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ION-ACOUSTIC MULTISOLITONS IN PLASMAS WITH ION BEAM

BudkerINP 95-3

НОВОСИБИРСК

Ion-acoustic multisolitons in plasmas with ion beam

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Abstract

It is shown that in plasmas with a supersonic ion beam there exist nonlinear waves, multisolitons, which are localized in the direction of the self propagation and are comprised of a finite set of equal plane solitons. Multisoliton stability is studied in the framework of the Korteveg-de Vries equation. Analysis is based on the description of the adiabatic interaction between solitons and ion beam.

1 Introduction

Ion-acoustic solitons in plasmas are objects that behave themselves, in many aspects, not as waves but rather as particles. Their dynamics is a matter of many studies (see, e.g., the survey [1] and cited literature). It is well-known [2] that a soliton interaction with resonant ions could lead to a coalescence of the solitons in a nonlinear structure referred to as a collisionless shock wave. With a neglect of a dissipation due to particle collisions, such a wave presents an infinite set of equal solitons located behind the shock front. The existence of this front is provided by a collisionless dissipation resulting in a partial transfer of energy to the resonant ions.

In the present paper the interaction between a finite set of solitons, cited below as multisoliton, and an ion beam is studied. Analysis is carried out for the case when the soliton velocities are close to the average beam velocity. The latter causes the resonant "multisoliton-ion beam" interaction. It is also presumed that the ion beam is sufficiently weak, so that to be unable to modify essentially the dispersion of an individual soliton.

The further discussion is scheduled as follows. In the next Section the ion-acoustic multisoliton is introduced and its basic characteristics are computed. Section 3 contains analysis of the adiabatic interaction between two KdV solitons with close amplitudes. In Section 4 the problem of multisoliton stability is treated. Multisoliton dynamics is shown to be similar to that of an oscillatory system with several degrees of freedom. Section 5 is devoted to final conclusions.

2 Ion-acoustic multisoliton

Multisoliton is a localized nonlinear wave composed of a finite set of coupled plane solitons with equal amplitudes (see Fig.1a). In plasma with an ion beam, the multisoliton propagates with a phase velocity equal to the average beam velocity. For more clarity of presentation, standard arguments of the ion-acoustic soliton theory are restored below.

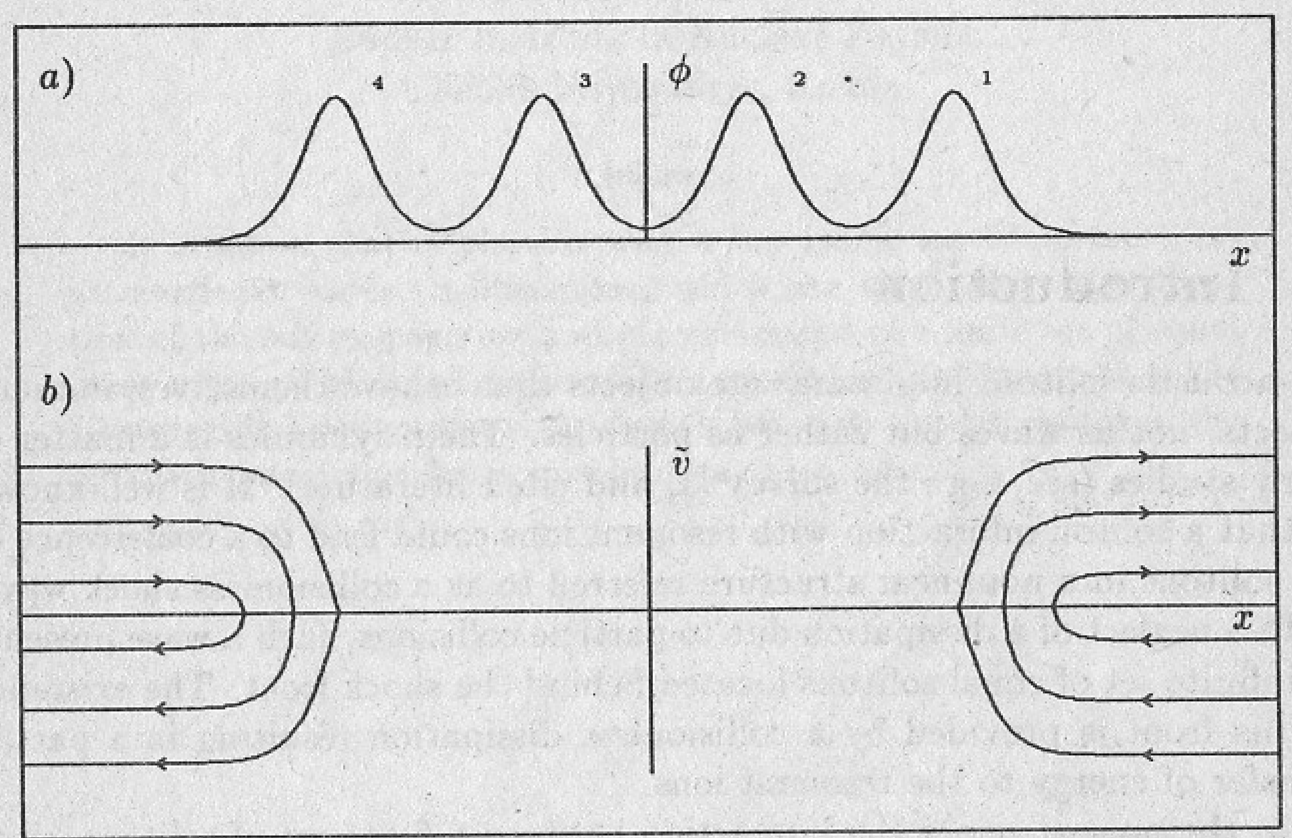

Figure 1: *Multisoliton composed of four solitons. (a) – Spatial distribution of the potential; (b) – Phase trajectories of the resonant beam paricles in the frame of reference associated with the multisoliton. Particles reflect from the leading and the back multisoliton fronts.*

Consider that plasma is neutral outside the multisoliton, so that the electron density n_{eo} is equal to the combined densities of the bulk plasma ions n_i and of the beam ions n_{b0}. The electron density is governed by the Boltzmann equation:

$$n_e = n_{e0}\exp(e\phi/T_e), \qquad (1)$$

here ϕ is the electrostatic potential; e, T_e are the ion charge ($e > 0$, $Z_i = 1$) and the electron temperature, respectively. Cold ions of the bulk plasma are

described by the hydrodynamic equations:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x}, \qquad (2)$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial n_i v}{\partial x} = 0\ , \qquad (3)$$

where m_i, v, n_i are the ion mass, the velocity and the density. For the wave propagating with the phase velocity u_0, the relationship between n_i and ϕ is given by

$$n_i = \frac{n_{i0}}{\sqrt{1 - 2e\phi/m_i u_0^2}}\ . \qquad (4)$$

To determine the density of the beam ions, consider the reference frame associated with the wave. Away from the multisoliton, the distribution function of the beam ions $f_0(v)$ is assumed to be symmetrical with respect to the average beam velocity. Suppose that the average beam velocity is equal to u_0 and that the following relations are obeyed:

$$e\phi_{max} \sim \frac{m_i\tilde{v}^2}{2} < e\phi_{max}. \qquad (5)$$

Here $\tilde{v} = v - u_0$ is a deviation from the average velocity and ϕ_{max} is the amplitude of the solitons, which compose the multisoliton. Inequality (5) states that all beam particles are reflected from the leading and the back multisoliton fronts (see Fig.1b), so that there are no beam ions in the "internal" region of the multisoliton. Besides, as it follows from (5), the velocity spread in the beam is taken to be not too small. The above-listed assumptions allow one to consider zero potential outside the multisoliton, to circumvent the treatment of passing beam particles, and to simplify the multisoliton dynamic analysis.

The potential ϕ is time independent in the chosen reference frame, and the stationary distribution function f can be found from the consideration of its constancy on the particle phase trajectories:

$$f = f_0(u_0 - \sqrt{\tilde{v}^2 + 2e\phi/m_i})\ .$$

Hence, the density of the beam ions is expressed as

$$n_b = 2\int_{-\tilde{v}_\phi}^{0} f_0(u_0 - \sqrt{\tilde{v}^2 + 2e\phi/m_i})\,d\tilde{v}\ ,\ \tilde{v}_\phi = \sqrt{2e(\phi_{max} - \phi)/m_i}\ . \qquad (6)$$

The potential ϕ is evaluated from the Poisson equation:

$$\frac{d^2\phi}{dx^2} = -4\pi e\left[\frac{n_{i0}}{\sqrt{1-2e\phi/m_i u_0^2}} - n_{e0}\exp\left(\frac{e\phi}{T_e}\right) + n_b\right]. \qquad (7)$$

Multiplying (7) by $\phi' \equiv d\phi/dx$ and integrating it from x to infinity, one comes to the equation:

$$\frac{\phi'^2}{8\pi} + W(\phi) = e\int_x^\infty n_b\phi' dx, \qquad (8)$$

$$W(\phi) \equiv n_{i0}m_i u_0^2\left(1-\sqrt{1-2e\phi/m_i u_0^2}\right) - n_{e0}T_e\left(\exp\left(\frac{e\phi}{T_e}\right)-1\right). \qquad (9)$$

In plasma without resonant beam particles, solitary waves, described by (8), present conventional ion-acoustic solitons with supersonic phase velocities, $c_s < u_0 < 1.6c_s$, where $c_s \equiv \sqrt{T_e/m_i}$ is a sound velocity. The presence of the ion beam leads to the qualitative changes. The matter is that the resonant particles, being reflected from the multisoliton leading front, increase their velocity in the laboratory frame of reference. The corresponding energy, which is taken off per unit time, can be evaluated as

$$q = \int_{-\infty}^{0} dv' v' f_0(u_0 - v') m_i[(u_0+v')^2 - (u_0-v')^2]/2\,. \qquad (10)$$

On the contrary, while reflecting from the back front of the multisoliton, the resonant particles give up the same energy q. The multisoliton presents a series of solitons that transmits the energy from the fast particles, which catch it up, to the slower particles, which multisoliton catches up with. Formally, this fact follows from (8) having regard that the integral I in the right side of (8) is a constant proportional to q,

$$I = -\int_0^{\phi_{max}} d\phi\, n_b = -\frac{q}{eu_0}, \qquad (11)$$

in the region between the tops of the first and the last solitons where $n_b = 0$. Equation (8) amounts to the implicit form of the energy balance equation. Its elementary integration shows that the potential is a periodic spatial function between the regions of the beam reflection. The maximum and the minimum potential values are to be found from the same equation

$$W(\phi) = -q/u_0\,, \qquad (12)$$

while the spatial period is given by

$$\lambda = \int_{\phi_{min}}^{\phi_{max}} \frac{d\phi}{\sqrt{8\pi(-W - q/u_0)}}. \qquad (13)$$

The total number of the potential oscillations is arbitrary being defined by initial conditions.

For an ion beam with a low density,

$$n_{b0}/n_{e0} \ll (u_0/c_s - 1)^2\,, \qquad (14)$$

that is actually treated in this paper, the value ϕ_{max} is close to the amplitude $\phi^{(0)}$ of the ion-acoustic soliton propagating with the phase velocity u_0 in plasma without the ion beam:

$$e\phi_{max} \simeq e\phi^{(0)} \sim (u_0/c_s - 1)T_e\,.$$

The value ϕ_{min} is equal to

$$\left(\frac{\phi_{min}}{T}\right)^2 \simeq \frac{2}{1-c_s^2/u_0^2}\left(\frac{q}{n_{e0}T_e u_0}\right) \ll \left(\frac{\phi_{max}}{T}\right)^2, \qquad (15)$$

and the spatial period is evaluated as

$$\lambda \simeq 2\int_{\phi_{min}/2}^{\phi_{max}} \frac{d\phi}{\sqrt{8\pi|W|}}. \qquad (16)$$

In the latter formula one can neglect the difference between n_{i0} and n_{e0} in the expression for W (see (9)). It is remarkable that for the fixed phase velocity u_0, all basic multisoliton parameters are determined by the intensity of the energy exchange q and are independent on the beam density. In the same way, contrary to the common considerations [3, 4], the parameters of a collisionless ion-acoustic shock wave are mainly governed by the intensity of a collisionless dissipation at the shock front and slightly depend on the density of the resonant ions.

3 Adiabatic interaction of two solitons

Before treating the multisoliton stability problem, let us clarify certain properties of the "soliton-soliton" interactions. For the computational simplicity, we restrict ourselves to the long wave limit, assuming soliton amplitudes to be small, $e\phi \ll T_e$. In this case equations (1-3), (7) reduce to the Korteveg-de Vries equation [5]:

$$\frac{\partial \psi}{\partial \tau} + \psi \frac{\partial \psi}{\partial \xi} + \frac{1}{2}\frac{\partial^3 \psi}{\partial \xi^3} = 0 , \qquad (17)$$

where the following dimensionless variables are introduced:

$$\psi = e\phi / T_e, \quad \xi = \omega_{pi}(x/c_s - t), \quad \tau = \omega_{pi} t; \quad \omega_{pi}^2 \equiv \frac{4\pi e^2 n_{i0}}{m_i}.$$

Consider a collision between two solitons with close amplitudes. At the initial moment the solitons are supposed to be widely spaced. All information about further evalution of such a system can be surely gained from the exact solution of the KdV equation. It is more convenient for our purposes, however, to invoke the direct description of the interaction between the solitons, based on the proximity of their amplitudes.

Let ϵ_i be a normalized soliton velocity defined as

$$\epsilon_i \equiv u_i / c_s - 1 .$$

Suppose that at the initial moment the soliton velocities ϵ_{10} and ϵ_{20}, $\epsilon_{10} < \epsilon_{20} \ll 1$, are close to each other,

$$\delta\epsilon \equiv \epsilon_{20} - \epsilon_{10} \ll \epsilon_{10} ,$$

so that the second soliton is moving to gain on the first one. In the reference frame, moving with the velocity $\epsilon = (\epsilon_{10} + \epsilon_{20}) / 2$, the potential ψ is a slowly varying function of time:

$$\frac{\partial \psi}{\partial \tau} \sim \delta\epsilon \frac{\partial \psi}{\partial \eta} \ll \epsilon \frac{\partial \psi}{\partial \eta},$$

with $\eta = \xi - \epsilon\tau$ as a new spatial coordinate. At every moment, the solution ψ of the equation for the potential

$$\frac{\partial \psi}{\partial \tau} - \epsilon \frac{\partial \psi}{\partial \eta} + \psi \frac{\partial \psi}{\partial \eta} + \frac{1}{2}\frac{\partial^3 \psi}{\partial \eta^3} = 0 \qquad (18)$$

could be approximately presented as a superposition of two terms, which relate individually to the pure soliton solutions:

$$\psi \approx \psi_1 + \psi_2 ,$$

$$\psi_i = \frac{3\epsilon_i(\tau)}{\cosh^2 \theta_i}, \quad \theta_i = \sqrt{\frac{\epsilon_i(\tau)}{2}} \left(\eta - \int_0^\tau [\epsilon_i(\tau) - \epsilon]\, d\tau - \eta_{i0} \right) , \qquad (19)$$

where η_{i0} are the initial coordinates of the soliton tops. The nonlinear interaction of the solitons manifests itself primarily in slow varying of their velocities and, hence, of their amplitudes. To compute this varying, one should invoke the energy conservation considerations and integrate the equation (18) from η' to $+\infty$, then multiply the result by $2\partial\psi/\partial\eta'$ and integrate it again from η to $+\infty$. Finally, one comes to the equation similar to (8):

$$\frac{1}{2}\left(\frac{\partial \psi}{\partial \eta}\right)^2 + W(\psi) = I(\eta), \qquad (20)$$

with

$$W(\psi) \equiv -\epsilon\psi^2 + \frac{\psi^3}{3} . \qquad (21)$$

The expression for $I(\eta)$ takes the following form:

$$I(\eta) = -2 \int_\eta^\infty d\eta' \frac{\partial \psi}{\partial \eta'} \int_{\eta'}^\infty d\eta'' \frac{\partial \psi}{\partial \tau} = 2\psi \int_\eta^\infty d\eta \frac{\partial \psi}{\partial \tau} - \int_\eta^\infty d\eta \frac{\partial \psi^2}{\partial \tau}. \qquad (22)$$

Let η in (20) be equal to η_m that corresponds to the minimum value ψ_{min} of the potential between the solitons. From what follows we'll see, if at the initial moment the solitons are located far from each other, so that

$$\psi_{min} \ll \psi_{max} \sim \epsilon ,$$

then this inequality will be satisfied for all ensuing moments. Acounting for this, one obtains, using (20-22), the relation similar to (15):

$$\epsilon\psi_{min}^2 \simeq \int_{\eta_m}^{+\infty} d\eta \frac{\partial \psi^2}{\partial \tau} \simeq \int_{-\infty}^{+\infty} d\eta \frac{\partial \psi_1^2}{\partial \tau} = 12\sqrt{2}\frac{d\epsilon_1^{3/2}}{d\tau} \simeq 18\sqrt{2\epsilon}\frac{d^2\eta_1}{d\tau^2} , \qquad (23)$$

where

$$\eta_1 = \int_0^\tau [\epsilon_1(\tau) - \epsilon]\, d\tau + \eta_{10}$$

is a coordinate of the first soliton top at the moment τ. Similarly, integrating from $-\infty$ to η_m, one derives for the second soliton:

$$\epsilon\psi_{min}^2 \simeq -\int_{-\infty}^{\eta_m} d\eta \frac{\partial \psi^2}{\partial \tau} \simeq -\int_{-\infty}^{+\infty} d\eta \frac{\partial \psi_2^2}{\partial \tau} \simeq -18\sqrt{2\epsilon}\frac{d^2\eta_2}{d\tau^2}, \qquad (24)$$

where η_2 is a coordinate of the second soliton top at the moment τ. As it follows from (23) and (24), the amplitude of the first soliton always grows up while the amplitude of the second one drops down. Using asymptotic behaviour of (19), one can express ψ_{min} as a function of the distance $l = \eta_1 - \eta_2$ between the solitons:

$$\psi_{min} \simeq 24\epsilon e^{-\kappa l} , \tag{25}$$

with $\kappa = \sqrt{\epsilon/2}$. Having regard to (25), equations (23), (24) can be rewritten as

$$\frac{d^2\eta_1}{d\tau^2} = -\frac{\partial V(\eta_1 - \eta_2)}{\partial \eta_1}, \tag{26}$$

$$\frac{d^2\eta_2}{d\tau^2} = -\frac{\partial V(\eta_1 - \eta_2)}{\partial \eta_2}, \tag{27}$$

where

$$V(l) = 16\epsilon^2 e^{-2\kappa l}. \tag{28}$$

The integration of these equations reinforces the well-known result that the collision of two KdV solitons leads to the velocity exchange between them. It should be particularly emphasized that the given above analysis allows one to extend this statement not only to KdV but to a wide class of solitons. [1]

Thus, the nonlinear interaction of two solitons with close amplitudes can be treated in terms of the repulsion potential between them. The distance of the soliton closest approach to each other is still far in excess of the soliton scale-length:

$$l_{min} = \kappa^{-1} \ln \frac{8\epsilon}{\delta\epsilon} \gg \kappa^{-1} . \tag{29}$$

This ensures the validity of the presented analysis. It should be also marked that since the interaction potential V declines steeply as the distance between solitons increases, the nonlinear interaction of several solitons reduces to the pair interactions between neighboring solitons.

4 Multisoliton stability

In the framework of the KdV equation, the solitons themselves are stable. The soliton interaction with an ion beam leads, in general, to a production of a set of baby solitons [7]. In the adiabatic limit [8], however, the only effect is a gradual changing of the soliton amplitude without emission of

[1] In particularly, to the solitons discussed in [6] for a collisionless plasma limit.

baby solitons. The requirement for the adiabatic limit to be valid implies that the energy, exchanged between the soliton and the beam in a time-scale of an ion-acoustic dispersion, should be small as compared with the energy of the soliton itself. In the conditions (5), this requirement is expressed by the inequality (14), which is assumed to be satisfied in our discussion. Hence, the analysis of the multisoliton stability amounts to the study of the temporal evolution of a system comprised of the ion beam and the interacting solitons whose amplitudes are close to that one in the equilibrium.

For small amplitudes, equations (1-3), (7) reduce to the equation [9]:

$$\frac{\partial \psi}{\partial \tau} + \psi \frac{\partial \psi}{\partial \xi} + \frac{1}{2}\frac{\partial^3 \psi}{\partial \xi^3} = -\frac{1}{2}\frac{\partial \tilde{n}_b}{\partial \xi}, \tag{30}$$

where $\tilde{n}_b = n_b/n_{0e}$ is a normalized beam density (6). Suppose that due to initial perturbations, the velocities ϵ_i of the solitons slightly differ from the equilibrium value ϵ_0. In the frame of reference, moving with the velocity ϵ_0, the energy balance equation for the first soliton in multisoliton can be written in the form similar to (23):

$$\epsilon_0 \psi^2_{min} \simeq 18\sqrt{2\epsilon_0}\frac{d^2\eta_1}{d\tau^2} + \tilde{q}_1 . \tag{31}$$

Here $\tilde{q}_1$ is a normalized energy transmitted per unit time to the beam ions at the leading front of the multisoliton. Note that $\tilde{q}_1$ depends on the velocity $\epsilon_1 - \epsilon_0$ of the first soliton. One can show that for the ion beam with the average velocity $u_0 = c_s(1+\epsilon_0)$, this relation may be approximated linearly as:

$$\tilde{q}_1 = \tilde{q}_0 + a(\epsilon_1 - \epsilon_0) ,$$

where $\tilde{q}_0$ relates to the energy transfer in the unperturbed multisoliton; the constant a defines the change of this value with the variation of the first soliton velocity:

$$\tilde{q}_0 \sim \tilde{n}_{b0}\epsilon_0 \ll \epsilon_0^3,$$

$$a \sim \tilde{q}_0/\sqrt{\epsilon_0} .$$

The same modification should be done in the energy balance equation for the last N-th soliton, with the only difference leading to

$$\tilde{q}_N = \tilde{q}_0 - a(\epsilon_N - \epsilon_0) .$$

Since there are no beam particles in the internal region of the multisoliton, it is sufficient to consider the interaction with the only neighbouring solitons for each "internal" soliton. Using (26)-(28), one can present the following set of equations describing the dynamics of the solitons in the multisoliton:

$$\frac{d^2\eta_1}{d\tau^2} = -\frac{\partial V(\eta_1 - \eta_2)}{\partial \eta_1} - \frac{1}{18\sqrt{2\epsilon_0}}\left(\tilde{q}_0 + a\frac{d\eta_1}{d\tau}\right), \tag{32}$$

$$\frac{d^2\eta_i}{d\tau^2} = -\frac{\partial V(\eta_{i-1} - \eta_i)}{\partial \eta_i} - \frac{\partial V(\eta_i - \eta_{i+1})}{\partial \eta_i}, \quad i = 2, \ldots N-1 \tag{33}$$

$$\frac{d^2\eta_N}{d\tau^2} = -\frac{\partial V(\eta_{N-1} - \eta_N)}{\partial \eta_N} + \frac{1}{18\sqrt{2\epsilon_0}}\left(\tilde{q}_0 - a\frac{d\eta_N}{d\tau}\right), \tag{34}$$

where $d\eta_i/d\tau = \epsilon_i - \epsilon_0$.

Equations (32)-(34) have a stationary multisoliton solution that has been derived in Section 1. For small perturbations from the stationary solution, these equations can be linearized to take the form:

$$\frac{d^2\delta\eta_1}{d\tau^2} = -\omega^2(\delta\eta_1 - \delta\eta_2) - \nu\frac{d\delta\eta_1}{d\tau}, \tag{35}$$

$$\frac{d^2\delta\eta_i}{d\tau^2} = -\omega^2(2\delta\eta_i - \delta\eta_{i-1} - \delta\eta_{i+1}), \quad i = 2, \ldots N-1 \tag{36}$$

$$\frac{d^2\delta\eta_N}{d\tau^2} = -\omega^2(\delta\eta_N - \delta\eta_{N-1}) - \nu\frac{d\delta\eta_N}{d\tau}. \tag{37}$$

Here $\delta\eta_i = \eta_i - \eta_{i0}$ is the soliton displacement from the equilibrium position. Constants ω^2 and ν are equal to:

$$\omega^2 = \frac{\tilde{q}_0}{18}, \quad \nu = \frac{a}{18\sqrt{2\epsilon_0}} \sim \frac{\tilde{q}_0}{\epsilon_0}.$$

Equations (35)-(37) present the multisoliton dynamics as the dynamics of a linear chain of equal bodies coupled with springs. Two outer bodies are also acted upon by a viscous force ($\nu^2 \ll \omega^2$). The spring rigidity as well as the viscosity coefficient are determined by the intensity of the exchanged energy q. Such a system is evidently stable. Its eigenfrequences can be easily computed, and the following estimate is valid for the characteristic damping time of the eigenmode:

$$\tau \sim N\nu^{-1}.$$

While damping, all energy of the multisoliton perturbation is transferred to the resonant beam particles. Finally, the multisoliton, being, in general, displaced as a whole, returns to its previous unperturbed state.

5 Conclusions

The interaction between of a finite set of solitons with close amplitudes and a resonant ion beam is discussed in the present paper. As it is demonstrated, such a system is dynamically equivalent to a trivial mechanical oscillatory system of equal bodies coupled with springs. One can conclude that the beam ions, while reflecting from the leading front of the first soliton and the back front of the last soliton, force the solitons to each other, so that despite the mutual repulsion between the solitons, they form a stable localized nonlinear wave that is a multisoliton. Though analysis is carried out for the KdV solitons, all basic conclusions concerning existance and stability of a multisoliton allow generalization to some other types of ion-acoustic solitons, in particular, to the solitons obtained in [6] for a collisionless plasma limit.

Authors are grateful to Dr.Beklemishev for the interest in this work.

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BudkerINP 95-3

Ответственный за выпуск С.Г. Попов
Работа поступила 11 января 1995 г.

Сдано в набор 18.01. 1995 г.
Подписано в печать 18.01 1995 г.
Формат бумаги 60×90 1/16 Объем 0,9 печ.л., 0,8 уч.-изд.л.
Тираж 200 экз. Бесплатно. Заказ N 3

Обработано на IBM PC и отпечатано на
ротапринте ГНЦ ИЯФ им. Г.И. Будкера СО РАН,
Новосибирск, 630090, пр. академика Лаврентьева, 11.