambda^a}{2} b) \right\}, \tag{62}$$

$$\bar L_c = g_{\mu\nu}\, \bar\lambda^2 \left(1 - \frac{M_c^2}{\bar\lambda^2}\right)^2 \left\{ S_d^o (\bar b\,\Gamma_\mu c)(\bar c\,\Gamma_\nu b) + O_d^o (\bar b\,\Gamma_\mu \tfrac{\lambda^a}{2} c)(\bar c\,\Gamma_\nu \tfrac{\lambda^a}{2} b) \right\}, \tag{63}$$

$$L_d = T_{\mu\nu}^{(c)}(\lambda) \left\{ S_u^o (\bar b\,\Gamma_\mu d)(\bar d\,\Gamma_\nu b) + O_u^o (\bar b\,\Gamma_\mu \frac{\lambda^a}{2} d)(\bar d\,\Gamma_\nu \frac{\lambda^a}{2} b) \right\}, \tag{64}$$

$$L_s = T_{\mu\nu}^{(\bar c c)}(\lambda) \left\{ S_u^o (\bar b\,\Gamma_\mu s)(\bar s\,\Gamma_\nu b) + O_u^o (\bar b\,\Gamma_\mu \frac{\lambda^a}{2} s)(\bar s\,\Gamma_\nu \frac{\lambda^a}{2} b) \right\}, \tag{65}$$

$$L_c = T_{\mu\nu}(\lambda) \left\{ S_s^o (\bar b\,\Gamma_\mu c)(\bar u\,\Gamma_\nu c) + O_s^o (\bar b\,\Gamma_\mu \frac{\lambda^a}{2} c)(\bar c\,\Gamma_\nu \frac{\lambda^a}{2} b) \right\}, \tag{66}$$

$$\widetilde L_c = T_{\mu\nu}^{(c)}(\lambda) \left\{ S_s^o (\bar b\,\Gamma_\mu c)(\bar c\,\Gamma_\nu b) + O_s^o (\bar b\,\Gamma_\mu \frac{\lambda^a}{2} c)(\bar c\,\Gamma_\nu \frac{\lambda^a}{2} b) \right\}, \tag{67}$$

where the dots in the above formulae denote the Cabibbo-suppressed contributions. Here ($x = M_c^2/\lambda^2$):

$$T_{\mu\nu}(\lambda) = \frac{1}{3} \left(\lambda_\mu \lambda_\nu - \lambda^2 g_{\mu\nu}\right), \tag{68}$$

$$T_{\mu\nu}^{(c)}(\lambda) = \frac{1}{3}(1 - x)^2 \left[\left(\lambda_\mu \lambda_\nu - \lambda^2 g_{\mu\nu}\right)(1 + 2x) + \frac{3}{2} g_{\mu\nu} \lambda^2 x \right], \tag{69}$$

$$T_{\mu\nu}^{(\bar c c)}(\lambda) = \frac{1}{3}\sqrt{1 - 4x} \left[(1 + 2x)\lambda_\mu \lambda_\nu - (1 - x)\lambda^2 g_{\mu\nu} \right]. \tag{70}$$

The coefficients in Eqs.(61-(67)):

$$\begin{pmatrix} S_d^o & O_d^o \\ S_u^o & O_u^o \\ S_s^o & O_s^o \end{pmatrix}_{\mu = M_b} \simeq \begin{pmatrix} -0.15 & 2.65 \\ 0.03 & 2.51 \\ 3.20 & 0.14 \end{pmatrix},$$

become after the renormalization, $M_b \to \mu_o$:
i) for the terms L_d, L_s, $\bar L_u$, for which the renormalization factor is

$$\tau_{bo} = \eta_{bc}^{27/50} \tau_{co} \simeq 1.715\,, \quad \eta_{bc} = \frac{\alpha_s(M_c^2)}{\alpha_s(M_b^2)} \simeq 1.52\,, \tag{71}$$

$$\begin{pmatrix} S_d & O_d \\ S_u & O_u \\ S_s & O_s \end{pmatrix}_{\mu=\mu_o} \simeq \tau_{bo} \begin{pmatrix} -0.31 & 1.71 \\ -0.12 & 1.57 \\ 3.04 & -0.80 \end{pmatrix} \simeq \begin{pmatrix} -0.53 & 2.93 \\ -0.21 & 2.69 \\ 5.21 & -1.37 \end{pmatrix},$$

ii) for the terms L_c, $\bar{L}_c$, $\widetilde{L}_c$, for which the renormalization stops at $\mu = M_c$ and the renormalization factor is:

$$\tau_{bc} = \eta_{bc}^{27/50} \simeq 1.253: \qquad (72)$$

$$\begin{pmatrix} S_d & O_d \\ S_s & O_s \end{pmatrix}_{\mu=M_c} \simeq \tau_{bc} \begin{pmatrix} -0.23 & 2.20 \\ 3.12 & -0.32 \end{pmatrix} \simeq \begin{pmatrix} -0.29 & 2.75 \\ 3.91 & -0.40 \end{pmatrix}.$$

The corresponding term $L^{(b)}_{PNV}$ has the form:

$$L^{(b)}_{PNV} \simeq \frac{G_F^2\,|V_{cb}|^2}{2\pi} M_b^2 \left\{ J^a_\rho(3) \left[\bar{b} \frac{\lambda^a}{2} \gamma_\rho (A + B\gamma_5)\, b \right] + J^a_\rho(c) \left[\bar{b} \tfrac{\lambda^a}{2} \gamma_\rho (C + D\gamma_5)\, b \right] \right\}, \qquad (73)$$

$$A = \lambda_1 (A^o_s - A^o_d) + \lambda_2 (A^o_u - A^o_d) \simeq 0.40; \qquad (74)$$

$$B = \lambda_1 \left(\frac{1}{3} A^o_s - A^o_d \right) + \lambda_2 \left(\frac{1}{3} A^o_u - A^o_d \right) \simeq 0.27, \qquad (75)$$

$$C = \lambda_3 (A^o_s + A^o_u - 2\,A^o_d) \simeq 0.31;\; D = \lambda_3 \left(\frac{1}{3} A^o_s + \frac{1}{3} A^o_u - 2\,A^o_d \right) \simeq 0.17 \qquad (76)$$

$$\lambda_3 = \frac{1 - \eta_{bc}^{-8/25}}{2} \tau_{bc}\,, \quad \lambda_1 = \lambda_3\, \eta_{co}^{-2/9}\, \tau_{co}\,,$$
$$\lambda_2 = \tfrac{2}{3} \left(1 - \tfrac{1 + 3\eta_{bc}^{-8/25}}{4} \eta_{co}^{-2/9} \right) \tau_{bo}\,, \qquad (77)$$

$$A^o_i = \left(S^o_i - \frac{1}{6} O^o_i \right), \quad J^a_\rho(3) = \bar{u} \frac{\lambda^a}{2} \gamma_\rho\, u + (d) + (s), \quad J^a_\rho(c) = \bar{c} \frac{\lambda^a}{2} \gamma_\rho\, c. \qquad (78)$$

For the leptonic decays, $B \to l + \nu + X$, $l = e, \mu$, we have:

$$\delta L^{lept}_{eff} = \Delta L^{lept} + L^{lept}_{PNV}\,, \qquad (79)$$

$$\Delta L^{lept} \simeq \frac{G_F^2\,|V_{cb}|^2}{2\pi} T_{\mu\nu}(\lambda) \left[1.225\, \bar{b}\, \Gamma_\mu c \cdot \bar{c}\, \Gamma_\nu b - 0.169\, \bar{b}\, \Gamma_\mu \frac{\lambda^a}{2} c \cdot \bar{c}\, \Gamma_\nu \frac{\lambda^a}{2} b \right]_{\mu_o}, \qquad (80)$$

$$L^{lept}_{PNV} \simeq \frac{G_F^2\,|V_{cb}|^2}{2\pi} \frac{\left(1 - \eta_{bc}^{-8/25}\right)}{4} \tau_{bo}\, \eta_{co}^{-2/9}\, M_b^2 \times$$
$$\left[\bar{b}\, \gamma_\rho \left(1 + \frac{1}{3}\gamma_5 \right) \frac{\lambda^a}{2} b \right] \cdot \left[J^a_\rho(3) + \tau_{co}^{-1}\, \eta_{co}^{2/9}\, J^a_\rho(c) \right]_{\mu_o} \simeq$$
$$\frac{G_F^2\,|V_{cb}|^2}{2\pi} 0.046\, M_b^2 \left[\bar{b}\, \gamma_\rho \left(1 + \frac{1}{3}\gamma_5 \right) \frac{\lambda^a}{2} b \right] \cdot \left[J^a_\rho(3) + 0.85\, J^a_\rho(c) \right]_{\mu_o}. \qquad (81)$$

4. $D^+ \to l\nu + X$. Determination of M_c

The purpose of this section is to find out the value of the c-quark "hard pole mass" (see sect.2), M_c, using the experimental data for the semileptonic width [25], [26]:

$$\Gamma_{sl}(D^+ \to l\nu + X) = (1.06 \pm 0.11) \cdot 10^{-13}\, GeV\,. \qquad (82)$$

The expression for Γ_{sl} can be represented in the form (see [9] and sect.3a):

$$\Gamma_{sl}(\, D^+ \,) \simeq \Gamma^o_{sl} \left\{ z_o\, I_{rad} \frac{< D|\,\bar{c}\, c\,|D >}{2\, M_D} \left[1 - 2\frac{z_1}{z_o} \Delta_G \right] + \delta^{(c)}_{lept} \right\}, \qquad (83)$$

$$\Gamma^o_{sl} = \frac{G_F^2\, M_c^5}{192\, \pi^3}\,, \qquad I_{rad} \simeq \left[1 - \frac{2}{3} \frac{\alpha_s(M_c^2)}{\pi} f_o\left(\frac{m_s^2}{M_c^2}\right) \right], \qquad (84)$$

where $\delta^{(c)}_{lept}$ in Eq.(83) is the contribution of δL^{lept}_{eff} into Γ_{sl}, and the explicit form of $f_o(x)$ can be found in [27],[28]. We use below [6]:

$$m_s \simeq 150\, MeV\,, \quad < (\vec{p}^{\,2})_c > \simeq 0.3\, GeV^2\,, \quad \Delta_G \simeq \frac{3}{4} \frac{M^2_{D^*} - M^2_D}{M_c^2}. \qquad (85)$$

[6]From our viewpoint, the value: $< \vec{p}^{\,2} > \simeq 0.6\, GeV^2$ available in the literature [29] is overestimated. Let us recall (see Eq.(42)) that the mean value of the vacuum quark 4-momentum squared is: $< -k_\mu^2 >_o = 4/3 < \vec{k}^2 >_o \simeq 0.4\, GeV^2$, and the quarks inside the pion have their momenta on the average somewhat less than those of the vacuum quarks [30]. Let us point out also that $< \vec{p}^{\,2} >$ enters here to $< D|\bar{c}c|D >$ only and plays no essential role.

The value of M_c can be determined now from a comparison of Eqs.(82) and (83). Because the dependence of Γ_{sl} on M_c is highly nonlinear, it is more convenient to proceed in an opposite way. Namely, let us show that Eq.(83) reproduces the experimental value at $M_c \simeq 1.65\, GeV$. We have:

$$\alpha_s(M_c^2) \simeq 0.310\,, \quad z_o \simeq 0.938\,, \quad z_1 \simeq 0.967\,, \tag{86}$$

$$\Delta_G \simeq 0.15\,, \quad \Delta_K \simeq 0.055\,, \quad f_o \simeq 3.25\,, \tag{87}$$

$$\Gamma_{sl}^o \simeq 2.80 \cdot 10^{-13}\, GeV\,, \tag{88}$$

$$I_{rad} \simeq 0.786\,, \quad \frac{< D|\bar{c}\, c\,|D>}{2\, M_D} \simeq 1.02\,, \quad \left[1 - 2\frac{z_1}{z_o}\Delta_G\right] \simeq 0.691\,. \tag{89}$$

Substituting all this into Eq.(83) one obtains:

$$\Gamma_{sl}(D^+) \simeq \Gamma_{sl}^o\,(0.52 + \delta_{lept}^{(c)})\,. \tag{90}$$

The quantity $\delta_{lept}^{(c)}$ is calculated in sect.11 and is:

$$\delta_{lept}^{(c)} \simeq -0.13\,. \tag{91}$$

So:

$$\Gamma_{sl}(D^+) \simeq 0.39\, \Gamma_{sl}^o \simeq 1.09 \cdot 10^{-13}\, GeV\,, \tag{92}$$

in agreement with Eq.(82).

5. Mass formulae. Determination of M_b and $\bar{\Lambda}$

To find out the value of the b-quark "hard pole" mass, M_b, we use the mass formula which has the form (see, for instance, [17] and sect.2):

$$M_D = M_c + \bar{\Lambda} + \frac{1}{M_c}\delta m^2 + O\left(\frac{\Lambda_{QCD}^3}{M_c^2}\right), \tag{93}$$

$$\delta m^2 \equiv \frac{1}{2}\frac{<D(p)|\bar{c}\,\Delta\, c\,|D(p)>}{2\, M_D} \simeq \frac{1}{2} < \vec{p}_c^{\,2} > -\frac{3}{8}\,(M_{D^*}^2 - M_D^2)\,, \tag{94}$$

$$\Delta = (i\vec{D})^2 - \frac{i}{2} g_s \sigma_{\mu\nu} G_{\mu\nu}^a \frac{\lambda^a}{2}\,. \tag{95}$$

As it was pointed out in sect.2, the difference between the "hard pole mass", M_c, and the pole mass, M_p, can be absorbed by a redefinition of Λ_o appearing in the matrix element:

$$\frac{1}{2\, M_D} < D|\, H_{light}\, |D> = \Lambda_o \left(1 + O(\frac{\Lambda_{QCD}^2}{M_c^2})\right), \tag{96}$$

or, equivalently, through the trace anomaly:

$$\frac{1}{2\, M_D} < D|\, \frac{\beta(\alpha)}{2\pi\alpha} G_{\mu\nu}^2\, |D> = \Lambda_o + \frac{2\, \delta\tilde{m}^2}{M_c} + O(\frac{\Lambda_{QCD}^3}{M_c^2})\,, \tag{97}$$

and by a redefinition of the next term, $(1/4M_D) < D|\bar{c}\Delta\, c\,|D> = \delta\tilde{m}^2 \to \delta m^2$, etc.

Turning now to concrete numbers, we use in Eq.(93):

$$M_c \simeq 1.65\, GeV\,, \quad M_D \simeq 1.867\, GeV\,, \\ < \vec{p}_c^{\,2} > \simeq 0.3\, GeV^2\,, \quad (M_{D^*}^2 - M_D^2) \simeq 0.543\, GeV^2\,, \tag{98}$$

and obtain:

$$1.867\, GeV \simeq 1.65\, GeV + \bar{\Lambda} + 91\, MeV - 123\, MeV\,, \tag{99}$$

and therefore:

$$\bar{\Lambda} \simeq 250\, MeV\,. \tag{100}$$

Using now this value of $\bar{\Lambda}$ and:

$$M_B \simeq M_b + \bar{\Lambda} + \frac{1}{2M_b} < \vec{p}_b^{\,2} > - \frac{3}{8M_b}(M_{B^*}^2 - M_B^2) + O(\frac{\Lambda_{QCD}^3}{M_b^2})\,, \tag{101}$$

$$M_B \simeq 5.28\, GeV\,, \quad < \vec{p}_b^{\,2} > \simeq 0.3\, GeV^2\,, \quad (M_{B^*}^2 - M_B^2) \simeq 0.488\, GeV^2\,, \tag{102}$$

one obtains from Eq.(101) [7]:

$$M_b \simeq 5.04\, GeV\,. \tag{103}$$

[7] Really, it is clear that we need not $\bar{\Lambda}$ at all to determine M_b.

6. $B \to l\nu + X$. Determination of $|V_{cb}|$

The semileptonic decay width "$B \to e\nu + X$" is obtained from Eq.(83) by evident replacements. We have:

$$\Gamma^o_{sl} = \frac{G_F^2 M_b^5 |V_{cb}|^2}{192\,\pi^3} \simeq 1.19 \cdot 10^{-13}\, GeV \left|\frac{V_{cb}}{0.040}\right|^2 , \qquad (104)$$

$$\alpha_s(M_b^2) \simeq 0.204\,, \quad z_o \simeq 0.460\,, \quad z_1 \simeq 0.635\,, \quad f_o \simeq 2.46\,, \qquad (105)$$

$$\frac{< B|\bar{b}\,b|B >}{2\,M_B} \simeq 1.00\,, \quad \left[1 - 2\,\frac{z_1}{z_o}\,\Delta_G\right] \simeq 0.96\,. \qquad (106)$$

$$I_{rad} \simeq \left[1 - \frac{2}{3}\,\frac{\alpha_s(M_b^2)}{\pi}\, f_o(\frac{M_c^2}{M_b^2})\right] \simeq 0.893\,. \qquad (107)$$

So, (see Eq.(83):

$$\Gamma(B \to e\nu + X) \simeq \Gamma^o_{sl}\left(0.394 + \delta^{(b)}_{lept}\right). \qquad (108)$$

The quantity $\delta^{(b)}_{lept}$ is calculated in sect.11:

$$\delta^{(b)}_{lept} \simeq -2 \cdot 10^{-3}, \qquad (109)$$

and is negligble. Therefore:

$$\Gamma(B \to e\nu + X) \simeq 0.47 \cdot 10^{-13}\, GeV \left|\frac{V_{cb}}{0.04}\right|^2 . \qquad (110)$$

If we take, for instance, [26] [8] :

$$\tau(B) = (1.6 \pm 0.04) \cdot 10^{-12}\, s\,, \quad \Gamma_{tot}(B) = (4.1 \pm 0.1) \cdot 10^{-13}\, GeV, \qquad (111)$$

[8] We use the LEP data for the b-quark lifetime and $Br(b \to e\nu + X)$. When comparing these with the $\Upsilon(4S)$ data, one sees that the absolute value of $\Gamma(B \to e\nu + X) \simeq \Gamma(b \to e\nu + X)$ is the same, while the LEP data give a smaller value for the total decay width. We prefer to use the LEP value for the following reasons. It gives the weighted average of the b-hadron widths. Because (see below) the B-meson widths are practically the same, and there is all the reason to expect that the b-baryon total widths are somewhat larger than those of the B-mesons (with the semileptonic widths being the same), the admixture of the b-baryons can only increase the weighted total decay width. So, the LEP data can be considered as giving the upper limit for the B-meson decay width and the lower limit for the semileptonic branching.

$$Br(B \to l\nu + X) = (11.4 \pm 0.5)\,\%\,, \qquad (112)$$

then:

$$\Gamma(B \to l\nu + X) = (0.47 \pm 0.03) \cdot 10^{-13}\, GeV \left[\frac{Br(B \to l\nu + X)}{11.4\%}\right]\left[\frac{1.6 \cdot 10^{-12} s}{\tau(B)}\right], \qquad (113)$$

and comparing with Eq.(110) we obtain:

$$|V_{cb}| \simeq 0.040 \left[\frac{Br(B \to l\nu + X)}{11.4\%}\right]^{1/2} \left[\frac{1.6 \cdot 10^{-12} s}{\tau(B)}\right]^{1/2} . \qquad (114)$$

7. Calculation of f_D, f_B

There is a large number of papers dealing with the calculation of the decay constants f_D and f_B with the help of the QCD sum rules. We point out here only early papers [31], [32], [33], and the paper [34] close in spirit to this work, were it was proposed to use the correlators of chiral currents in the QCD sum calculations.

Due to reasons which are explained below in detail (see also [34]), our approach here rests heavily on the use of the chiral currents correlators. One of the main difficulties which prevents the calculation of reliable results from the correlator of the pseudoscalar currents, is the very large radiative correction to the Born approximation for this correlator [35], [36], [37]. To avoide this difficulty, let us consider the following correlator of the chiral currents:

$$K_1(q^2) =$$
$$i \int d\,x\, e^{iqx} < 0|T\left\{\bar{Q}(x) i(1+\gamma_5) q(x)\,, \bar{q}(0) i(1+\gamma_5) Q(0)\right\}|0 > \equiv$$
$$i \int d\,x\, e^{iqx} < 0|T\left\{P(x)\,P^+(0) - S(x)\,S^+(0)\right\}|0 >,$$
$$P(x) = \bar{Q} i\gamma_5\, q(x)\,, \quad S(x) = \bar{Q}(x)\,q(x)\,. \qquad (115)$$

Because $K_1(q^2)$ is the difference of the pseudoscalar and scalar current correlators, the pure perturbative contributions cancel completely in all orders of the perturbation theory (in the chiral limit). On the other hand, there appear additional scalar state contributions in the spectral density. Let us emphasize, however, that the mass differences between the lowest lying pseudoscalar and scalar resonances are sufficiently large nevertheless, both for the D and B mesons. So (after the Borel transformation), the scalar meson contributions are sufficiently suppressed in the sum rules.

The diagrams giving the main contribution to $K_1(q^2)$ are shown in fig.5. The spectral density (for the D-meson) has the form:

$$\delta K(s) = r_D^2\, \delta(S - M_D^2) + \cdots , \quad r_D = f_D\, M_D^2 \,/\, M_c \,. \tag{116}$$

The sum rules obtained in a standard way have the form [9]:

$$r_D^2(\overline{M^2}) = 2\, M_c \, < 0|q\,\bar{q}|0 >_{\overline{M^2}} \, \Phi_c(M^2)\,, \tag{117}$$

$$\Phi_c(M^2) = exp\left\{\frac{M_D^2 - M_c^2}{M^2}\right\}\left[1 - \frac{m_o^2\, M_c^2}{4\, M^4}\left(1 - 2\,\frac{M^2}{M_c^2}\right)\right], \tag{118}$$

$$\frac{m_o^2}{2} = \frac{< 0|\bar{q}\frac{i}{2}\sigma_{\mu\nu}\, G^a_{\mu\nu}\, \frac{\lambda^a}{2} q|0 >_{\overline{M^2}}}{< 0|\bar{q}\, q\,|0 >_{\overline{M^2}}} = \frac{< 0|\bar{q}\,(\,i\,D_\mu\,)^2\, q\,|0 >_{\overline{M^2}}}{< 0|\bar{q}\, q\,|0 >_{\overline{M^2}}} \,. \tag{119}$$

In Eqs.(117) and (119) $\overline{M^2} \simeq 1\, GeV^2$ (see below) is the normalization point of all operators and of the D-meson residue, r_D, and the number m_o^2 is also determined at this point. We will use for it the standard value determined previously [24] at this scale: $m_o^2 \simeq 0.8\, GeV^2$.

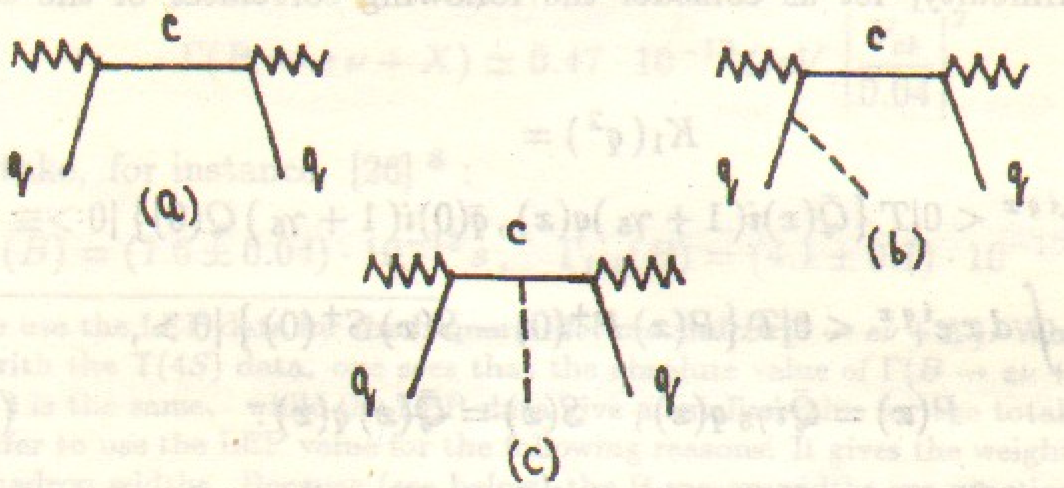


Fig. 5a,b,c. The diagrams contributing to the sum rule Eq.(118).

[9] We neglected in Eq.(118) the small contribution $\sim < \bar{q}\, q >^2$. The anomalous dimensions of the operators $\bar{q}D^2q$ and $\bar{q}q$ are respectively: $(-2/3b_o)$ and $(4/b_o)$.

The power corrections due to the figs.5b, c diagrams do not exceed $\simeq 35\%$ of the fig.5a contribution in the region $0.8\, GeV^2 \le M^2 \le 1.5\, GeV^2$ and, on the other hand, the values of M^2 are sufficiently small for the contributions of higher states in the spectral density to be exponentially suppressed. The function $\Phi_c(M^2)$ varies only slightly in this region and the characteristic value of M^2 (at the extremum of $\Phi_c(M^2)$) is:

$$\overline{M^2} \simeq 1.15\, GeV^2 , \quad \Phi_c(\overline{M^2}) \simeq 1.8 \,. \tag{120}$$

Because we need in what follows the value of f_D at the low normalization point μ_o, we renormalize now both r_D^2 and $< \bar{q}\, q >$ to this point and (because they have the same anomalous dimension) obtain:

$$r_D^2(\mu_o^2) \simeq 2\, M_c \, < 0|\, q\,\bar{q}\,|0 >_{\mu_o} 1.8 \,. \tag{121}$$

Finally, to obtain the concrete answer we use

$$< 0|\, q\,\bar{q}\,|0 >_{\mu_o} \simeq (\,0.25\, GeV\,)^3 , \quad \mu_o^2 \simeq 0.5\, GeV^2 \,, \tag{122}$$

for the value of the quark condensate at the low normalization point. So,

$$r_D(\mu_o^2 = 0.5\, GeV^2) \simeq 0.30\, GeV^2 , \quad f_D(\mu_o^2) \simeq 144\, MeV ,$$

$$f_D(M_c^2) = \left(\frac{\alpha_s(\mu_o^2)}{\alpha_s(M_c^2)}\right)^{2/9} f_D(\mu_o^2) \simeq 165\, MeV \,. \tag{123}$$

For the B-meson, the corresponding sum rule has the form:

$$r_B^2(\overline{M^2}) = 2\, M_b \, < 0|q\,\bar{q}|0 >_{\overline{M^2}} \, \Phi_b(M^2)\,, \tag{124}$$

$$\Phi_b(M^2) = exp\left\{\frac{M_B^2 - M_b^2}{M^2}\right\}\left[1 - \kappa\, \frac{m_o^2\, M_b^2}{4\, M^4}\left(1 - 2\,\frac{M^2}{M_b^2}\right)\right]. \tag{125}$$

The factor κ in Eq.(125) ($\overline{M^2} \simeq 4\, GeV^2$, $\alpha_s(4\, GeV^2) \simeq 0.284$) :

$$\kappa = \left(\frac{\alpha_s(\overline{M^2})}{\alpha_s(M_c^2)}\right)^{14/25} \left(\frac{\alpha_s(M_c^2)}{\alpha_s(1\, GeV^2)}\right)^{14/27} \simeq 0.81, \tag{126}$$

is due to different anomalous dimensions of the operators $\bar{q}ig_s\sigma Gq$ and $\bar{q}q$.

The corresponding interval (see above) of M^2 is: $3\, GeV^2 \le M^2 \le 5\, GeV^2$, the function $\Phi_b(M^2)$ varies only slightly in this interval, and the characteristic value of M^2 (at the extremum) is:

$$\overline{M^2} \simeq 4\, GeV^2 , \quad \Phi_b(\overline{M^2}) \simeq 1.53 \,. \tag{127}$$

Proceeding now in the same way as for the D-meson, we obtain:

$$r_B(\mu_o) \simeq 0.49\, GeV^2\,, \quad f_B(\mu_o) \simeq 89\, MeV\,,$$

$$f_B(M_b) = \left(\frac{\alpha_s(M_c^2)}{\alpha_s(M_b^2)}\right)^{6/25} \left(\frac{\alpha_s(\mu_o^2)}{\alpha_s(M_c^2)}\right)^{6/27} f_B(\mu_o) \simeq 113\, MeV\,. \quad (128)$$

It is of interest to compare the above results with those obtained in the static limit: $M_q \to \infty$. Let us define the constant f_o (a static analog of f_D) as:

$$f_D(\mu_o) = \frac{1}{\sqrt{M_c}}\, f_o\,, \quad (129)$$

$$M_D \simeq M_c + \bar\Lambda\,, \quad M^2 \equiv 2\, M_c\, E\,, \quad \bar\Lambda \simeq 250\, MeV\,. \quad (130)$$

Then, in the limit $M_c \to \infty$, the sum rule Eq.(118) takes the form [10]:

$$f_o^2 \simeq 2 < 0|q\,\bar q|0>_{\mu_o} \phi(E)\,, \quad \phi(E) = e^{\bar\Lambda/E}\left(1 - \frac{m_o^2}{16\, E^2}\right) \quad (131)$$

The corresponding interval (see above) of E is: $400\, MeV \leq E \leq 700\, MeV$, the function $\phi(E)$ varies only slightly in this region, and the characteristic value of E (at the extremum) is:

$$\overline{E} \simeq 550\, MeV\,, \quad \phi(\overline{E} \simeq 550\, MeV) \simeq 1.32\,. \quad (132)$$

Substituting Eq.(132) into Eq.(131) we have:

$$f_o \simeq 0.20\, GeV^{3/2}, \quad f_D(M_c) \simeq \left[\frac{\alpha_s(\mu_o^2)}{\alpha_s(M_c^2)}\right]^{2/9} \frac{1}{\sqrt{M_c}}\, f_o \simeq 180\, MeV. \quad (133)$$

Comparing Eq.(133) with Eq.(123) we see that the result obtained for f_D in the static limit does not differ considerably from those obtained for the real value of the c-quark mass.

Although we succeeded in using the correlator Eq.(115) in which both the Born contribution and all radiative corrections to it are absent, for the calculation of the decay constants f_D and f_B, the analogous trick, unfortunately, turns out to be useless for the calculation of nonfactorizable contributions to the matrix elements of the 4-quark operators which we will need in what follows. The reason is as follows. In the sum rules for the correlator Eq.(115) the

[10] Really, we drop out power corrections but keep the value of m_o^2 fixed.

leading contribution is proportional to the known quark condensate $< \bar q\, q >$, and the main correction is proportional to the condensate $< \bar q \sigma G q >$ which is also known. However, if we will try to calculate the matrix elements like (see below): $< D|\bar c \Gamma_\mu \frac{\lambda^a}{2} q \cdot \bar q \Gamma_\nu \frac{\lambda^a}{2} c|D >$ in an analogous way, then even the leading contribution is expressed through the high dimension condensate $< \bar q q \bar q q G G >$ which is unknown. Therefore, the only way to calculate such matrix elements with the help of the QCD sum rules is to use such interpolating currents that the leading contribution is pure perturbative, and the leading corrections are expressed then through the known low dimension condensates like $< G_{\mu\nu}^2 >$, etc.

As it was pointed out above, to deal with such sum rules we have to overcome in some way the difficulties originating from very large radiative corrections. We will describe now the way we used here to deal with this problem and which is used then in sect.9.

Let us return to a consideration of the decay constant f_D and consider now the correlator:

$$K_2(q^2) =$$
$$i \int d\,x\, e^{i\,q\,x} < O|T\left\{\bar Q(x)(1+\gamma_5)q(x),\, \bar q(0)(1-\gamma_5)\, Q\,(0)\right\}|O> \equiv$$
$$i \int d\,x\, e^{i\,q\,x} < O|T\left\{P(x)\, P^+(0) + S(x)\, S^+(0)\right\}|O>, \quad (134)$$

The sum rule obtained from Eq.(134) in a standard way has the form (in the chiral limit):

$$r_D^2(\mu_o) \simeq \frac{3}{4\pi^2}\int_{M_c^2}^{S_o} \frac{d\,S\,(\,S - M_c^2\,)^2}{S}\, exp\left\{\frac{M_D^2 - S}{M^2}\right\} \times$$
$$\left[1 + \tfrac{4}{3}\tfrac{\alpha_s(\mu_o^2)}{\pi} X\left(\tfrac{M_c^2}{S}\right) + O(\alpha_s^2)\right], \quad (135)$$

$$X(z) = \left[\frac{9}{4} + 2\, L(z) + \log(z)\, \log(1-z) + \frac{3}{2}\log\frac{z}{1-z} + \log\frac{1}{1-z} - \right.$$
$$\left. z\log\frac{z}{1-z} + \frac{z}{1-z}\log\frac{1}{z} - \frac{3}{4}\log\frac{M_c^2}{\mu_o^2}\right], \quad L(z) = -\int_0^z \frac{d\,t}{t}\log(1-t). \quad (136)$$

Let us emphasize that the chirality odd condensates like: $< \bar q q >$, $< \bar q \sigma G q >$ etc., give no contribution to the correlator Eq.(134). [11]

[11] We neglect the small contribution to Eq.(135) due to $< G^2 >$ and $< \bar q q >^2$.

Let us try now, as a first approximation, to neglect all radiative corrections in Eq.(135) and let us choose the value of S_o (the effective parameter which models the beginning of the perturbative continuum in a given correlator) in Eq.(135) to obtain a fit in M^2. There is a good fit in the standard region (see Eq.(120)): $0.8\,GeV^2 \le M^2 \le 1.5\,GeV^2$ at $S_o = 3.81\,GeV^2$, which gives:

$$r_D(\mu_o) \simeq 0.096\,GeV^2\,, \quad f_D(\mu_o) \simeq 45\,MeV\,. \tag{137}$$

For the B-meson, there is a good fit in the standard region $3\,GeV^2 \le M^2 \le 5\,GeV^2$ at $S_o = 28.8\,GeV^2$ which gives:

$$r_B(\mu_o) \simeq 0.19\,GeV^2\,, \quad f_B(\mu_o) \simeq 35\,MeV\,. \tag{138}$$

It is seen that the Born approximation to the sum rule Eq.(135) gives (at $M_c = 1.65\,GeV$, $M_b = 5.04\,GeV$) very small values for f_D, f_B (compare with Eqs.(123),(128)). Let us account now for the one loop correction to the Born approximation in the sum rule Eq.(135) and make anew the fits in M^2. We will obtain good fits both for D- and B-mesons in the standard regions of M^2-values at $S_o = 3.85\,GeV^2$ and $S_o = 28.9\,GeV^2$ respectively with the results:

$$r_D(\mu_o) \simeq 0.149\,GeV^2\,, \quad f_D(\mu_o) \simeq 71\,MeV\,. \tag{139}$$

$$r_B(\mu_o) \simeq 0.324\,GeV^2\,, \quad f_B(\mu_o) \simeq 59\,MeV\,. \tag{140}$$

It is seen from a comparison of Eqs.(139),(140) and Eqs.(137),(138) that if, in the Born approximation, we obtain the values of f_D, f_B which are $\simeq 3$ times smaller than the right values Eqs.(123), (128), on account of the first radiative correction the results increase $\simeq 1.6 - 1.7$ times. It is clear that, in such a situation, one needs either to account for all radiative corrections, or to use some trick in the hope that it can help to account effectively for a summary effect of all radiative corrections with reasonable accuracy. We have chosen the second way, of course.

The large radiative corrections in the correlator Eq.(134) look reasonable as they tend to increase the Born contribution which is too small. On the other hand, these large corrections indicate that the values of the parameters entering the spectral density in Eq.(134) are not chosen properly. In the given case, the only parameter is the quark mass, M_c, (we used everywhere above the "hard pole masses"), and the correlator Eq.(134) is very sensitive to the

precise value of the quark mass (with the D-meson mass fixed). The right hand side of Eq.(135) increases greatly when the quark mass decreases.

Therefore, it looks natural to try to describe the main effect of radiative corrections by using the effective quark masses: $\mu_c \le M_c$, $\mu_b \le M_b$. So, let us define:

$$M_c = \mu_c \left(1 + \frac{\alpha_s(\mu_o^2)}{4\pi}\, C_D + O(\alpha_s^2)\right) \tag{141}$$

(and analogously for M_b), and let us express M_c through μ_c at the right hand side of Eq.(135). We obtain:

$$r_D^2(\mu_o) \simeq \frac{3}{4\pi^2}\int_{\mu_c^2}^{S_o} \frac{dS\,(S-\mu_c^2)^2}{S}\, exp\left\{\frac{M_D^2 - S}{M^2}\right\} \times \left[1 + \frac{4}{3}\frac{\alpha_s(\mu_c^2)}{\pi}\bar{X}\left(\frac{\mu_c^2}{S}\right) + O(\alpha_s^2)\right], \tag{142}$$

$$\bar{X}(z) = X(z) - \frac{3}{4}\, C_D\, \frac{z}{1-z} \tag{143}$$

Let us try now to find a value of μ_c which will give the answer for f_D close to the right one, Eq.(123), and for which the radiative corrections will remain reasonably small at the same time. It is clear that there is no guaranty that it is possible to succeed in this way.

Nevertheless, the results are very encouraging. For instance, let us suppose that the radiative corrections are reasonably small (for a properly chosen value of μ_c), put $\bar{X}(z) = 0$ in Eq.(142) and find the value of μ_c which reproduces the right answer, Eq.(123). For: $\mu_c = 1.40\,GeV$, $S_o = 4.4\,GeV^2$ one obtains a good fit in the standard region $0.8\,GeV^2 \le M^2 \le 1.5\,GeV^2$ with the result $r_D(\mu_o) \simeq 0.3\,GeV^2$ (compare with Eq.(123)). To elucidate the role of radiative corrections in the sum rule Eq.(142), let us calculate it now in the same region and with the same papameters: $\mu_c = 1.40\,GeV$, $S_o = 4.4\,GeV^2$, and $C_D = 3.87$ (see Eq.(141)). The fit is not optimal but sufficiently good, and when the right hand side of Eq.(142) is taken at the characteristic value $M^2 = 1.15\,GeV^2$ it gives: $r_D(\mu_o) \simeq 0.29\,GeV^2$, in good agreement with Eq.(123).

For the B-meson, the situation is analogous but slightly worse. Namely, let us choose $\mu_b = 4.84\,GeV$ and put $\bar{X}(z) = 0$. We obtain then a good fit in the standard region: $3\,GeV^2 \le M^2 \le 5\,GeV^2$ at $S_o = 30.0\,GeV^2$ with the result: $f_B(\mu_o) \simeq 89\,MeV$, which reproduces Eq.(128). Accounting now for the radiative correction, using $\mu_b = 4.84\,GeV$, $S_o = 30\,GeV^2$, and $C_B = 0.895$ (see Eq.(141)), and taking the right hand side at the characteristic value $\overline{M^2} = 4\,GeV^2$, we obtain the result for $r_B(\mu_o)$ which is $\sim 15\%$ higher than the right value, Eq.(128). Fortunately, because the power corrections to the B-meson decay widths are small (see below), such accuracy will be sufficient for our purposes.

In summary, it is possible to reproduce the right values of f_D and f_B by neglecting the radiative corrections in the sum rule Eq.(142) and using the effective quark masses:

$$\mu_c = 1.40\,GeV\,, \quad \mu_b = 4.84\,GeV\,. \tag{144}$$

The residual effect of radiative corrections remains reasonably small in this case.

As for the correlator Eq.(134) and the decay constants f_D and f_B, there was no need to perform all the above manipulations because the answer was obtained previously in this section. As it was pointed out above, our real purpose is to calculate more complicated matrix elements of the 4-quark operators (see sect.9). And our main assumption is that for the correlators used below in sect.9 it is possible to obtain reasonable results by neglecting radiative corrections and using μ_c and μ_b instead of M_c and M_b in the corresponding spectral densities.

8. Difficulties with naive estimates.

Let us turn now to Γ^o_{nl} defined in Eq.(25). All the quantities entering are known (because the radiative corrections are known here for $m_s = 0$ only and we neglect below the SU(3)-symmetry breaking corrections, we put $m_s = 0$ in nonleptonic calculations):

$$\left(\frac{2C_+^2 + C_-^2}{3}\right) \simeq 1.43, \ (1-2\Delta_G) \simeq 0.7, \ 4\left(\frac{C_-^2 - C_+^2}{2C_+^2 + C_-^2}\right)\Delta_G \simeq 0.36, \tag{145}$$

$$\frac{< D|\bar{c}c|D >}{2\,M_D} \simeq 1.02, \ I_{rad} \simeq 1.05, \ \Gamma_o = \frac{G_F^2 M_c^5}{64\,\pi^3} \simeq 8.4 \cdot 10^{-13}\,GeV, \tag{146}$$

$$\Gamma^o_{nl} \simeq$$

$$\Gamma_o[1.43]_{rad}\,[1.02]_{\bar{c}c}\left\{[1.05]_{rad}\left(1-[0.3]_{\sigma G}\right) + [0.36]_{\sigma G}\right\} \simeq 1.59\Gamma_o. \tag{147}$$

Let us try now to obtain a rough estimate of the matrix element $< D^+|\delta L_{eff}|D^+ >$, see Eq.(45), by putting: $\lambda \simeq P_D$ in L_s, L_u (see fig.3, P_D is the D-meson momentum), $\bar{\lambda}^2 \simeq P_c^2 \simeq M_c^2$ in L_d and using the factorization approximation [7], [38]. Then, L_u and L_s give zero contributions and:

$$< D^+(p)|\,\bar{c}\Gamma_\mu\, d \cdot \bar{d}\Gamma_\nu\, c\,|D^+(p) >_{\mu_o} \simeq p_\mu\, p_\nu\, f_D^2(\mu_o)\,, \tag{148}$$

$$< D^+|\,L_d\,|D^+ >^{factor}_{\mu_o} \simeq -1.1\,M_c^2\, f_D^2(\mu_o)\, M_D^2\,, \tag{149}$$

$$\Delta\Gamma_{factor}(D^+) \simeq \left[-1.1\cdot 16\,\pi^2\,\frac{f_D^2(\mu_o)\,M_D}{M_c^3}\right]\Gamma_o \simeq -1.50\,\Gamma_o\,. \tag{150}$$

The contribution from L_{PNV} in Eq.(51) is obtained by using:

$$< D|\, J^a_\rho \cdot \bar{c}\gamma_\rho(1\pm\gamma_5)\frac{\lambda^a}{2}\, c\,|D >^{factor}_{\mu_o} \simeq -\frac{4}{9}\, r_D^2(\mu_o)\left(1-\frac{M_c^2}{2M_D^2}\right), \tag{151}$$

$$\frac{\Delta\Gamma_{PNV}}{\Gamma_o} \simeq -\frac{64}{9}\pi^2\,\frac{f_D^2(\mu_o)\,M_D^3}{M_c^5}\,N_v(1-\eta_{co}^{-2/9})(1-\frac{M_c^2}{2\,M_D^2}) \simeq -0.15. \tag{152}$$

On the whole, one obtains:

$$\Gamma_{nl}(D^+) \simeq (1.59 - 1.50 - 0.15)\;\Gamma_o \simeq -0.06\,\Gamma_o\,, \tag{153}$$

which does not make much sense. It is clear that the above approximations are too rough and some estimates are essentially wrong. It is the purpose of subsequent sections to improve the above described naive estimates.

9. Nonfactorizable contributions: gluon condensates

We calculate in this section the nonfactorizable gluon contributions to the matrix elements of the 4-quark operators. Let us begin with the operator $O_{\mu\nu} = \bar{c}\Gamma_\mu(\lambda^a/2)q \cdot \bar{q}\Gamma_\nu(\lambda^a/2)c$, which gives zero matrix elements in the factorization approximation. We want to calculate the contribution of the fig.6 diagrams with the help of the QCD sum rules.

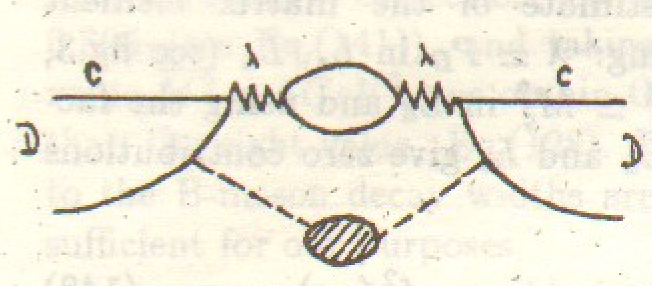


Fig.6a. The nonfactorizable gluon contribution to the weak annihilation.

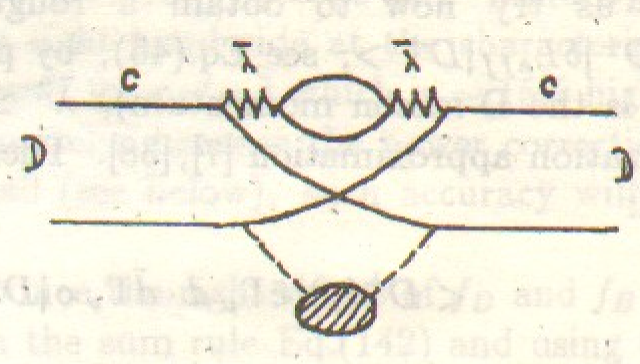
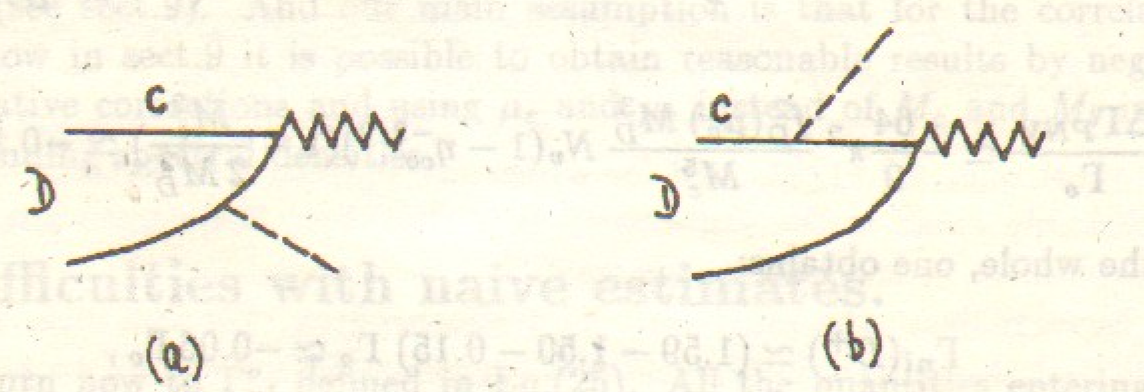


Fig.6b. The nonfactorizable gluon contribution to the cross weak annihilation.

Practically, however, it is more convenient to calculate the nonperturbative gluon emission amplitude, fig.7, and to obtain then the contribution to the decay width by averaging its modulus squared over the gluon field fluctuations in the QCD vacuum.



Figs.7a,b. The diagrams for the meson transition into a current with an emission of a nonperturbative gluon.

To calculate the fig.7 amplitude, we replace the D-meson by the interpolating current and consider the correlator:

$$T_1^\mu = i \int d\,x\, exp\,\{ipx\}\; < 1\,gl\,|T\, J^a_\mu(x)\, J_P(0)\,|0>_{\mu_o}\,, \qquad (154)$$

$$J^a_\mu(x) = \bar{q}\,(x)\,\Gamma_\mu \frac{\lambda^a}{2}\, c(x)\,, \quad J_P(0) = \bar{c}\,(0) i (1+\gamma_5\,)\, q(0)\,. \qquad (155)$$

Calculating the contributions of figs.7a,b diagrams to the discontinuity of T_1^μ in p^2, integrating it from the threshold up to S_o with the weight $exp\{-p^2/M^2\}$

and equating to the D-meson contribution, we obtain the sum rule for the D-meson transition amplitude into the current J^a_μ with an emission of a nonperturbative gluon [12]:

$$N_\mu \simeq \frac{4\pi M_c\, g_s}{(2\,\pi\,)^4\, r_D(\mu_o)} \left\{ i\, \tilde{G}^a_{\mu\,\alpha}\, \tilde{I}_G + G^a_{\mu\,\alpha}\, I_G \right\} p_\alpha\,, \qquad (156)$$

$$\tilde{I}_G = \int_{\mu_c^2}^{S_o} \frac{d\,S}{S}\, exp\left\{ \frac{M_D^2 - S}{M^2} \right\}; $$

$$I_G = \int_{\mu_c^2}^{S_o} \frac{d\,S\,(S-\mu_c^2)}{S^2}\, exp\left\{ \frac{M_D^2 - S}{M^2} \right\}. \qquad (157)$$

$$\tilde{G}_{\mu\nu} = \frac{1}{2}\, \epsilon_{\mu\,\nu\,\lambda\,\sigma}\, G_{\lambda\,\sigma}\,.$$

(In accordance with the discussion in sect.7, we replaced M_c by μ_c in the spectral density, see Eq.(144), supposing this accounts for the major effect of radiative corrections).

Unfortunately, the attempt fails to obtain a good fit for $\tilde{I}^c_G$ in the standard region of M^2: $0.8\,GeV^2 \le M^2 \le 1.5\,GeV^2$, and with S_o in the reasonable vicinity of its optimal value $S_o = 4.4\,GeV^2$ (see sect.7), because $\tilde{I}^c_G$ is nearly independent of S_o. For instanse, $\tilde{I}^c_G$ varies as: $2.0 \ge \tilde{I}^c_G \ge 1.2$ in this region at $S_o = 4.4\,GeV^2$. Therefore, the best we can do is to take for $\tilde{I}^c_G$ its value at the characteristic point $\overline{M^2} = 1.15\,GeV^2$ (see sect.7):

$$\tilde{I}^c_G \simeq \tilde{I}^c_G(\overline{M^2} = 1.15\,GeV^2) \simeq 1.4\,. \qquad (158)$$

Analogously, $0.4 \ge I^c_G \ge 0.3$ in the standard interval of M^2 and at $S_o = 4.4\,GeV^2$. So, we take:

$$I^c_G \simeq I^c_G(\overline{M^2} = 1.15\,GeV^2) \simeq 0.33\,. \qquad (159)$$

On the whole, we estimate:

$$N_\mu \simeq \frac{4\pi M_c\, g_s}{(2\,\pi\,)^4\, r_D(\mu_o)} \left\{ 1.4 i\, \tilde{G}^a_{\mu\alpha} + 0.33\, G^a_{\mu\,\alpha} \right\} p_\alpha\,. \qquad (160)$$

---

[12]The term I_G in Eq.(156) is due to the fig.7b diagram and is parametrically smaller than those from fig.7a.

Taking now the product $N_\mu N_\nu^+$ and averaging the gluon fields over the vacuum, we have finally [13]:

$$< D(p)| \left[\bar c \Gamma_\mu \frac{\lambda^a}{2} q \cdot \bar q \Gamma_\nu \frac{\lambda^a}{2} c\right]_{\mu_o} |D(p) >_{gl} \simeq \left(\frac{p_\mu p_\nu}{p^2} - g_{\mu\nu}\right) C_G^c \,, \qquad (161)$$

$$C_G^c \simeq \frac{M_D^2 M_c^2}{48\pi^2 r_D^2(\mu_o)} < 0|\frac{\alpha_s}{\pi} G^2_{\mu\nu}|0 > \times$$
$$[(\tilde I_G^c)^2 - (I_G^c)^2] \simeq 0.48\cdot 10^{-2} GeV^4. \qquad (162)$$

The situation is qualitatively the same for the B-meson ($\tilde I_G^b$ varies as: $0.46 \geq \tilde I_G^b \geq 0.35$ in the standard region $3\,GeV^2 \leq M^2 \leq 5\,GeV^2$, $S_o = 30\,GeV^2$), and proceeding in the same way we obtain $\tilde I_G^b \simeq 0.4$, $I_G^b \simeq 0.03$, so that:

$$C_G^b \simeq 1.15 \cdot 10^{-2}\, GeV^4 \,. \qquad (163)$$

Let us point out that the parametric behaviour of $\tilde I_G$ is: $\tilde I_G = 0(\mu_o/M_c)$, $I_G = 0(\mu_o^2/M_c^2)$. So, the matrix element Eq.(161) which describes the non-perturbative contribution to WA (weak annihilation, fig.6a) and CWA (cross weak annihiltion, fig.6b) behaves as: $0(\mu_o^3 M_c)$, i.e. in the same way as the factorizable contribution Eq.(148). This gives a relative correction to the decay width: $\delta\Gamma/\Gamma = 0(\mu_o^3/M_c^3)$, as it should be.

In order to understand to what extent the factorization approximation is good, let us compare Eqs.(161)-(163) with the factorizable matrix elements:

$$< D(p)|\bar c\Gamma_\mu q \cdot \bar q \Gamma_\nu c|D(p) >^{fact}_{\mu_o} \simeq$$
$$\left(\tfrac{p_\mu p_\nu}{p^2}\right) f_D^2(\mu_o) M_D^2 \simeq \left(\tfrac{p_\mu p_\nu}{p^2}\right) 7.2 \cdot 10^{-2}\, GeV^4 \,. \qquad (164)$$

$$< B(p)|\bar b\Gamma_\mu q \cdot \bar q \Gamma_\nu b|B(p) >^{fact}_{\mu_o} \simeq$$
$$\left(\tfrac{p_\mu p_\nu}{p^2}\right) f_B^2(\mu_o) M_B^2 \simeq \left(\tfrac{p_\mu p_\nu}{p^2}\right) 22.0 \cdot 10^{-2}\, GeV^4 \,.$$

It is seen that the nonfactorizable contributions are $\simeq 15 - 20$ times smaller than the corresponding factorizable one, so that the factorization approximation works very well even for the D-mesons.

[13] It is implied, see figs.6,7, that the quark flavour in Eq.(161) is such that it is a valent one.

It seems therefore that there are no chances to change essentially the results obtained in the previous section. As will be shown below, this is not the case really for the following reasons:

1) Although the matrix element Eq.(161) is small, it enters δL_{eff} with much larger coefficients in comparison with the factorizable contributions;

2) As will be shown in sect.12, the characteristic value of $\bar\lambda^2$ in L_d , Eq.(31) is: $< \bar\lambda^2 > \simeq 0.35\, M_D^2$, while those of λ^2 in L_u, L_s is: $< \lambda^2 > \simeq M_D^2$, and this effect suppresses strongly the large factorizable contribution, Eq.(164).

Let us perform now an estimate of the nonfactorizable contribution to the matrix element of the operator $S_{\mu\nu} = \bar c\Gamma_\mu q \cdot \bar q \Gamma_\nu c$. For this, let us consider the correlator:

$$T_2^\mu = i \int d\,x\, exp\,\{ipx\} < 2\,gl\,|T\, J_\mu(x)\, J_P(0)\,|0 >_{\mu_o} , \qquad (165)$$

$$J_\mu(x) = \bar q\,(x)\,\Gamma_\mu\, c(x)\,, \quad J_P(0) = \bar c\,(0) i(\,1 + \gamma_5\,)\, q(0) \qquad (166)$$

and calculate the fig.8a contribution. [14] We have:

$$T_2^\mu = \frac{8i\, M_c\, g_s^2}{(2\,\pi)^4} \int \frac{d\,k}{k^8\,[(p+k)^2 - M_c^2]}$$
$$(k^2\, k_\nu\, g_{\mu\rho} - k_\mu\, k_\nu\, k_\rho) \left[G^a_{\rho\lambda} G^a_{\lambda\nu} \right] . \qquad (167)$$

One obtains from Eq.(167) with logarithmic accuracy:

$$T_2^\mu \simeq \Omega_\mu\, \frac{2}{3}\, M_c\, \log\left(\frac{\mu^2_{max}}{\mu^2_{min}}\right) \frac{\partial}{\partial p^2} \frac{1}{p^2 - M_c^2} , \qquad (168)$$

$$\Omega_\mu = \frac{\alpha_s}{\pi} \left[G^a_{\mu\lambda}\, G^a_{\rho\lambda}\, p_\rho - \frac{1}{4}\, p_\mu\, G^a_{\rho\lambda}\, G^a_{\rho\lambda} \right] . \qquad (169)$$

Proceeding now in the standard way, one obtains the sum rule for the D-meson transition amplitude into the current J_μ and two gluons:

$$M_\mu \simeq \Omega_\mu \frac{2}{3} M_c\, \frac{1}{r_D(\mu_o)} \log\left(\frac{\mu^2_{max}}{\mu^2_{min}}\right) \frac{1}{M^2}\, exp\left\{\frac{M_D^2 - M_c^2}{M^2}\right\} + \cdots \qquad (170)$$

[14] Because we can expect beforehand (see the above discussion) that the nonfactorizable correction will be small, we confine ourselves to the main contribution from the fig.8a diagram, neglecting even smaller contributions from those diagrams where gluons are emitted by the c-quarks.

As we need here a rough estimate only, let us put: $\overline{M^2} = 1\,GeV^2$ in Eq.(170) (see sect.7). Besides, because both μ^2_{max} and μ^2_{min} in Eq.(170) remain finite at $M_c \to \infty$, we put:

$$\log\left(\frac{\mu^2_{max}}{\mu^2_{min}}\right) \simeq 1. \qquad (171)$$

Therefore,

$$M_\mu \sim (\,7.9\,GeV^{-3})\,\Omega_\mu\,. \qquad (172)$$

Now, taking the product $M_\mu\,M^+_\nu$ and averaging the gluon fields, one has:

$$< D(p)|\bar{c}\Gamma_\mu\,q\cdot\bar{q}\Gamma_\nu\,c\,|D(p)>_{nonfact}\simeq (62\,GeV^{-6}) < 0|\Omega_\mu\,\Omega_\nu\,|0>. \qquad (173)$$

Unfortunately, this 4-gluon vacuum condensate is unknown. So, let us obtain first an estimate in the factorization approximation, see fig.8b.

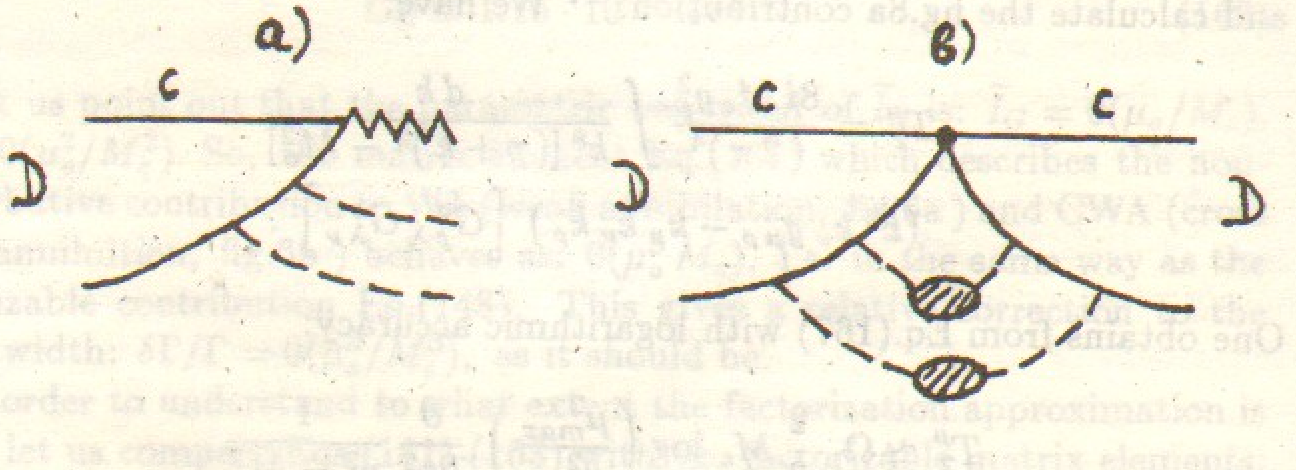


Fig. 8a,b. The nonfactorizable contribution to the matrix element $< D|S_{\mu\nu}|D>$.

One has:

$$< 0|\,\Omega_\mu\,\Omega_\nu\,|0>\sim \frac{1}{576}\,g_{\mu\nu}M^2_D < 0|\frac{\alpha_s}{\pi}\,G^2_{\mu\nu}|0>^2 + O(p_\mu\,p_\nu)\,, \qquad (174)$$

so that:

$$< D(p)|\,\bar{c}\,\Gamma_\mu\,q\cdot\bar{q}\,\Gamma_\nu\,c\,|D(p)>_{nonfact}\sim g_{\mu\nu}\,(0.5\cdot 10^{-4}\,GeV^4)\,, \qquad (175)$$

(and the term $\sim p_\mu\,p_\nu$ is of the same order). Comparing Eq.(175) with Eqs.(164) and (161) we see that the nonfactorizable correction is very small in this matrix element. Therefore, even if the approximations made above give the right order of magnitude only, we can safely neglect this correction.

10. Nonfactorizable contributions: quark condensates

The contribution $< D|\delta L_{PNV}|D>$ was calculated in sect.8 in the factorization approximation. Because it is not large by itself and nonfactorizable corrections are also small (see sect.9), it does not make much sense to account for them in this matrix element. So, we will use for it the expression Eq.(152).

There are, however, analogous corrections of the "penguin" type in the matrix elements of the operators L_u, L_d, L_s in Eqs.(45)-(48), see figs.9, 10, and we proceed now to their calculation.

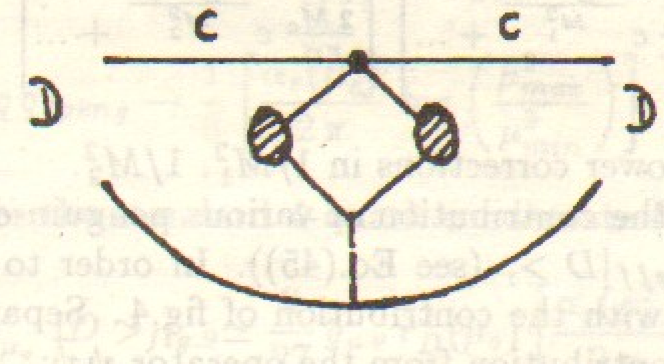


Fig.9. The nonvalence nonfactorizable penguin contribution (NV).

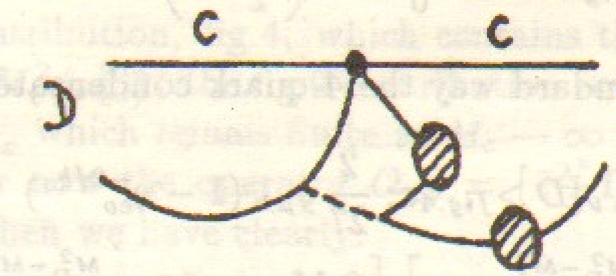


Fig.10. The valence nonfactorizable penguin contribution.

For a calculation of the diagrams in figs.9, 10 (see also fig.4) let us consider the correlator:

$$T_{\mu\nu} = i\int d\,x\,e^{ip_1x}\,i\int d\,y\,e^{-ip_2y} < 0|T\,J^+_P(x)\,S_{\mu\nu}(0)\,J_P(y)|0>\,, \qquad (176)$$

$$J_P = \bar{c}\,i\,(1+\gamma_5)q, \quad J^+_P = \bar{q}\,i\,(-1+\gamma_5)c, \quad S_{\mu\nu} = \bar{c}\Gamma_\mu\psi\cdot\bar{\psi}\Gamma_\nu c\,, \qquad (177)$$

where q and ψ are light quark fields

One can neglect the co-ordinate dependence of the light quark fields in Eq.(176) for calculations with logarithmic accuracy, and obtain:

$$T_{\mu\nu} \simeq \left(\frac{2M_c}{p_1^2 - M_c^2}\right)\left(\frac{2M_c}{p_2^2 - M_c^2}\right) K_{\mu\nu} + \cdots, \tag{178}$$

$$K_{\mu\nu} = < 0|\bar{q}(0)\Gamma_\mu\psi(0)\cdot\bar{\psi}(0)\Gamma_\nu q(0)|0>. \tag{179}$$

Proceeding in the usual way one obtains now from Eq.(178) the sum rule:

$$< D(p)|S_{\mu\nu}(0)|D(p)>_{peng} \simeq$$

$$K_{\mu\nu}\left[\frac{2M_c}{r_D} e^{\frac{M_D^2 - M_c^2}{M_1^2}} + \ldots\right]\left[\frac{2M_c}{r_D} e^{\frac{M_D^2 - M_c^2}{M_2^2}} + \ldots\right], \tag{180}$$

where the dots denote power corrections in $1/M_1^2$, $1/M_2^2$.

Let us consider now the contribution of various penguin-diagrams to the matrix element $< D|\delta L_{eff}|D >$, (see Eq.(45)). In order to check the normalization, let us start with the contribution of fig.4. Separating in $K_{\mu\nu}$, Eq.(179), the penguin contribution from the operator $\psi\bar{\psi}$:

$$[\psi\bar{\psi}]_{peng} \to \frac{1 - \eta_{co}^{-2/b_o}}{6}\left(\frac{\lambda^a}{2}\gamma_\rho\right) J_\rho^a(3), \tag{181}$$

and factorizing in the standard way the 4-quark condensate, we obtain:

$$< D|S_{\mu\nu}|D>_{fig.4} \simeq \frac{2}{27} g_{\mu\nu}(1 - \eta_{co}^{-2/b_o})$$

$$\left[\frac{2M_c < q\bar{q}>}{r_D} e^{\frac{M_D^2 - M_c^2}{M_1^2}} + \ldots\right]\left[\frac{2M_c < q\bar{q}>}{r_D} e^{\frac{M_D^2 - M_c^2}{M_2^2}} + \ldots\right]. \tag{182}$$

On the other hand, to calculate this contribution there is no need to use the sum rules at all because, after the replacement Eq.(181), this contribution to the matrix element can be obtained directly through factorization:

$$< D|S_{\mu\nu} \to \left(\frac{1 - \eta_{co}^{-2/b_o}}{6}\right)\bar{c}\Gamma_\mu\frac{\lambda^a}{2}\gamma_\rho\Gamma_\nu c\cdot J_\rho^a|D>_{fig.4} \simeq$$

$$\frac{2}{27} g_{\mu\nu}(1 - \eta^{-2/b_o}) r_D^2. \tag{183}$$

Comparing with Eq.(182) we see that we simply reproduced the sum rule used before (see Eqs.(117),(118)):

$$2M_c < 0|q\bar{q}|0> e^{\frac{M_D^2 - M_c^2}{M^2}}[1 + \cdots] \simeq r_D^2. \tag{184}$$

Therefore, we can rewrite Eq.(180) in the form:

$$< D|S_{\mu\nu}|D> \simeq \frac{r_D^2}{< 0|\bar{q}q|0>^2} K_{\mu\nu}. \tag{185}$$

It is not difficult to obtain now other penguin contributions from Eq.(185). Namely, the contribution of fig.9 is obtained by separating out the penguin contribution from the operator $[q\bar{q}]$ in $K_{\mu\nu}$:

$$[q\bar{q}]_{peng} \to \frac{1}{6}\left[\frac{\alpha_s(\mu_o^2)}{2\pi}\log\left(\frac{\mu_{max}^2}{\mu_{min}^2}\right)\right]\left(\frac{\lambda^a}{2}\right) J_\rho^a, \tag{186}$$

which gives after the standard 4-quark condensate factorization:

$$< D|[S_{\mu\nu}]_{\mu_o}|D>_{fig.9} \simeq \frac{2}{27} g_{\mu\nu} r_D^2(\mu_o)\left[\frac{\alpha_s(\mu_o^2)}{2\pi}\log\left(\frac{\mu_{max}^2}{\mu_{min}^2}\right)\right]\Delta_s \tag{187}$$

where $\Delta_s = 1$ if the ψ in $S_{\mu\nu}$ is u- or d-quark, and $\Delta_s = (< \bar{s}s > / < \bar{u}u >)^2 \simeq 0.64$ if ψ is the s-quark. Let us point out that, unlike the standard penguin contribution, fig.4, which contains the factor: $(1 - \eta_{co}^{-2/b_o}) \simeq (\alpha_s(\mu_o^2)/2\pi)\log(M_c^2/\mu_o^2))$, the fig.9 contribution contains as an upper cut off the quantity μ_{max}^2 which remais finite at $M_c \to \infty$ (see also Eq.(171)).[15]

If we consider now the operator $O_{\mu\nu} = [\bar{c}\frac{\lambda^a}{2}\Gamma_\mu\psi\cdot\bar{\psi}\frac{\lambda^a}{2}\Gamma_\nu c]$ instead of $S_{\mu\nu}$, Eq.(177), then we have clearly:

$$< D|O_{\mu\nu}|D>_{fig.9} = -\frac{1}{6} < D|S_{\mu\nu}|D>_{fig.9}. \tag{188}$$

For the operator $O_{\mu\nu}$, however, there is an additional (valence) penguin contribution originating from the diagram fig.10 (plus the mirror one). It can be obtained easily from the relation analogous to Eq.(185):

$$< D|O_{\mu\nu}|D> \simeq \frac{r_D^2}{< 0|\bar{q}q|0>^2} P_{\mu\nu}. \tag{189}$$

[15] It is clear that the contributions of figs.4,9 are nonvalence, i.e. the same for all $D^{o,\pm,s}$-mesons; they only shift the position of the "decay width centre" Γ_{nl}^o.

$$P_{\mu\nu} = < 0|\,\bar q\,\Gamma_\mu \frac{\lambda^a}{2} q \cdot \bar q\,\Gamma_\nu \frac{\lambda^a}{2} q\,|0>, \tag{190}$$

by separating out the penguin contribution:

$$\left[\bar q \Gamma_\mu \frac{\lambda^a}{2} q\right]_{peng} \rightarrow \left[-\frac{\alpha_s(\mu_o^2)}{6\,\pi} \log\left(\frac{\mu^2_{max}}{\mu^2_{min}}\right)\right] J^a_\mu\,, \tag{191}$$

and factorizing the 4-quark condensate. We obtain (the factor 2 accounts for the mirror diagram):

$$< D|\,[O_{\mu\nu}]_{\mu_o}\,|D>_{fig.10} \simeq \frac{2}{27}\, g_{\mu\nu}\, r^2_D(\mu_o)\left[\frac{\alpha_s(\mu_o^2)}{2\,\pi}\,\log\left(\frac{\mu^2_{max}}{\mu^2_{min}}\right)\right]. \tag{192}$$

Let us point out finally that, within the logarithmic approximation, we can put: $\lambda \simeq \bar\lambda \simeq P_c \simeq M_c$ for the contribution of fig.4, and $\lambda \simeq \bar\lambda \simeq P_D$ for those of figs.9, 10.

For the B-mesons, it is sufficient to replace: $r_D(\mu_o) \rightarrow r_B(\mu_o)$ in the above formulae Eqs.(187), (188), (192) to obtain contributions to the matrix elements of $\bar L_u$, L_d, L_s in Eq.(61) (for the $B^{\pm,o,s}$ mesons there are no contributions from L_c, $\bar L_c$, $\tilde L_c$ in Eq.(61)).

11. Corrections to semileptonic widths

We are ready now to explain how the value of $\delta^{lept}(D^+)$ used in sect.3 was obtained.

Let us recall (see sects.8, 10 and fig.9) that:

$$< D^+(p)|\,\bar c\,\Gamma_\mu\, s \cdot \bar s\,\Gamma_\nu\, c\,|D^+(p)>_{\mu_o} \simeq \frac{2}{27} g_{\mu\nu}\,\rho_o \left(\frac{<\bar s s>}{<\bar u u>}\right)^2 r^2_D(\mu_o)\,,$$

$$< D^+(p)|\,\bar c \frac{\lambda^a}{2}\Gamma_\mu\, s\,\bar s \frac{\lambda^a}{2}\Gamma_\nu\, c\,|D^+(p)>_{\mu_o} \simeq$$

$$-\tfrac{1}{6} < D^+(p)|\,\bar c\,\Gamma_\mu\, s \cdot \bar s\,\Gamma_\nu\, c\,|D^+(p)>_{\mu_o}\,,$$

$$\rho_o = \frac{\alpha_s(\mu_o^2)}{2\,\pi}\,\log\left(\frac{\mu^2_{max}}{\mu^2_{min}}\right) \simeq 0.1\,, \qquad \frac{<0|\bar s s|0>}{<0|\bar u u|0>} \simeq 0.8\,, \tag{193}$$

$$< D^+(p)|\,\bar c\,\Gamma_\mu\, d \cdot \bar d\,\Gamma_\nu\, c\,|D^+(p)>_{\mu_o} \simeq p_\mu\, p_\nu\, f^2_D(\mu_o) + \frac{2}{27} g_{\mu\nu}\,\rho_o\, r^2_D(\mu_o) \tag{194}$$

Therefore (see Eq.(55) in sect.2):

$$\Delta\Gamma^{lept}(D^+) = \frac{1}{2\,M_D} < D^+(p)|\,\Delta L^{lept}(\mu_o)\,|D^+(p)>_{\mu_o} \simeq -\frac{G_F^2}{54\pi M_D}\times$$

$$M^2_D\,\rho_o\,\tau_{co}\, r^2_D(\mu_o)\Psi_o\,, \quad \Psi_o = \left[\tfrac{<\bar s s>^2}{<\bar u u>^2}|V_{cs}|^2 + |V_{cd}|^2\right] \simeq 0.656\,, \tag{195}$$

$$\frac{\Delta\Gamma^{lept}(D^+)}{\Gamma^{lept}_o} \simeq -\frac{32}{9}\pi^2 \frac{f^2_D(\mu_o)\, M^5_D}{M^7_c}\,\rho_o\,\tau_{co}\,\Psi_o \simeq -0.045\,. \tag{196}$$

In addition (see sects.3a, 8 and fig.4):

$$\delta\Gamma^{lept}_{PNV} \equiv \frac{1}{2\,M_D} < D|\,L^{lept}_{PNV}(\mu_o)\,|D>_{\mu_o} =$$

$$\frac{G_F^2}{12\pi\, M_D}(1 - \eta_{co}^{-2/9})\,\tau_{co}\, M_c^2 < D|\left[J^a_\rho(3)\cdot \bar c \frac{\lambda^a}{2}\gamma_\mu\,(1 + \frac{1}{3}\gamma_5\,)\,c\right]|D>_{\mu_o} \simeq$$

$$\frac{G_F^2}{12\pi\, M_D}(1 - \eta_{co}^{-2/9})\,\tau_{co}\, M_c^2\left[-\frac{4}{9}\, r^2_D(\mu_o)\left(1 - \frac{M^2_c}{2\,M^2_D}\right)\right], \tag{197}$$

$$\frac{\delta\Gamma^{lept}_{PNV}}{\Gamma^{lept}_o} \simeq -\frac{64}{9}\pi^2 \frac{f^2_D(\mu_o)\, M^3_D}{M^5_c}(1 - \eta_{co}^{-2/9})\tau_{co}\left(1 - \frac{M^2_c}{2\,M^2_D}\right) \simeq -0.085. \tag{198}$$

On the whole:

$$\delta^{lept}(D^+) \equiv \frac{1}{\Gamma^o_{sl}} \frac{< D^+|\,\delta L^{lept}_{eff}\,|D^+>}{2 M_D} \simeq (-0.085 - 0.045) \simeq -0.13\,. \tag{199}$$

For the $B^{\pm,o,s}$ mesons (see sect.3b), the term $\Delta L^{lept}(\mu_o)$ gives no contribution, and the analog of eq.(198) looks as:

$$\frac{\delta\Gamma^{lept}_{PNV}}{\Gamma^{lept}_o} \simeq$$

$$-\tfrac{16}{3}\pi^2 \tfrac{f^2_B(\mu_o)\, M^3_B}{M^5_b}(1 - \eta_{bc}^{-8/25})\,\tau_{bo}\,\eta_{co}^{-2/9}\left(1 - \tfrac{M^2_b}{2\,M^2_B}\right) \simeq -2\cdot 10^{-3}\,. \tag{200}$$

All the above described contributions are nonvalence. There is also a sizeable valence contribution to the D_s leptonic width (see Eq.(56) and sect.9):

$$\Delta\Gamma^{lept}(D_s) \simeq \frac{G_F^2}{4\pi\, M_D}|V_{cs}|^2\,(-0.245)\,T_{\mu\nu} < D_s|\bar c \Gamma_\mu \frac{\lambda^a}{2}\, s\times$$

$$\bar s \Gamma_\nu \tfrac{\lambda^a}{2}\, c|D_s>_{\mu_o} \simeq \tfrac{G_F^2}{4\pi\, M_D}|V_{cs}|^2\,(-0.245)\, M^2_D\,(\,C^c_G - P^c_V\,)\,, \tag{201}$$

$$\frac{\Delta\Gamma^{lept}(D_s)}{\Gamma_o^{lept}} \simeq 48\pi^2|V_{cs}|^2(-0.245)\frac{M_D}{M_c^5}(C_G^c - P_V^c) \simeq -7\% . \qquad (202)$$

12. λ and $\bar\lambda$

As it was indicated before, the values of λ and $\bar\lambda$ (in terms of the c-quark and spectator quark momenta) are clear from each diagram. Namely (P_c is the c-quark 4-momuntum, k_i are the spectator quark momenta, P_D is the D meson momentum, $P_D = P_{c1} + k_1 = P_{c2} + k_2$ for the initial and final D mesons):

a) $\lambda \simeq (P_c + k_1) = P_D$ for the figs. 3a, 3b, 6a contributions;
b) $\lambda \simeq \bar\lambda \simeq P_D$ (within logarithmic accuracy) for the figs.9, 10 contributions;
c) $\lambda \simeq \bar\lambda \simeq P_c \simeq M_c$ (within logarithmic accuracy) for the fig.4 contribution;
d) $\bar\lambda \simeq ((P_D - k_1 - k_2)$ for the figs.3c, 6b contributions.

The differences between all of the cases above disappear in the formal limit $M_Q \to \infty$ but, as will be shown below, they are of great importance for D mesons and are sizeable even for B mesons.

Because the D meson momentum, P_D, is not an operator but a fixed number, we have to deal practically with the case "d" only. So, let us consider in detail the matrix element (see fig.3c and sect.3a):

$$I_D \equiv < D(P_D)|\,\bar\lambda^2 \bar c \Gamma_\mu d \cdot \bar d \Gamma_\mu c\,|D(P_D) > \equiv$$
$$< \bar\lambda_D^2 >< D|\,\bar c \Gamma_\mu d \cdot \bar d \Gamma_\mu c\,|D >, \qquad (203)$$

where $\bar\lambda = (P_D - k_1 - k_2)$ and k_1, k_2 are understood as the 4-momentum operators of the initial and final spectator quarks.

As it was shown above (see sects.9, 10), the factorization approximation works very well even for the D mesons, and the non-factorizable contributions are much smaller than factorizable ones. So, we can reliably estimate I_D as:

$$I_D \simeq P_D^2 < D|\,\bar c \Gamma_\mu d\,|0 >< 0|\,\bar d \Gamma_\mu c\,|D > -$$
$$4(P_D)_\alpha < D|\,\bar c \Gamma_\mu d\,|0 >< 0|\,\bar d k_\alpha \Gamma_\mu c\,|D > +2 < D|\,\bar c \Gamma_\mu k_\alpha d\,|0 > \times$$
$$< 0|\,\bar d k_\alpha \Gamma_\mu c\,|D > +2 < D|\,\bar c \Gamma_\mu d\,|0 >< 0|\,\bar d k^2 \Gamma_\mu c\,|D > . \qquad (204)$$

Let us define the matrix elements in Eq.(204) as:

$$< 0|\,\bar q\, k_\nu i \gamma_5\, c\,|D(P) >= r_D < x >_P \; P_\nu\,, \qquad (205)$$

$$< 0|\,\bar q\, k_\nu \gamma_\mu \gamma_5\, c\,|D(P) >= i\, f_D < x >_A \left(P_\mu P_\nu - \frac{1}{4} g_{\mu\nu} P^2 \right), \qquad (206)$$

$$< 0|\,\bar q\, k^2\, \gamma_\mu \gamma_5\, c\,|D(P) >= i\, f_D\, P_\mu \;\; < k^2 >_A . \qquad (207)$$

The quantity $< x >$ has the meaning of the mean momentum fraction carried by the light quark (in the $P_z \to \infty$ frame), and $< k^2 >$ is the characteristic value of the light quark 4-momentum squared inside the D meson. So:

$$\frac{< \bar\lambda^2 >}{M_D^2} \simeq \left(1 - 3 < x >_A + \frac{3}{2} < x >_A^2 - 2\frac{< -k^2 >_A}{M_D^2} \right) . \qquad (208)$$

Using the equations of motion for the matrix elements Eqs.(205), (206) it is not difficult to obtain:

$$< x >_P = \frac{1}{2}\left(1 - \frac{(M_c - m_q)^2}{M_D^2} \right),$$
$$< x >_A = \tfrac{4}{3} < x >_P \left[1 + O\left(\tfrac{\Lambda_{QCD}}{M_c} \right) \right] . \qquad (209)$$

As for the value of $< k^2 >$, the estimates obtained from the corresponding QCD sum rules for this quantity show that it is not far from its value for the vacuum quarks (see Eq.(42)). So, we have [16]:

$$< x >_A \simeq 0.15, \quad < -k^2 >_A \simeq 0.4\, GeV^2, \quad < \bar\lambda_D^2 > \simeq 0.35\, M_D^2 . \quad (210)$$

It is seen that it is of great importance here to account for the spectator quark momenta, in spite of the fact that these are power corrections only in the formal limit $M_c \to \infty$. In what follows, we use the same estimate Eq.(210) also for the fig.6b contribution:

$$< D|\; \bar\lambda^2\; \bar c \Gamma_\mu \frac{\lambda^a}{2} d \cdot \bar d \Gamma_\mu \frac{\lambda^a}{2} c\,|D > \simeq$$
$$0.35\, M_D^2 < D|\, \bar c \Gamma_\mu \tfrac{\lambda^a}{2} d \cdot \bar d \Gamma_\mu \tfrac{\lambda^a}{2} c\,|D > . \qquad (211)$$

The corresponding expressions for the B mesons look as:

$$< x >_A \simeq 6\% , \quad < -k^2 > \simeq 0.4\, GeV^2, \quad < \bar\lambda_B^2 > \simeq 0.8\, M_B^2 , \quad (212)$$

so that the effect discussed is sizeable even for B mesons.

[16] That the effect is large for the D mesons can be seen from a simplest rough estimate: $k_1 \simeq k_2 \simeq < x >_A P_D$, $\bar\lambda^2 \simeq (P_D - 2 < x >_A P_D)^2 \simeq (1 - 2 < x >_A)^2 M_D^2 \simeq 0.5 M_D^2$.

13a. Calculation of the $D^{\pm,o,s}$ decay widths

Let us collect now the results obtained in previous sections (see also Appendix). As it was shown in sects.3a, 8, keeping only the leading term and first corrections $O(1/M_c^2)$, we obtain a common nonleptonic decay width for all $D^{\pm,o,s}$ mesons:

$$\Gamma^o_{nl} \simeq 1.59\, \Gamma_o\,, \quad \Gamma_o = \frac{G_F^2\, M_c^5}{64\,\pi^3} \simeq 8.4 \cdot 10^{-13}\, GeV. \tag{213}$$

Accounting for the contributions of the four-fermion operators, we obtained:

I) The nonvalence contributions (which are the same for all $D^{\pm,o,s}$ mesons (see sects.3a,10)) from the diagram of fig.4:

$$\delta\Gamma_{PNV} \equiv \frac{1}{2\,M_D} < D|\, L_{PNV}(\mu_o)\,|D >=$$

$$\frac{G_F^2}{4\pi\, M_D}(1-\eta_{co}^{-2/9})\, M_c^2 < D| \left[J^a_\rho(3)\cdot \bar{c}\,\frac{\lambda^a}{2}\gamma_\rho\,(N_v + N_a\gamma_5)\, c \right] |D>_{\mu_o} \simeq$$

$$\frac{G_F^2}{4\pi\, M_D}(1-\eta_{co}^{-2/9})\, M_c^2 \left[-\frac{4}{9}\, r_D^2(\mu_o)\, N_v \left(1 - \frac{M_c^2}{2\,M_D^2} \right) \right], \tag{214}$$

$$\frac{\delta\Gamma_{PNV}}{\Gamma_o} \simeq -\frac{64}{9}\,\pi^2\,\frac{f_D^2(\mu_o)\, M_D^3}{M_c^5}(1-\eta_{co}^{-2/9})\, N_v \left(1 - \frac{M_c^2}{2\,M_D^2} \right) \simeq -0.15. \tag{215}$$

II) The nonvalence contribution from the diagram of fig.9 (see sect.10):

$$\delta\Gamma_{NV} \equiv \frac{1}{2\,M_D} < D|\Delta L(\mu_o)|D>_{NV} \simeq \frac{G_F^2}{4\pi\, M_D}\,\frac{2}{27}\, M_D^2\, \rho_o\, \Omega_{NV}$$

$$\Omega_{NV} = [-A_u + A_d(|V_{ud}|^2 + \xi_s |V_{us}|^2) - A_s(\xi_s |V_{cs}|^2 + |V_{cd}|^2)] \simeq -9.3,$$

$$\xi_s = \frac{<\bar{s}s>^2}{<\bar{u}u>^2} \simeq 0.64\,, \tag{216}$$

where the subscript NV means nonvalence contributions,

$$\frac{\delta\Gamma_{NV}}{\Gamma_o} \simeq \frac{32}{27}\,\pi^2\,\frac{f_D^2(\mu_o)\, M_D^5}{M_c^7}\,\rho_o\,\Omega_{NV} \simeq -0.15\,. \tag{217}$$

So, the total nonvalence contribution is:

$$\frac{\Delta\Gamma_{NV}}{\Gamma_o} \equiv \frac{[\delta\Gamma_{PNV}] + [\delta\Gamma_{NV}]}{\Gamma_o} \simeq [-15\%] + [-15\%] = -30\%\,. \tag{218}$$

III) We have for the valence contributions (see sects.3a, 9, 10 and figs.6, 10):

$$\Delta\Gamma_{nl}(D^o) \equiv \frac{1}{2\,M_D} < D^o|\Delta L^{(c)}(\mu_o)|D^o >_V \simeq$$

$$\frac{G_F^2}{4\pi\, M_D}\, O_u\, M_D^2\, (\, C_G^c - P_V^c\,)\,, \tag{219}$$

$$\frac{\Delta\Gamma_{nl}(D^o)}{\Gamma_o} \simeq 16\,\pi^2\,\frac{M_D}{M_c^5}\, O_u\,(\, C_G^c - P_V^c\,) \simeq +33\%\,. \tag{220}$$

$$\frac{\Delta\Gamma_{nl}(D^+)}{\Gamma_o} = \frac{1}{\Gamma_o} < D^+|\Delta L^{(c)}(\mu_o)|D^+ >_V \simeq$$

$$\left[16\,\pi^2\,\frac{f_D^2(\mu_o)\, M_D^3}{M_c^5} |V_{ud}|^2\, S_d\, \frac{<\bar{\lambda}_D^2>}{M_D^2} \right] -$$

$$\left[48\,\pi^2\,\frac{M_D}{M_c^5} |V_{ud}|^2\, O_d \left(\frac{<\bar{\lambda}_D^2>}{M_D^2}\, C_G^c - \frac{4}{3}\, P_V^c \right) \right] \simeq$$

$$[-62\%] + [-21\%] \simeq -83\%\,, \tag{221}$$

where the subscript V means valence contributions.

For the D_s meson [17]:

$$\frac{\Delta\Gamma_{nl}(D_s)}{\Gamma_o} = \frac{1}{\Gamma_o} < D_s|\Delta L^{(c)}(\mu_o)|D_s >_V \simeq$$

$$\left[16\,\pi^2\, |V_{us}|^2\, \frac{f_D^2(\mu_o)\, M_D^3}{M_c^5}\, S_d\, \frac{<\bar{\lambda}_D^2>}{M_D^2} \right] +$$

$$\left[16\,\pi^2\, |V_{cs}|^2\, \frac{M_D}{M_c^5}\, O_s\,(\, C_G^c - P_V^c\,) \right] +$$

$$\left[-48\,\pi^2\, |V_{us}|^2\, \frac{M_D}{M_c^5}\, O_d \left(\frac{<\bar{\lambda}_D^2>}{M_D^2}\, C_G^c - \frac{4}{3}\, P_V^c \right) \right] \simeq$$

$$[-3\%] + [-3\%] + [-1\%] \simeq -7\%\,. \tag{222}$$

We have therefore for the nonleptonic widths (the experimental values are given in brackets, all values are given below in units of $10^{-13}\, GeV$):

$$\Gamma_{nl}(D^+) \simeq \Gamma_o\,[1.59 - 0.30 - 0.83] \simeq 0.46\,\Gamma_o \simeq 3.9 \quad \{4.05\}\,, \tag{223}$$

$$\Gamma_{nl}(D^o) \simeq \Gamma_o\,[1.59 - 0.30 + 0.33] \simeq 1.62\,\Gamma_o \simeq 13.6 \quad \{13.5\}\,, \tag{224}$$

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[17] I am indebted to N.G. Uraltsev who pointed out an arithmetical mistake in the original calculation of this correction.

$$\Gamma_{nl}(D_s) \simeq \Gamma_o\,[1.59 - 0.30 - 0.07] \simeq 1.22\,\Gamma_o \simeq 10.2 \quad \{\ ?\ \}\,, \qquad (225)$$

We have for the semileptonic widths (see sects.4, 11):

$$\Gamma^{lept}(D^+) = 1.06 \quad (input)\,, \qquad (226)$$

$$\Gamma^{lept}(D^o) \simeq \Gamma^{lept}(D^+)\,, \qquad (227)$$

$$\tilde{\Gamma}^{lept}(D_s) \simeq \Gamma^{lept}(D^+)\,[1 - 7\%] \simeq 1.0\,. \qquad (228)$$

We have to add also to $\Gamma^{lept}(D_s)$ the "$D_s \to \tau\,\nu$" contribution:

$$\frac{\Gamma(D_s \to \tau\,\nu)}{\Gamma_o^{lept}} = 24\,\pi^2 \frac{f^2_{D_s}(M_D)\,M_{D_s}\,M_\tau^2}{M_c^5}\left[1 - \frac{M_\tau^2}{M_{D_s}^2}\right]^2 \simeq 0.16\left(\frac{f_{D_s}(M_D)}{200\,MeV}\right)^2,$$

$$\Gamma(D_s \to \tau\nu) \simeq 0.4\,, \quad \Gamma^{lept}_{tot}(D_s) \simeq 2\cdot 1 + 0.4 \simeq 2.4\,. \qquad (229)$$

So, we have finally for the total decay widths:

$$\Gamma_{tot}(D^+) \simeq [3.9 + 2.1] \simeq 6.0\,, \quad \{6.2\} \qquad (230)$$

$$\Gamma_{tot}(D^o) \simeq [13.6 + 2.1] \simeq 15.7\,, \quad \{15.6\} \qquad (231)$$

$$\Gamma_{tot}(D_s) \simeq [10.2 + 2.4] \simeq 12.6\,, \quad \{13.8\}\,. \qquad (232)$$

13b. Calculation of the $B^{\pm,o,s}$ lifetime differences

Substituting the corresponding numbers into the formula Eq.(25), we obtain:

$$\frac{2C_+^2 + C_-^2}{3} \simeq 1.127\,, \quad \frac{< B|\,\bar{b}\,b\,|B >}{2\,M_B} \simeq 1.00\,, \quad I_{rad} \simeq 0.98\,, \qquad (233)$$

$$\Gamma^o_{nl} \simeq 1.10\,\Gamma_o\,z_o\,, \quad \Gamma_o = \frac{G_F^2\,M_b^5\,|V_{cb}|^2}{64\,\pi^3}\,. \qquad (234)$$

The values of the phase space factors z_o^i [39] (for $M_b = 5.04\,GeV$, $M_c = 1.65\,GeV$, $M_\tau = 1.784\,GeV$) are:

$$z_o^{cud} \simeq 0.460\,, \quad z_o^{ccs} \simeq 0.131\,, \quad z_o^{c\tau\nu} \simeq 0.105\,. \qquad (235)$$

So, neglecting the 4-fermion operator contributions (which are small, see below) we have for the relative yields:

$$n_B^{eff} \simeq \left\{(1)_{ud} + (0.285)_{cs} + 2\,(0.260)_{e\nu+\mu\nu} + (0.060)_{\tau\nu}\right\} \simeq 1.87, \quad (236)$$

and the branching fractions are respectively:

$$Br\,(b \to c\bar{c}s) \simeq 15\%\,, \quad Br\,(b \to ce\nu) \simeq 13.9\%\,,$$
$$Br\,(b \to c\tau\nu)) \simeq 3.2\%\,, \quad Br\,(\frac{\tau}{e}) \simeq 0.23\,. \qquad (237)$$

The above value of $Br\,(b \to ce\nu)$ is $\simeq 20\%$ larger than the LEP data [26], and this is a well known difficulty [40], [41], [42]. What we can add is that the 4-fermion operator contributions calculated below are too small and can not help here. From our viewpoint, one of the important reasons for this discrepancy is that the radiative corrections to the nonleptonic widths are known for massless quarks only, and there are reasons to expect that accounting for nonzero final quark masses will increase considerably the nonleptonic radiative correction. [18] Another evident reason which comes to mind is that the energy release is not large in the $b \to c\bar{c}s$ mode, so that it can be enhanced due to nonperturbative effects. Although the present data do not support this, it seems that the experimental numbers can change here with time.

Let us proceed now to the calculation of the B meson lifetime differences. Using results obtained in preceeding sections (see sects. 3b,9,10) , we have for the valence contributions:

$$\frac{\Delta\Gamma(B^-)}{\Gamma_o} \simeq 16\pi^2 \frac{<\bar{\lambda}_B^2>}{M_B^2}\left(1 - \frac{M_c^2}{<\bar{\lambda}_B^2>}\right)^2 \times$$
$$\left\{S_d \frac{f_B^2(\mu_o)\,M_B^3}{M_b^5} - 3\,O_d \frac{M_B}{M_b^5}\left(C_G^B - \frac{4}{3}\xi_B\,P_V^B\right)\right\} \simeq$$
$$(-1.9\%) + (-1.3\%) \simeq -3.2\%, \qquad (238)$$

$$\xi_B = \frac{M_B^2}{<\bar{\lambda}_B^2>}\frac{\left(1 - \frac{M_c^2}{M_B^2}\right)^2}{\left(1 - \frac{M_c^2}{<\bar{\lambda}_B^2>}\right)^2} \simeq 1.31\,. \qquad (239)$$

[18] To illustrate possible changes, we can put $M_c = 0$ in the leptonic radiative correction and obtain: $Br\,(b \to ce\nu) \simeq 12.9\%$, in comparison with $\simeq 11.4\%$ from the LEP data.

$$\frac{\Delta\Gamma(B^o)}{\Gamma_o} \simeq 16\pi^2(1-x)^2\frac{M_B}{M_b^5}O_u\left[(1+\frac{x}{2})C_G^B - P_V^B\right] \simeq 0.6\%\,, \quad (240)$$

$$\frac{\Delta\Gamma(B_s)}{\Gamma_o} \simeq 16\pi^2(1-4x)^{1/2}\frac{M_B}{M_b^5}O_u\times$$
$$[(1-x)C_G^B - (1-2x)P_V^B] \simeq 0.5\%, \quad (241)$$

where: $x = M_c^2/M_B^2$. The nonvalence penguin contributions are also small, both factorizable (see fig.4) and nonfactorizable (see fig.9):

$$\frac{\Delta\Gamma_{PNV}}{\Gamma_o} \simeq -\frac{64}{9}\pi^2\frac{f_B^2(\mu_o)M_B^3}{M_b^5}\left(1-\frac{M_b^2}{2M_B^2}\right)\cdot A \simeq -0.5\%\,. \quad (242)$$

$$\frac{\Delta\Gamma_{NV}}{\Gamma_o} \simeq -\frac{32}{27}\pi^2\frac{f_b^2(\mu_o)M_B^5}{M_b^7}\rho_o\times$$
$$\left[4(1-x)^2A_d - A_u - \sqrt{1-4x}(1-2x)\frac{<\bar{s}s>^2}{<\bar{u}u>^2}A_u\right] \simeq -0.1\%\,. \quad (243)$$

It is seen that, analogously to the D mesons, the largest effect is the negative contribution to the B^- width due to Pauli interference, but it is only $\simeq -3\%$ here, while the B^o and B_s meson widths receive both only $\simeq 0.5\%$ corrections.

Let us comment finally on the accuracy of all the above calculations. It is extremely difficult to give a reliable estimate of the accuracy. Too many issues are involved, and estimates of various contributions vary in their accuracy. So, the accuracy of predictions for various quantities (the quark masses, $|V_{cb}|$, the D meson decay widths, the B meson lifetime differences, etc.) differ from each other considerably. So, we do not even try to give here any concrete numbers, except for some "educated guesses" like: a) the c- and b-quark masses can hardly be less than 1.6 GeV and 5.0 GeV respectively; b) $f_B(M_b)$ can hardly be larger than 120 MeV; c) the value of $|V_{cb}|$ can hardly deviate more than ± 0.002 from 0.040 (with the experimental value of the semileptonic width taken at its central value, see Eq.(114)); d) the lifetime difference between the B^- and B^o mesons can hardly be more than $\simeq 5\%$.

14. $B^o - \bar{B}^o$ mixing

The effective Lagrangian which determines the mixing width, Γ_{12}, is obtained directly from Eqs.(61),(65) by the substitution: $(\bar{b}...s) \to (\bar{s}...b)$ [19]:

$$L^{(mix)}_{width}(M_b) = \frac{G_F^2(V_{cb}V_{cq}^*)^2}{4\pi}T^{(\bar{c}c)}_{\mu\nu}L^{(mix)}_{\mu\nu}(M_b)\,,$$
$$L^{(mix)}_{\mu\nu}(M_b) = S_u^o(\bar{s}\Gamma_\mu b)(\bar{s}\Gamma_\nu b) + O_u^o(\bar{s}\frac{\lambda^a}{2}\Gamma_\mu b)(\bar{s}\frac{\lambda^a}{2}\Gamma_\nu b)\,, \quad (244)$$

where q denotes the s or d quark field. Because the difference between λ and $\bar{\lambda}$ (see sect.12) is not of great importance for the B mesons but complicates considerably the renormalization formulae, it will be neglected in what follows. Then, using the relation:

$$(\bar{s}\frac{\lambda^a}{2}\Gamma_\mu b)(\bar{s}\frac{\lambda^a}{2}\Gamma_\nu b) = -\frac{2}{3}(\bar{s}\Gamma_\mu b)(\bar{s}\Gamma_\nu b) + \frac{1}{4}g_{\mu\nu}(\bar{s}\Gamma_\rho b)(\bar{s}\Gamma_\rho b)\,, \quad (245)$$

we can rewrite Eq.(244) in the form:

$$L^{(mix)}_{\mu\nu}(M_b) = \left(S_u^o - \frac{2}{3}O_u^o\right)\left[\bar{s}\Gamma_\mu b\cdot\bar{s}\Gamma_\nu b - \frac{1}{4}g_{\mu\nu}\bar{s}\Gamma_\rho b\cdot\bar{s}\Gamma_\rho b\right]_{M_b} +$$
$$\frac{1}{4}\left(S_u^o + \frac{1}{3}O_u^o\right)g_{\mu\nu}\left[\bar{s}\Gamma_\rho b\cdot\bar{s}\Gamma_\rho b\right]_{M_b}. \quad (246)$$

The operators in the square brackets in Eq.(246) renormalize multiplicatively [43], so that we obtain:

$$L^{(mix)}_{\mu\nu}(\mu_o) = \alpha\left[\bar{s}\Gamma_\mu b\cdot\bar{s}\Gamma_\nu b\right]_{\mu_o} + \beta\left[g_{\mu\nu}\bar{s}\Gamma_\rho b\cdot\bar{s}\Gamma_\rho b\right]_{\mu_o}\,,$$
$$\alpha = \Lambda_{bo}^{1/3}\left(S_u^o - \frac{2}{3}O_u^o\right) \simeq -1.93\,,$$
$$\beta = \frac{1}{4}\left[\Lambda_{bo}\left(\frac{1}{3}O_u^o + S_u^o\right) + \Lambda_{bo}^{1/3}\left(\frac{2}{3}O_u^o - S_u^o\right)\right] \simeq 0.83\,,$$
$$\Lambda_{bo} = \left(\frac{\alpha_s(M_c)}{\alpha_s(M_b)}\right)^{12/25}\left(\frac{\alpha_s(\mu_o)}{\alpha_s(M_c)}\right)^{4/9} \simeq 1.614\,. \quad (247)$$

[19] It is implied in Eq.(244) that when calculating the matrix element each of two b-operators acts on both sides, and this gives the additional factor 2 which is compensated by the additional factor 1/2 introduced into Eq.(244)

Using now (see sects.8, 9):

$$< \bar{B}^o(p)|\, L^{(mix)}_{\mu\nu}(\mu_o)\,|B^o(p) >_{factor} \simeq 2\,\alpha\, f_B^2(\mu_o)\left(\frac{2}{3}\,p_\mu\, p_\nu + \frac{1}{6}\,p^2\, g_{\mu\nu}\right) +$$
$$2\,\beta\,\frac{4}{3} f_B^2(\mu_o)\, M_B^2\, g_{\mu\nu}\,,$$
$$< \bar{B}^o(p)|\, L^{(mix)}_{\mu\nu}(\mu_o)\,|B^o(p) >_{nonfactor} \simeq 2\left[-2\,\alpha\,\frac{p_\mu\, p_\nu}{p^2}\, C_G^b\right] +$$
$$2\, g_{\mu\nu}\left[\alpha\,(-C_G^b + 2\,P_V^b) + \beta\,(-6\,C_G^b + 8\,P_V^b)\right], \qquad (248)$$

and contracting with:

$$T^{(\bar{c}c)}_{\mu\nu} = \sqrt{1-4\,x}\left\{\frac{1+2\,x}{3}\,(p_\mu\, p_\nu - p^2\, g_{\mu\nu}) + p^2\, x\, g_{\mu\nu}\right\}, \quad x \simeq \frac{M_c^2}{M_b^2}, \qquad (249)$$

we have:

$$T^{(\bar{c}c)}_{\mu\nu} < \bar{B}^o(p)|\, L^{(mix)}_{\mu\nu}(\mu_o)\,|B^o(p) >_{factor} \simeq$$
$$2\,(-0.57)\, f_B^2(\mu_o)\, M_B^4 \simeq -7.0\, GeV^6\,,$$
$$T^{(\bar{c}c)}_{\mu\nu} < \bar{B}^o(p)|\, L^{(mix)}_{\mu\nu}(\mu_o)\,|B^o(p) >_{nonfactor} \simeq 1.2\, GeV^6\,. \qquad (250)$$

So, we have finally for Γ_{12}:

$$\frac{\Gamma_{12}(B)}{\Gamma_o} = \frac{1}{\Gamma_o}\frac{< \bar{B}^o|\, L^{(mix)}_{width}\,|B^o >}{2\, M_B} \simeq$$
$$\xi_{KM}\, 16\pi^2\,\frac{f_B^2(\mu_o)\, M_B^3}{M_b^5}\,[-0.57\,(1-17\%)] \simeq (-2.7\%)\,\xi_{KM}\,, \qquad (251)$$

where ξ_{KM} is the ratio of the Kobayashi-Maskawa factors. It is seen that the nonfactorizable contributions appear to be surprisingly large here and decrease the mixing by $\simeq 17\%$.

An analogous situation takes place for the mixing mass of the B^o and $\bar{B}^o$ mesons. The effective Lagrangian can be found in any review and has the form:

$$L^{(mix)}_{mass}(M_b) = C_o\,[\bar{s}\,\Gamma_\rho\, b\cdot \bar{s}\,\Gamma_\rho\, b]_{M_b}\,, \qquad (252)$$

where C_o is a known coefficient (see sect.15). The above operator renormalizes multiplicatively [22], so that there appears only the factor Λ_{bo} (see

Eq.(247)) when it is renormalized to the point μ_o. The matrix element is given by Eq.(248), so that we have:

$$< \bar{B}^o|\, L^{(mix)}_{mass}(M_b)\,|B^o >_{factor} \simeq \Lambda_{bo}\,\frac{8}{3} f_B^2(\mu_o) M_B^2\, C_o \simeq 0.95\, GeV^4\, C_o\,,$$
$$< \bar{B}^o|\, L^{(mix)}_{mass}(M_b)\,|B^o >_{nonfactor} \simeq$$
$$2\Lambda_{bo}\left[-6C_G^b + 8P_V^b\right] C_o \simeq -0.176\, GeV^4\, C_o\,. \qquad (253)$$

On the whole:

$$< \bar{B}^o|\, L^{(mix)}_{mass}(M_b)\,|B^o >_{M_b} \simeq \Lambda_{bo}\,\frac{8}{3} f_B^2(\mu_o) M_B^2\,(1-18\%)\, C_o \simeq$$
$$\frac{8}{3} f_B^2(M_b) M_B^2\,(1-18\%)\, C_o \simeq 0.8\, GeV^4\, C_o\,;$$
$$B_B(M_b) \simeq (1-0.18) \simeq 0.82\,, \qquad (254)$$

and the corrections to the factorization approximation are also very significant here.

15. The unitarity triangle

The purpose of this section is to show that the results obtained above are marginally consistent with the available data and can be used to determine the parameters of the unitarity triangle (we use below in this section the Wolfenstein parametrization and the notations from [45]).

We would like to emphasize that we have not tried in this section to account for all the various predictions available in the literature for the parameters involved (f_B, B_B, B_K, etc.). In fact, there are a number of papers which try to account with an equal weight for all the values available in the literature for these parameters. This results in large uncertainties which prevent to obtain more or less definite results from the available experimental data. Instead, we prefer to use only those results which, from our viewpoint, are more reliable and this allows us to obtain the concrete predictions for the unitarity triangle parameters.

1). Using: $V_{cb} = A\,\lambda^2$, $\lambda \simeq 0.22$ and (see Eq.(114)) $V_{cb} \simeq 0.040$, one has:

$$A \simeq 0.825\,. \qquad (255)$$

2) . The $B_d - \bar{B}_d$ mass difference is given by:

$$x_d \equiv \frac{\Delta M_d}{\Gamma_B} \simeq$$

$$\tau_B \frac{G_F^2}{6\pi^2} M_W^2 M_B \left[f_B^2 B_B\right]_{M_B} \left[\bar{\eta}^*_{2B} S(x_t^*)\right] |V_{td}|^2 = (0.71 \pm 0.07) , \quad (256)$$

$$S(x) = x\left[\frac{1}{4} - \frac{9}{4(x-1)} - \frac{3}{2(x-1)^2}\right] + \frac{3}{2}\left(\frac{x}{x-1}\right)^3 \log x ,$$

$$x_t^* = \left(\frac{M_t^*}{M_W}\right)^2 ,$$

$$|V_{td}|^2 = A^2 \lambda^6 \left[(1-\rho)^2 + \eta^2\right] \simeq 0.77 \cdot 10^{-4} \left[(1-\rho)^2 + \eta^2\right] . \quad (257)$$

Using in Eq.(256) (see Eqs.(128), (254) and [45]) [20]:

$$M_t^* \simeq 180\, GeV , \quad [\bar{\eta}^*_{2B} S(x_t^*)] \simeq 0.85 \cdot 2.7 \simeq 2.30 , \quad (258)$$

$$\tau_B \simeq 1.6 \cdot 10^{-12}\, s , \quad f_B(M_b) \simeq 113\, MeV , \quad B_B(M_b) \simeq 0.82 , \quad (259)$$

one obtains:

$$\left[(1-\rho)^2 + \eta^2\right] \simeq \frac{x_d}{0.35} \simeq 2.0 . \quad (260)$$

3) . Using also:

$$\left|\frac{V_{ub}}{V_{cb}}\right|^2 = \lambda^2 (\rho^2 + \eta^2) \simeq 1 \cdot 10^{-2} , \quad (\rho^2 + \eta^2) \simeq 0.2 , \quad (261)$$

one obtains then from Eqs.(260), (261):

$$\rho \simeq -0.4 , \quad \eta \simeq 0.2 , \quad \delta = tg^{-1}(\frac{\eta}{\rho}) \simeq 0.85\,\pi , \quad (262)$$

$$\sin(2\alpha) \simeq 0.60 , \quad \sin(2\beta) \simeq 0.28 , \quad \sin(2\gamma) \simeq -0.80 . \quad (263)$$

[20] In order to use the numericals from [45] we use the mass M_t^* which is the so-called $\overline{MS}$ mass; $M_t^* = 180\, GeV$ corresponds to the pole mass: $M_t^{pole} \simeq 190\, GeV$, which is marginally consistent with the CDF and LEP data, [46].

4) . The CP-violating part of the $K^o - \bar{K}^o$ mixing can be written in the form [45]:

$$e^{-i\pi/4}\, \epsilon_K \simeq C_\epsilon\, B_K\, A^2 \lambda^6\, \eta \left[P_o + A^2 \lambda^4 (1-\rho) \eta^*_{2K} S(x_t^*)\right] , \quad (264)$$

$$C_\epsilon = \frac{G_F^2 F_K^2 M_K M_W^2}{6\sqrt{2}\,\pi^2\, \Delta M_K} \simeq 3.85 \cdot 10^4 , \quad (265)$$

$$P_o = x_c\, (\, \eta_3\, S_3(x_t) - \eta_1\,) ,$$

$$S_3(x_t) = \left[\log\frac{x_t}{x_c} - \frac{3}{4}\frac{x_t}{x_t-1}\left(\frac{x_t \log x_t}{x_t-1} - 1\right)\right] , \quad (266)$$

$$x_i = \left(\frac{M_i}{M_W}\right)^2 , \quad M_c \simeq 1.65\, GeV , \quad M_t^* \simeq 180\, GeV , \quad (267)$$

$$\eta_1 \simeq 0.80 , \quad \eta^*_{2K} \simeq 0.57 , \quad \eta_3 \simeq 0.36 . \quad (268)$$

Substituting all this into Eq.(264), one has:

$$e^{-i\pi/4}\, \epsilon_K = 2.26\, 10^{-3} \simeq 7.3 \cdot 10^{-3}\, B_K\, \eta \left[(1-\rho) + \omega_o\right] , \quad (269)$$

$$\omega_o = \frac{P_o}{|V_{cb}|^2\, \eta^*_{2K}\, S(x_t^*)} \simeq 0.38$$

$$\eta\,(1 - \rho + \omega_o) \simeq \frac{0.31}{B_K} \quad (270)$$

Let us recall [45] that, unlike $B_B(M_b)$ in Eq.(254), B_K in Eq.(264) is defined as:

$$< \bar{K}^o |\, \bar{s}\, \Gamma_\nu\, d \cdot \bar{s}\, \Gamma_\nu\, d\, | K^o >_\mu \equiv \frac{8}{3}\, B_K(\mu)\, f_K^2\, M_K^2 , \quad (271)$$

$$B_K = B_K(\mu)\, \alpha_s^{-2/9}(\mu) . \quad (272)$$

The characteristic value of B_K obtained from lattice calculations is: $B_K = (0.9 \pm 0.1)$ [47] , and substituting $B_K \simeq 0.9$ into Eq.(270), one obtains:

$$\eta\,(1 - \rho + \omega_o) \simeq 0.345 , \quad (273)$$

Using now: $\rho \simeq -0.40$, $\omega_o \simeq 0.38$ one obtains $\eta \simeq 0.19$, in agreement with Eq.(262).

5). Clearly, we can proceed in the opposite way: using $B_K \simeq 0.9$ we obtain from ϵ_K and x_d:

$$\eta\,(1-\rho+0.38) \simeq 0.345\,, \quad (1-\rho)^2+\eta^2 \simeq 2.0\,. \tag{274}$$

This gives us then: $\rho \simeq -0.4$, $\eta \simeq 0.2$ and $|V_{ub}/V_{cb}| \simeq 0.10$.

With the above parameters, the CP-violating asymmetry in $B^o \to \Psi K_S$ decay is:

$$A(B^o \to \Psi K_S) \simeq \frac{x_d}{1+x_d^2}\sin(2\beta) \simeq 0.13\,.$$

6). The quantity ϵ'/ϵ can be represented in the form [48]:

$$\frac{\epsilon'}{\epsilon} = \frac{Im\,\lambda_t}{1.7}\left[P^{(1/2)} - P^{(3/2)}\right] \simeq 0.4\cdot 10^{-4}\left[P^{(1/2)} - P^{(3/2)}\right]\,, \tag{275}$$

$$Im\,\lambda_t = \eta\,A^2\,\lambda^5 \simeq 0.7\cdot 10^{-4}\,, \tag{276}$$

where $P^{(1/2)}$ and $P^{(3/2)}$ are expressed through the known coefficients $a_i(M_t)$ and the parameters $B_6^{(1/2)}$, $B_8^{(3/2)}$. These latter are determined from the corresponding matrix elements of the penguin operators, see [48]. In the factorization approximation: $B_6^{(1/2)} = B_8^{(3/2)} \equiv 1$, and the values obtained from both the lattice calculations and the $1/N_c$ expansion agree with the factorization approximation within ±20%. Using the values of $a_i(LO)$ from the Tables 14, 15 in [48] and $B_6^{(1/2)} \simeq B_8^{(3/2)} \simeq 1$, one obtains:

$$P^{(1/2)} \simeq 5.2\,, \quad P^{(3/2)} \simeq 6.2\,, \quad \frac{\epsilon'}{\epsilon} \simeq -0.4\cdot 10^{-4}\,. \tag{277}$$

The above value can not be taken too literally because it is a result of strong cancelations. [21] To be conservative, the realistic value of ϵ'/ϵ can be considered to lie in the interval: $\sim \pm(1-2)\cdot 10^{-4}$. In any case, this is inconsistent with the NA31-result: $\epsilon'/\epsilon = (23\pm 7)\cdot 10^{-4}$, while there is no contradiction with the E731-result: $\epsilon'/\epsilon = (7\pm 6)\cdot 10^{-4}$, [50], [51], [52].

7). The box diagram contribution to $(K_L - K_S)$ the mass difference, $\Delta M_K = M(K_L) - M(K_S)$, is [53], [54]:

$$\{\Delta M_K\}_{box} = \frac{G_F^2}{6\pi^2}\,f_K^2\,M_K\,M_c^2\,\lambda^2\,B_K\,\eta_1\left[1+O(10^{-2})\right]\,, \tag{278}$$

[21] These are due to the large value of the t-quark mass which enhances the electroweak penguin contribution $P^{(3/2)}$, see [49].

where the correction $O(10^{-2})$ represents the t-quark contribution and other small corrections. Using: $f_K \simeq 162\,MeV$, $M_c \simeq 1.65\,GeV$, $B_K \simeq 0.9$, one obtains:

$$\{\Delta M_K\}_{box} \simeq (4.0\cdot 10^{-15}\,GeV)\,B_K\,\eta_1$$
$$\simeq (3.6\cdot 10^{-15}\,GeV)\,\eta_1 \simeq 2.9\cdot 10^{-15}\,GeV\,, \tag{279}$$

for $\eta_1 \simeq 0.8$. The experimental value is: $\Delta M_K = 3.5\cdot 10^{-15}\,GeV$, so that there is not much room for additional large distance contributions.

16. Summary and conclusions

Let us summarize first in short the results obtained for the D mesons.

It is seen that the overall picture is sufficiently complicated: there are a number of contributions, all of the same order and of different signs. As to the four-fermion operator contributions we were mainly interested in, the qualitative picture is as follows.

Although they are formally $0(1/M_c^3)$ corrections, these contributions are very important numerically and comparable with the Born term. In particular, their final effect in the nonleptonic widths is much larger than those of the leading $0(1/M_c^2)$ corrections. There are two reasons for this.

i) Various $0(1/M_c^2)$ corrections, being only a few times smaller than the Born term, cancel each other strongly, see Eq.(147);

ii) The four-fermion operator contributions are the first to gain a large numerical factor from the larger two-particle phase space. Otherwise they would have been much smaller than separate $0(1/M_c^2)$ terms. It is clear that this effect operates one time only. So, there are reasons to expect that all other $O(1/M^3)$ and higher order corrections are much smaller, and those considered in this paper are the main ones. Some simple estimates confirm this.

As for the relative significance of various contributions, the picture is as follows.

1) Most significant is the destructive Pauli interference effect (see fig.3c) of two d-quarks in D^- decay ($\sim -60\%$ of the Born term, Γ_o).

2) The (cross) annihilation contribution (see fig.6b) ensured by the nonperturbative nonfactorizable gluon interaction decreases further ($\sim -20\%$ of Γ_o) the D^- nonleptonic width.

3) Both the above contributions could have appeared much larger, but are strongly suppressed by the much smaller two particle phase space. The

reason is that the "normal" total 4-momentum of the quark pair is (see fig.3a): $\lambda \simeq (P_c + k_1) = P_D$ (k is the momentum of the spectator quark). However, it is: $\bar{\lambda} \simeq (P_c - k_2) = (P_D - k_1 - k_2)$ for both these "cross-contributions". Because the charm quark is not very heavy and spectator quarks carry a significant fraction of the charmed meson momentum, this leads to a strong suppression: $\lambda^2 \gg \bar{\lambda}^2$. This effect remains noticeable even for the B mesons.

4) The (direct) weak annihilation contribution (see fig.6a) increases significantly ($\sim 30\%$ of Γ_o) the D^o nonleptonic width [22].

5) The nonvalence penguin contributions, both factorizable (see fig.4) and nonfactorizable (see fig.9)), are very significant and, on the whole, diminish considerably ($\sim -(20-25)\%$) both semileptonic and nonleptonic widths.

6) There are no noticeable positive valence contributions into the D_s meson width. Those which are available are negative and decrease ($\sim -7\%$ of the Born terms) both semileptonic and nonleptonic widths. As a result, there is a sizeable difference ($\simeq 10\%$) between the above calculated and the experimental numbers. A possible explanation may be due to the SU(3) symmetry breaking effects which were neglected in these calculations. At first sight, however, most of them tend to increase the discrepancy rather than to decrease it. Clearly, this subject requires careful investigation which is out the scope of this paper.

One of the important results of all the calculations above is that the factorization approximation works well for the matrix elements, i.e. the nonfactorizable contributions which have the same parametrical behaviour at large M_Q are, in comparison, an order of magnitude smaller. Nevertheles, they are of importance due to specific features of the problem considered. Firstly, these nonfactorizable parts enter the effective Lagrangian with much larger coefficients. Secondly, the factorizable contributions are additionally suppressed by the smaller phase space (see point 3 above).

Let us add a few words about baryon lifetimes. It seems clear that the pattern here is very unlike those for the pseudoscalar mesons. The entire structure of the matrix elements of L_{eff} is quite different, and the reasons connected with the helicity suppression of factorizable contributions are not operative here. So, one can expect that: 1) the scale of the 4-fermion operator contributions to the baryon lifetimes is potentially a few times larger (term by term) than those for the pseudoscalar mesons; 2) the significance of nonfactorizable contributions will be smaller for baryons. Therefore, it

[22] Let us emphasize that the annihilation contribution $\simeq 30\%$ into the inclusive width does not contradict that it can be $\sim 100\%$ in separate exclusive modes, see [55].

seems, the main problem here will be to calculate reliably the factorized parts of the matrix elements.

For the B mesons, all the above qualitative properties remain true but , clearly, the role of all power corrections to the Born term becomes much smaller. It seems clear now also that the factorization approximation will be especially good for the B_c mesons.

It is important that we understand now the properties of all the main contributions giving rise the lifetime differences and that there is sufficiently good agreement between the calculations for the D mesons and the experimental data. This gives us confidence that the predictions obtained above for the B mesons are reliable. Therefore, we can insist now that the lifetime difference between the $B^{\pm}$ and B^o mesons will not exceed $\simeq 5\%$, while those between the B^o and B_s mesons have to be even smaller. As it was pointed out above, the scale of the 4-fermion operator contributions for the b-baryons can be a few times larger.

Surprisingly large corrections to the factorization approximation ($\simeq -18\%$) are found for $B^o - \bar{B}^o$ mixing. This reduction of the $B_d - \bar{B}_d$ mixing mass is of importance, in particular, for the parameters of the unitarity triangle. These latter are calculated in sect.15 and are:

$$\lambda \simeq 0.22\,, \quad A \simeq 0.825\,, \quad \rho \simeq -0.40\,, \quad \eta \simeq 0.20\,, \tag{280}$$

$$\sin(2\alpha) \simeq 0.60\,, \quad \sin(2\beta) \simeq 0.28\,, \quad \sin(2\gamma) \simeq -0.80\,. \tag{281}$$

We would like to emphasize finally that the experimental value of $Br(D \to e\nu + X)$ requires a noticeably larger value of the c-quark mass ($M_c \simeq 1.65\, GeV$), in comparison with those ($M_c \simeq 1.4 - 1.5\, GeV$) in common use. As a result, because the mass formulae tell us that the quark mass difference is close to those of the mesons, this leads to a value of the b-quark mass ($M_b \simeq 5.04\, GeV$) which is also considerably larger than those in common use ($M_b \simeq 4.6 - 4.8\, GeV$). Further, the chain of the argument is as follows. The large value of M_b leads to a small value for f_B : $f_B(M_B) \simeq 113\, MeV$. Together with (see sect.14) $B_B \simeq 0.82$, this requires a large value of the t-quark $\overline{MS}$-mass: $M_t^* \simeq 180\, GeV$, to reproduce the experimental value of $x_d \simeq 0.70$. This results finally in the rather small value of $\epsilon'/\epsilon \sim (\pm 1 \cdot 10^{-4})$, which is incompatable with the NA31-group result but does not contradict to those from the E731-group.

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Appendix

For the convenience of the reader, we list below the notations and numerical values of various parameters used in the text.

Parameters:

$$M_b = 5.04\,GeV\,[Eq.(103)],$$
$$M_c = 1.65\,GeV[sect.4],\ \ \mu_o^2 = 0.5\,GeV^2\,[Eq.(43)]; \qquad (282)$$

$$\alpha_s(M_b^2) = 0.204,\ \ \alpha_s(M_c^2) = 0.310,\ \ \alpha_s(\mu_o^2) = 0.580\,[Eq.(49)]; \qquad (283)$$

$$<0|\,\bar q i\gamma_5\,Q|D> = r_D = f_D\,\frac{M_D^2}{M_c}; \qquad (284)$$

$$f_D(\mu_o) = 144\,MeV,\quad f_D(M_c) = 165\,MeV\quad [Eq.(123)]; \qquad (285)$$

$$f_B(\mu_o) = 89\,MeV,\quad f_B(M_b) = 113\,MeV\quad [Eq.(128)]; \qquad (286)$$

$$\eta_{co} = \frac{\alpha_s(\mu_o^2)}{\alpha_s(M_c^2)} = 1.87,\quad \tau_{co} = \eta_{co}^{1/2} = 1.37; \qquad (287)$$

$$\eta_{bo} = \frac{\alpha_s(\mu_o^2)}{\alpha_s(M_b^2)} = 2.84,\ \eta_{bc} = \frac{\alpha_s(M_c^2)}{\alpha_s(M_B^2)} = 1.52,$$
$$\tau_{bo} = \eta_{bc}^{27/50}\,\tau_{co} = 1.715; \qquad (288)$$

$$\left(1-\eta_{co}^{-2/9}\right) = 0.13,\ \ \rho_o = \frac{\alpha_s(\mu_o^2)}{2\pi}\log\frac{\mu_{max}^2}{\mu_{min}^2} = 0.1; \qquad (289)$$

$$<\vec p_c^{\,2}> = \frac{<D|\,\bar c\,(i\vec D)^2\,c\,|D>}{<D|\,\bar c\,c\,|D>} = 0.3\,GeV^2,\quad \Delta_K = \frac{<\vec p_c^{\,2}>}{2\,M_c^2}; \qquad (290)$$

$$\Delta_G = \frac{1}{M_c^2}\,\frac{<D|\,\bar c\frac{i}{2}\,g_s\,\sigma_{\mu\nu}\,G^a_{\mu\nu}\,\frac{\lambda^a}{2}\,c\,|D>}{<D|\,\bar c\,c\,|D>} = \frac{3}{4}\,\frac{M_D^2 - M_D^2}{M_c^2} = 0.15; \qquad (291)$$

$$N_v = 2.54 \quad [Eqs.(51),(52)], \quad A = 0.40 \quad [Eqs.(73),(74)]\,. \tag{292}$$

The matrix elements (sects.9,10):

$$< D(p)|\bar{c}\Gamma_\mu\, q\cdot\bar{q}\Gamma_\nu\, c\,|D(p)>_{\mu_o} = p_\mu\, p_\nu\, f_D^2(\mu_o)\,\delta_V + \frac{2}{27}\rho_o\, g_{\mu\nu}\, r_D^2(\mu_o), \tag{293}$$

$$< D(p)|\,\bar{c}\,\Gamma_\mu\frac{\lambda^a}{2}q\cdot\bar{q}\frac{\lambda^a}{2}\Gamma_\nu\, c\,|D(p)>_{\mu_o}=$$

$$\left(\tfrac{p_\mu\, p_\nu}{p^2} - g_{\mu\nu}\right) C_G\,\delta_V + g_{\mu\nu} P_V\,\delta_V - \tfrac{1}{81}\,\rho_o\, g_{\mu\nu} r_D^2(\mu_o)\,, \tag{294}$$

Here: δ_V is unity if the quark field in the matrix element has a valence flavour and is zero otherwise.

$$C_G^c = 0.48\cdot 10^{-2} GeV^4\,[Eq.(162)],\; C_G^b = 1.15\cdot 10^{-2} GeV^4\,[Eq.(163)]; \tag{295}$$

$$P_V^c = 0.685\cdot 10^{-3}\, GeV^4\,,\;\; P_V^b = 0.18\cdot 10^{-2}\, GeV^4 \quad [Eq.(192)]; \tag{296}$$

$$< D(p)|(J^a_\rho\cdot\bar{c}\gamma_\rho(1\pm\gamma_5)\frac{\lambda^a}{2}c\,|D(p)>_{\mu_o}=$$

$$-\tfrac{4}{9}\, r_D^2(\mu_o\left(1-\tfrac{M_c^2}{2M_D^2}\right)+\cdots, \tag{297}$$

where the dots mean that we neglected corrections because this matrix element appears itself as a correction in the above calculations.

In addition to the 4-quark operators considered in the text, there is also the nonvalence 4-quark operator contribution originating from the Born diagram, and we show below that it is negligibly small. Using the Fock-Schwinger gauge:

$$x_\mu A_\mu(x) = 0, \quad A_\mu(x) = \frac{1}{2}x_\nu G_{\nu\mu}(0) + \frac{1}{3}x_\nu x_\lambda D_\lambda G_{\nu\mu}(0) + \cdots\,, \tag{298}$$

the Born diagram contribution can be represented in the form:

$$\Gamma_{Born} \sim < D(p)|\left\{\bar{c}(x)\overleftarrow{\partial}_\mu^{\,2}\,(1-\gamma_5)\cdot\partial_\nu^2\,(i\hat{\partial})\,c(x)\right\}_{x=0}|D(p)>\,. \tag{299}$$

Using the equations of motion: $i\hat{\partial}c = M_c\, c - g_s\hat{A}\,c$, it is not difficult to obtain:

$$\Gamma_{Born} \sim M_c^5 < D(p)|\bar{c}(0)(1-\gamma_5)\,(1-ig_s\sigma_{\mu\nu}G_{\mu\nu}(0))\;c(0)|D(p)> -$$
$$\frac{1}{6}g_s^2\, M_c^2 < D(p)|\bar{c}(0)\gamma_\mu(1+\gamma_5)\frac{\lambda^a}{2}c(0)\cdot J^a_\mu(0)|D(p)> +$$
$$\frac{1}{2}(\delta_V+\delta_A)+\cdots, \tag{300}$$

where "…" denotes higher dimension terms. Also ($\overleftrightarrow{\partial}_\mu = \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu$),

$$\delta_V = < D(p)|\bar{c}(0)\gamma_\mu\frac{\lambda^a}{2}\overleftrightarrow{\partial}_\nu c(0)\cdot G^a_{\mu\nu}(0)|D(p)> = 0 \tag{301}$$

due to C-parity, and

$$\delta_A = < D(p)|\bar{c}(0)\gamma_\mu\gamma_5\frac{\lambda^a}{2}\overleftrightarrow{\partial}_\nu c(0)\cdot G^a_{\mu\nu}(0)|D(p)> = 0 \tag{302}$$

due to P-parity. Therefore, the correction due to the 4-fermion operator is:

$$\delta_4 = 1 - \frac{2}{3}\pi\,\alpha_s(M_c)\frac{1}{2\,M_D\,M_c^3} < D|\bar{c}(0)\gamma_\mu(1+\gamma_5)\frac{\lambda^a}{2}c(0)\cdot J^a_\mu(0)|D> \tag{303}$$

This becomes after the factorization of the matrix element:

$$\delta_4 \simeq 1 + \frac{4}{27}\pi\,\alpha_s(M_c)\frac{f_D^2(M_c)\,M_D^3}{M_c^5}\left(1-\frac{M_c^2}{2\,M_D^2}\right) \simeq 1 + 1.3\cdot 10^{-3}\,, \tag{304}$$

so that it is negligble even for the D-meson.

For the semileptonic decays the above described 4-quark operator contribution is the only one, in addition to the $\bar{c}c$, $\bar{c}\sigma_{\mu\nu}G_{\mu\nu}\,c$ and 4-quark operators described in the text, which is capable to give $\sim O(\Lambda^3_{QCD}/M_c^3)$ correction. For the nonleptonic decays, in addition, it is not difficult to see that it is sufficient to replace Δ_G in the last term in Eq.(25) (which is due to the fig.2 diagram) by its original expression:

$$\Delta_G \to -\frac{< D|\bar{c}\frac{\lambda^a}{2}\gamma_\mu\gamma_5\, i\overleftrightarrow{\partial}_\nu\, c\cdot\widetilde{G}^a_{\mu\nu}|D>}{2 < D|\bar{c}c|D > M_c^3}\,. \tag{305}$$