

Государственный научный центр Российской Федерации

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CS-TH 12/95
June 1995

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AMPLITUDE IN QCD

ИЯФ 95-49

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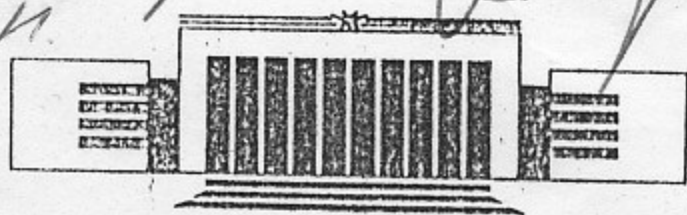
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ИЯФ № 95-49

В.С. Фадин, Р. Фиоре, А. Квартароло

РЕДЖЕЗАЦИЯ АМПЛИТУДЫ РАССЕЯНИЯ КВАРКА
НА КВАРКЕ В КХД

В печать и в свет.
У секретаря,
г.р. м.п. *Дир. Тюнов С.Г.*



НОВОСИБИРСК

1995

REGGEIZATION OF QUARK-QUARK SCATTERING
AMPLITUDE IN QCD *

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Abstract

s-channel discontinuity of quark-quark scattering amplitude with gluon quantum numbers in *t* channel and negative signature is calculated in the Regge kinematical region in the two-loop approximation. Using this discontinuity and assuming that the Regge asymptotic behaviour is given by the Reggeized gluon contribution, we calculate the gluon trajectory in the two-loop approximation. Remarkable cancellations lead to independence of the trajectory on properties of the scattered quarks, confirming the gluon Reggeization.

* Work supported in part by ISF under grant RAK-000, in part by the Ministero italiano dell'Università e della Ricerca Scientifica e Tecnologica and in part by the EEC Programme "Human Capital and Mobility", Network "Physics at High Energy Colliders", contract CHRX-CT93-0537 (DG 12 COMA)

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1. INTRODUCTION

Perturbative QCD is widely used nowadays for the description of semihard [1] as well as hard processes [2]. However, whereas for the later ones the theory is well developed and understood, for the formers we have a lot of unsolved theoretical problems. An applicability of perturbation theory improved by the renormalization group for a hard process having a large typical virtuality Q^2 is justified by a smallness of the strong coupling constant $\alpha_s(Q^2)$. Contrary, for semihard processes another essential parameter appears: the ratio of the typical virtuality Q^2 to the square of the c.m.s energy s of colliding particles. At sufficiently high energy the value $x = Q^2/s$ becomes so small that it is necessary to sum up terms of the type $\alpha_s^n (\ln(1/x))^m$, with $m \leq n$ (for the scattering channel which is considered here). Up to now this problem is solved [3] in the leading logarithmic approximation (LLA) only, which means summation of the terms with $m = n$.

The results of LLA have two serious disadvantages. Firstly, the Froissart bound $\sigma_{tot} < c \ln^2 s$ is violated in LLA. In fact, calculated in LLA, the total cross section σ_{tot}^{LLA} grows at large c.m.s. energies as a power of s :

$$\sigma_{tot}^{LLA} \sim \frac{s^{\omega_0}}{\sqrt{\ln s}}, \quad (1)$$

where, for the gauge group $SU(N)$ ($N = 3$ for QCD), with coupling g ($\alpha_s = \frac{g^2}{4\pi}$),

$$\omega_0 = \frac{4\alpha_s}{\pi} N \ln 2. \quad (2)$$

In terms of structure functions this means their strong power increase in the small x region. The Froissart bound is violated in LLA because the s -channel unitarity constraints for scattering amplitudes are not completely fulfilled in this approximation. The problem of *unitarization* of LLA results is extremely important from

a theoretical point of view. It is concerned in a lot of papers (see, for example, Ref. [4]).

Another disadvantage seems to be even more important from a practical point of view, since the results of LLA are applied to the small x phenomenology (see, for instance, Ref. [5]). There is an uncertainty of the argument of the running coupling constant which appears because the scale dependence of α_s is beyond of the accuracy of LLA. This uncertainty diminishes the predictive power of LLA, permitting to change strongly presents numerical results by changing a scale.

Therefore, the problem of calculation of radiative corrections to LLA becomes very important now, as it gives us the possibility to fix the scale dependence of the coupling constant, to reduce the uncertainty of the predictions of LLA and to determine a region of its applicability.

A solution of this problem can be strongly simplified [6] by using the Reggeization property of the non-Abelian SU(N) gauge theories. It was proved [3, 7] in LLA that gauge bosons are Reggeized in these theories with the trajectory

$$j(t) = 1 + \omega(t) , \quad (3)$$

where in the leading approximation

$$\omega(t) = \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}} \frac{N}{2} \int \frac{d^{D-2} k_{\perp}}{k_{\perp}^2 (q - k)_{\perp}^2} . \quad (4)$$

Here q is the momentum transfer, $t = q^2 \approx q_{\perp}^2$, and $D = 4 + \varepsilon$ is the space-time dimension. A non zero ε is introduced to regularize Feynman integrals. The integration in Eq.(4) is performed over the $(D-2)$ -dimensional momenta orthogonal to the initial particle momentum plane.

The results of LLA are summarized [3] in the Bethe-Salpeter type equation for the t -channel partial amplitudes. The problem of the calculation of corrections to LLA, therefore, can be set up as the problem of the calculation of corrections to the kernel of this equation [6]. The kernel is expressed in terms of the gluon trajectory and the Reggeon-Reggeon-gluon (RRG) vertex. The corrections to the vertex are known [8, 9], therefore the calculation of the contribution $\omega^{(2)}(t)$ to the trajectory in the next (two-loop) approximation has become most urgent.

In this paper we present results and details of the calculation of the two-loop correction to the trajectory for the real case of QCD with massive quark flavours. The result for $\omega^{(2)}(t)$ in the massless quark case was published earlier [10]. The paper is organized as follows. In Sec. II we discuss the method of calculation. In Sec. III we calculate the contribution of the two-particle intermediate state to the s -channel discontinuity of the quark-quark scattering amplitude. An analogous calculation is performed in Sec. IV for the contribution of the three particle intermediate state. The final expression for the correction $\omega^{(2)}(t)$ is obtained and discussed in Sec. V.

2. METHOD OF CALCULATION

The method is based on using s -channel unitarity. The two-loop contribution to the gluon trajectory can be obtained from the s -channel discontinuity of an elastic scattering amplitude with gluon quantum numbers and negative signature in the t channel calculated in the two-loop approximation with the accuracy up to a constant. Indeed, let us consider such an amplitude for a process of the type $A + B \rightarrow A' + B'$ at large s and fixed t . Assuming the gluon Reggeization, the amplitude takes the factorized form

$$\left(\mathcal{A}_s^{(-)}\right)_{AB}^{A'B'} = \Gamma_{A'A}^i \frac{s}{t} \left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)} \right] \Gamma_{B'B}^i, \quad (5)$$

where $\Gamma_{A'A}^i$ are the particle-particle-Reggeon (PPR) vertices. They can be written as

$$\Gamma_{A'A}^i = g \langle A'|T^i|A \rangle (\Gamma_{A'A}^{(0)} + \Gamma_{A'A}^{(1)}) , \quad (6)$$

where $\langle A'|T^i|A \rangle$ stands for a matrix element of the colour group generator in the corresponding representation (i.e. fundamental for quarks and adjoint for gluons), $\Gamma_{A'A}^{(0)}$ and $\Gamma_{A'A}^{(1)}$ are respectively the Born and the one-loop contributions to the vertices. Using Eq.(6) and the decomposition

$$\omega(t) = \omega^{(1)}(t) + \omega^{(2)}(t) \quad (7)$$

one can present the two-loop contribution to the s -channel discontinuity $\left[\left(\mathcal{A}_s^{(-)} \right)_{AB}^{A'B'} \right]_S$ in the the form

$$\begin{aligned} & \left[\left(\mathcal{A}_s^{(-)} \right)_{AB}^{A'B'} \text{ (two-loop)} \right]_S = \left(-\frac{2\pi i s}{t} \right) g^2 \langle A'|T^i|A \rangle \langle B'|T^i|B \rangle \\ & \times \Gamma_{A'A}^{(0)} (\omega^{(1)}(t))^2 \ln \left(\frac{s}{-t} \right) \Gamma_{B'B}^{(0)} + (\Gamma_{A'A}^{(1)} \Gamma_{B'B}^{(0)} + \Gamma_{A'A}^{(0)} \Gamma_{B'B}^{(1)}) \omega^{(1)}(t) + \Gamma_{A'A}^{(0)} \omega^{(2)}(t) \Gamma_{B'B}^{(0)} . \end{aligned} \quad (8)$$

Since the one-loop corrections $\Gamma_{A'A}^{(1)}$ to the PPR vertices became available [8, 11, 12], the only unknown quantity in the RHS of Eq.(8) is the two-loop contribution $\omega^{(2)}(t)$ to the gluon trajectory. Consequently, one may obtain it from the expression of the discontinuity $\left[\left(\mathcal{A}_s^{(-)} \right)_{AB}^{A'B'} \text{ (two-loop)} \right]_S$ calculated in the two-loop approximation with accuracy up to a constant. The calculation of this discontinuity is the main content of this paper.

By definition the trajectory should not depend on a particular type of scattered particles, therefore we have a freedom in choosing these particles. In this paper we consider the process of the quark-quark scattering for calculating the correction to the trajectory.

In LLA helicities of each of the scattered particles are conserved, so that in the helicity basis we have

$$\Gamma_{A'A}^{(0)} = \delta_{\lambda_{A'}\lambda_A} . \quad (9)$$

On the contrary, in higher orders PPR vertex $\Gamma_{A'A}^{(i)}$ may contain another spin structure. Because of the parity conservation, the one-loop correction $\Gamma_{A'A}^{(1)}$ can be written as [12]

$$\Gamma_{A'A}^{(1)} = \delta_{\lambda_{A'}\lambda_{A'}}\Gamma_{A'A}^{(+)} + \delta_{\lambda_{A'}-\lambda_{A'}}\Gamma_{A'A}^{(-)} \quad (10)$$

if the relative phases of the states with opposite helicity are appropriately chosen. Both quantities $\Gamma_{A'A}^{(\pm)}$ are calculated in Ref. [12]. For our purpose the knowledge of $\Gamma_{A'A}^{(+)}$ is sufficient. In fact, the correction $\omega^{(2)}(t)$ in Eq.(8) is multiplied by the helicity conserving vertices, then for its determination only the helicity conserving parts of the amplitudes entering Eq.(8) are necessary.

Let us write the helicity conserving part of the discontinuity of the amplitude in the LHS of Eq.(8) in the following form:

$$\left[\left(\mathcal{A}_s^{(-)} \right)_{AB}^{A'B'} \text{ (two-loop)} \right]_S^{(+)} = g^2 \langle A'|T^i|A \rangle \langle B'|T^i|B \rangle \left(-\frac{2\pi i s}{t} \right) \Delta_s , \quad (11)$$

where the superscript (+) in the LHS means *helicity conserving part*. Then Eq.(8), together with Eqs.(9) and (10), gives us

$$\omega^{(2)}(t) = \Delta_s - \left(\omega^{(1)}(t) \right)^2 \ln \left(\frac{s}{-t} \right) - \left(\Gamma_{A'A}^{(+)} + \Gamma_{B'B}^{(+)} \right) \omega^{(1)}(t) . \quad (12)$$

The only unknown term in the RHS of this Eq.(12) is the discontinuity Δ_s . We calculate it below by using the s -channel unitarity condition. In the two-loop approximation only two- and three-particle intermediate states do contribute. This allows to divide the discontinuity under consideration into two parts respectively:

$$\Delta_s = \Delta_s^{(2)} + \Delta_s^{(3)} . \quad (13)$$

3. TWO-PARTICLE CONTRIBUTION TO THE DISCONTINUITY

Let us consider the two-particle contribution in the unitarity relation:

$$\left[\mathcal{A}_{AB}^{A'B'} \right]_{S(2)} = i \int d\Phi_2 (p_A + p_B; p_{A_1}, p_{B_1}) \sum_{A_1, B_1} \mathcal{A}_{AB}^{A_1 B_1} \mathcal{A}_{A'B'}^{*A_1 B_1}. \quad (14)$$

Since we are considering the quark-quark scattering amplitude, the intermediate particles can be only quarks and the summation is performed over their spin and colour states; $d\Phi_2$ is the two-body phase space element. In general case

$$d\Phi_n (P; p_1, \dots, p_n) = (2\pi)^D \delta^{(D)} \left(P - \sum_{i=1}^n p_i \right) \prod_{i=1}^n \frac{d^{(D-1)} p_i}{2E_i (2\pi)^{D-1}}. \quad (15)$$

In order to obtain the two-loop contribution to the discontinuity we need to take one of the two amplitudes in the RHS of Eq.(14) in the Born approximation and the other one in the one-loop approximation (see Fig. 1). Both amplitudes should be calculated in the region of asymptotically large c.m.s. energies \sqrt{s} and fixed momentum transfers. In this region the amplitudes acquire a very simple structure in the Born approximation:

$$\mathcal{A}_{AB}^{A_1 B_1} (\text{Born}) = \langle A_1 | T^i | A \rangle \langle B_1 | T^i | B \rangle \delta_{\lambda_A, \lambda_{A_1}} \delta_{\lambda_B, \lambda_{B_1}} \frac{2g^2 s}{t_1}, \quad (16)$$

where $t_1 = (p_A - p_{A_1})^2$. The amplitude $\mathcal{A}_{A'B'}^{A_1 B_1}$ may be obtained from Eq.(16) by evident substitutions.

The one-loop contributions to the amplitudes $\mathcal{A}_{AB}^{A_1 B_1}$ and $\mathcal{A}_{A'B'}^{A_1 B_1}$ have a much more complicated form. Fortunately, it is sufficient to take into account only those parts of these contributions which conserve the helicity of each particle and have negative signature in t_1 and t'_1 channels correspondingly ($t'_1 = (p_{A'} - p_{A_1})^2$). Indeed, let us consider the spin structure. We are interested in the helicity conserving part of the discontinuity, so that only such a part should be kept in the product of the

Born and the one-loop contributions in Eq.(14). Since in the Born approximation the helicities are conserved, as Eq.(16) shows, the only helicity conserving part must be taken in the one-loop contribution. Now let's turn to the colour structure and to the signature. In the Born approximation (16) the amplitudes evidently have gluon quantum numbers in t_1 and t_2 channels and negative signature (in the region of asymptotically large s it simply means that they are odd functions of s). The one-loop contribution to the amplitudes $\mathcal{A}_{AB}^{A_1 B_1}$ and $\mathcal{A}_{A'B'}^{A_1 B_1}$, besides such a part, has also a positive signature part, with colour singlet as well as octet state in the t_1 and t'_1 channels. This part, however, is purely imaginary [12], whereas the Born amplitude is real; therefore, their product cancel in the unitarity equation (14).

As for the result, to calculate the discontinuity $\Delta_s^{(2)}$ we may use, for the amplitudes $\mathcal{A}_{AB}^{A_1 B_1}$ and $\mathcal{A}_{A'B'}^{A_1 B_1}$ in Eq.(14), the representation (5) with the trajectory $\omega(t)$ and vertices $\Gamma_{A'A}^c$ given by Eq.(4) and Eqs. (6), (9) and (10) respectively. In the one-loop approximation we get

$$\begin{aligned} \mathcal{A}_{AB}^{A_1 B_1}(\text{one-loop}) &= \langle A_1 | T^i | A \rangle \langle B_1 | T^i | B \rangle \delta_{\lambda_A, \lambda_{A_1}} \delta_{\lambda_B, \lambda_{B_1}} \\ &\times \frac{g^2 s}{t_1} \left\{ \omega^{(1)}(t_1) \left[\ln \left(\frac{s}{-t_1} \right) + \ln \left(\frac{-s}{-t_1} \right) \right] + 2 \left(\Gamma_{A_1 A}^{(+)}(t_1) + \Gamma_{B_1 B}^{(+)}(t_1) \right) \right\} + \dots, \quad (17) \end{aligned}$$

where dots denote terms which do not contribute to the discontinuity $\Delta_s^{(2)}$. Here we write explicitly the arguments of the vertex functions in order to stress that they are not constant, but depend on t_1 . The amplitude $\mathcal{A}_{A'B'}^{A_1 B_1}(\text{one-loop})$ is obtained from Eq.(17) by evident substitutions. Using Born (16) and one-loop (17) amplitudes and summing up in Eq.(14) over spin and colour states of the particles A_1 and B_1 leads to the spin and colour factor

$$\langle A' | T^i T^j | A \rangle \langle B' | T^i T^j | B \rangle \delta_{\lambda_A, \lambda_{A'}} \delta_{\lambda_B, \lambda_{B'}}.$$

Let us remind that we need to calculate not the discontinuity (14) itself, but the corresponding discontinuity for the amplitude with the octet colour state in the t channel and negative signature. The calculation of the discontinuity for this amplitude requires antisymmetrization with respect to the change $A \rightarrow \bar{A}'$, $A' \rightarrow \bar{A}$ or, equivalently, $B \rightarrow \bar{B}'$, $B' \rightarrow \bar{B}$, which in our case reduces to change of the order of the group generators into the factor $\langle A'|T^i T^j|A \rangle$. Using the relations

$$[T^i, T^j] = i f^{ij}{}_k T^k, \quad i f_{ij}{}^k T^i T^j = -\frac{N}{2} T^k, \quad (18)$$

where the coefficients f^{ijk} are the group structure constants, the colour factor becomes

$$-\frac{N}{4} \langle A'|T^i|A \rangle \langle B'|T^i|B \rangle.$$

Notice that the negative signature automatically leads here to the colour octet state in the t channel. Using the two-body phase space element

$$d\Phi_2(p_A + p_B; p_{A_1}, p_{B_1}) = \frac{1}{2s} \frac{d^{(D-2)}q_{1\perp}}{(2\pi)^{D-2}}, \quad (19)$$

with $q_1 = p_{A_1} - p_A$, from Eqs.(14), (16) and (17) we get

$$\begin{aligned} & \left[\left(\mathcal{A}_s^{(-)} \right)_{AB}^{A'B'} \text{ (two-loop)} \right]_{S(2)}^{(+)} = g^2 \langle A'|T^i|A \rangle \langle B'|T^i|B \rangle \left(-\frac{2\pi i s}{t} \right) \frac{g^2 N t}{(2\pi)^{D-1}} \\ & \times \int \frac{d^{(D-2)}q_{1\perp}}{(q_1 - q)_\perp^2 q_{1\perp}^2} \left[\omega^{(1)}(q_{1\perp}^2) \ln \left(\frac{s}{-q_{1\perp}^2} \right) + \Gamma_{A'A}^{(+)}(q_{1\perp}^2) + \Gamma_{B'B}^{(+)}(q_{1\perp}^2) \right]. \end{aligned} \quad (20)$$

Comparing Eq.(20) with Eq.(11) and the relation (13), we obtain

$$\begin{aligned} \Delta_s^{(2)} = & \frac{g^2 N t}{(2\pi)^{D-1}} \int \frac{d^{(D-2)}q_{1\perp}}{q_{1\perp}^2 (q_1 - q)_\perp^2} \left[\omega^{(1)}(q_{1\perp}^2) \ln \left(\frac{s}{-q_{1\perp}^2} \right) + \Gamma_{A'A}^{(+)}(q_{1\perp}^2) + \Gamma_{B'B}^{(+)}(q_{1\perp}^2) \right]. \end{aligned} \quad (21)$$

The helicity conserving part of the one-loop correction to the quark-quark-Reggeon vertices $\Gamma_{QQ}^{(+)}(t)$ is calculated in Ref. [12]. For our purpose it is convenient to express it as a sum of three parts having different flavour-colour dependence:

$$\Gamma_{QQ}^{(+)}(t) = a_f(t) + a_Q(t, m_Q^2) + a_g(t, m_Q^2). \quad (22)$$

Here the first term

$$a_f(t) = -2 \frac{g^2}{(4\pi)^{\frac{D}{2}}} \Gamma\left(2 - \frac{D}{2}\right) \sum_f \int_0^1 dx \frac{x(1-x)}{[m_f^2 - x(1-x)t]^{2-\frac{D}{2}}} \quad (23)$$

is the contribution of the quark loop, the summation being over the quark flavours.

The second term

$$a_Q(t, m_Q^2) = \frac{g^2}{(4\pi)^{\frac{D}{2}}} \frac{1}{2N} \Gamma\left(2 - \frac{D}{2}\right) \left\{ \int_0^1 \frac{dx}{[m_Q^2 - x(1-x)t]^{3-\frac{D}{2}}} \right. \\ \left. \times \left[t \left(\frac{1}{D-3} + \frac{D-4}{4} \right) - \frac{2m_Q^2}{(D-3)} \right] + \frac{2}{D-3} (m_Q^2)^{\frac{D}{2}-2} \right\} \quad (24)$$

is connected with the vertex and the quark self-energy diagrams. Eq.(24) is obtained from Eqs.(39) and (68) of Ref. [12] with the help of the identity

$$(D-3) \int_0^1 dx \frac{tx(1-x)}{[m^2 - x(1-x)t]^{3-\frac{D}{2}}} = \frac{1}{4} \int_0^1 dx \frac{t(D-4) + 4m^2}{[m^2 - x(1-x)t]^{3-\frac{D}{2}}} - (m^2)^{\frac{D}{2}-2}, \quad (25)$$

which in turn follows from the evident identity

$$\int_0^1 dx \frac{d}{dx} \left(\frac{x}{[m^2 - x(1-x)t]^{2-\frac{D}{2}}} \right) = (m^2)^{\frac{D}{2}-2}. \quad (26)$$

Finally, the last term, proportional to N, comes from the two gluon exchange and quark self-energy diagrams; it can be written as

$$a_g(t, m_Q^2) = a_g(t, 0) + \delta_g(t, m_Q^2). \quad (27)$$

Here $a_g(t, 0)$ is given by Eq.(63) of Ref. [12],

$$a_g(t, 0) = \frac{g^2 N}{(4\pi)^{\frac{D}{2}}} \frac{\Gamma\left(2 - \frac{D}{2}\right) \Gamma^2\left(\frac{D}{2} - 1\right)}{(-t)^{2 - \frac{D}{2}} \Gamma(D - 2)} \left\{ (D - 3) \left[\psi\left(3 - \frac{D}{2}\right) - 2\psi\left(\frac{D}{2} - 2\right) + \psi(1) \right] + \frac{1}{4(D - 1)} - \frac{2}{D - 4} - \frac{7}{4} \right\}, \quad (28)$$

where $\psi(x)$ is the logarithmic derivative of the gamma function:

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}.$$

In turn, the second term $\delta_g(t, m_Q^2)$ comes from Eq.(64) and from the part proportional to N in Eq.(67) of Ref. [12]. It is convenient to modify its form using the identity

$$\int_0^1 \int_0^1 dx_1 dx_2 \theta(1 - x_1 - x_2) \left(\frac{\partial}{\partial x_2} - 2 \frac{\partial}{\partial x_1} \right) \frac{x_1}{[m^2 x_1^2 - t x_2 (1 - x_1 - x_2)]^{2 - \frac{D}{2}}} = -\frac{2}{D - 2} (m^2)^{\frac{D}{2} - 2}, \quad (29)$$

which provides us the relation

$$2 \int_0^1 \int_0^1 dx_1 dx_2 \frac{t x_2 (1 - x_1 - x_2) \theta(1 - x_1 - x_2)}{[m^2 x_1^2 - t x_2 (1 - x_1 - x_2)]^{3 - \frac{D}{2}}} + \frac{2}{D - 2} (m^2)^{\frac{D}{2} - 2} = \int_0^1 \int_0^1 dx_1 dx_2 \frac{\theta(1 - x_1 - x_2) \left[\left(\frac{D}{2} - 2\right) t x_1 (1 - x_1) + 2(D - 3) m^2 x_1^2 \right]}{[m^2 x_1^2 - t x_2 (1 - x_1 - x_2)]^{3 - \frac{D}{2}}}. \quad (30)$$

Using this relation one arrives at

$$\delta_g(t, m_Q^2) = \frac{g^2 N}{(4\pi)^{\frac{D}{2}}} \Gamma\left(3 - \frac{D}{2}\right) \left\{ \int_0^1 \int_0^1 dx_1 dx_2 \theta(1 - x_1 - x_2) \times \left[\frac{t(1 - x_1)}{x_1} \left(1 - x_1 + \frac{D - 2}{4} x_1^2\right) \left(\frac{1}{[m_Q^2 x_1^2 - t x_2 (1 - x_1 - x_2)]^{3 - \frac{D}{2}}} \right) \right. \right.$$

$$\left. \begin{aligned} & - \frac{1}{[-tx_2(1-x_1-x_2)]^{3-\frac{D}{2}}} \right) - \frac{2m_Q^2 x_1}{[m_Q^2 x_1^2 - tx_2(1-x_1-x_2)]^{3-\frac{D}{2}}} \\ & + \frac{2(m_Q^2)^{\frac{D}{2}-2}}{(D-3)(D-4)} \end{aligned} \right\} \quad (31)$$

4. THREE-PARTICLE CONTRIBUTION TO THE DISCONTINUITY

Let us now turn to the calculation of the contribution coming from the three-particle intermediate state in the s -channel unitarity condition. Since we are considering the discontinuity of the quark-quark scattering amplitude, in the intermediate state we face the same quarks with an additional gluon. The unitarity condition in this case reads

$$\left[\mathcal{A}_{AB}^{A'B'} \right]_{S(3)} = i \int d\Phi_3(p_A + p_B; p_{A_1}, p_G, p_{B_1}) \sum_{A_1, G, B_1} \mathcal{A}_{AB}^{A_1 G B_1} \mathcal{A}_{A'B'}^{*A_1 G B_1} . \quad (32)$$

where G is the produced gluon. For calculating the discontinuity it is convenient to use Sudakov variables. Let us introduce light-like momenta p_- and p_+ close to the initial momenta p_A and p_B respectively:

$$\begin{aligned} p_A &= p_- + \frac{m_A^2}{s} p_+ , & p_B &= p_+ + \frac{m_B^2}{s} p_- , \\ p_-^2 &= p_+^2 = 0 , & 2p_+ p_- &= s . \end{aligned} \quad (33)$$

Thus any vector p_i can be decomposed as

$$p_i = \beta_i p_- + \alpha_i p_+ + p_{i\perp} , \quad (34)$$

so that

$$p_i^2 = s\alpha_i\beta_i + p_{i\perp}^2 .$$

In the region under consideration, i.e. for

$$s \gg |t| \sim m_A^2 \sim m_B^2, \quad (35)$$

the essential kinematics in Eq.(32) is defined by three conditions:

$$i) \quad |p_{A_1\perp}^2| \sim |p_{B_1\perp}^2| \sim |p_{G\perp}^2| \sim |t|, \quad (36)$$

which means that all transverse momenta are limited;

$$ii) \quad \beta_{A_1} \sim \alpha_{B_1} \sim 1, \quad \alpha_{A_1} \sim \beta_{B_1} \sim \frac{|t|}{s}, \quad (37)$$

that states that the order of magnitude of quark longitudinal momenta does not change; and

$$iii) \quad \frac{|t|}{s} \lesssim \beta_G \lesssim 1, \quad \frac{|t|}{s} \lesssim \alpha_G \lesssim 1, \quad (38)$$

according to which the range of variation of the gluon rapidity is large. Since

$$s\alpha_G\beta_G = -p_{G\perp}^2 \sim |t|,$$

only one independent variable can take values in a large region, leading to a contribution proportional to $\ln(s)$ [3].

In LLA, where the discontinuity should be calculated with logarithmic accuracy, only the multi-Regge kinematics could contribute so that the region was defined by Eqs.(36) and by

$$\begin{aligned} \beta_{A_1} \approx 1, \quad \alpha_{B_1} \approx 1, \quad \alpha_{A_1} \sim \beta_{B_1} \sim \frac{|t|}{s}, \\ \frac{|t|}{s} \ll \beta_G \ll 1, \quad \frac{|t|}{s} \ll \alpha_G \ll 1. \end{aligned} \quad (39)$$

Now we need to keep a constant term together with the logarithmic one, therefore, besides the contributions of the region (38) we must also calculate those ones of the

fragmentation region of the particles A, A' and B, B' . Let us start with the first of them. The region is determined, together with Eqs.(36), by the relations

$$\begin{aligned} \beta_{A_1} \approx 1, \quad \alpha_{B_1} \approx 1, \quad \alpha_{A_1} \sim \beta_{B_1} \sim \frac{|t|}{s}; \\ \beta_G \sim 1, \quad \alpha_G \sim \frac{|t|}{s}. \end{aligned} \quad (40)$$

Within the accuracy here required, the gauge invariant expression for the amplitude $\mathcal{A}_{AB}^{A_1GB_1}$ in this region is given by

$$\begin{aligned} \mathcal{A}_{AB}^{A_1GB_1} = & \frac{2g^3}{s} \bar{u}(p_{A_1}) \left\{ \langle A_1 | T^j T^c | A \rangle \not{v}_+ \frac{(\not{v}_A - \not{v}_G + m_A)}{-2(p_{APG})} \not{v}_G \right. \\ & + \langle A_1 | T^c T^j | A \rangle \not{v}_G \frac{(\not{v}_{A_1} + \not{v}_G + m_A)}{2(p_{A_1PG})} \not{v}_+ - i f_{ijc} \langle A_1 | T^i | A \rangle \\ & \times \frac{2}{(p_{A_1} - p_A)^2} \left[\not{v}_G (p_+ p_G) - \not{v}_G (p_+ e_G^*) - \not{v}_+ \left(\left(p_A - p_{A_1} + \frac{(p_B - p_{B_1})^2}{2p_B p_G} p_B \right) e_G^* \right) \right] \left. \right\} \\ & \times u(p_A) \epsilon_G^{*c} \langle B_1 | T^j | B \rangle \frac{1}{(p_{B_1} - p_B)^2} \bar{u}(p_{B_1}) \not{v}_- u(p_B). \end{aligned} \quad (41)$$

All Feynman diagrams for the gluon emission in the quark-quark scattering (see Fig. 2) contribute to the amplitude (41) and one may easily recognize the contribution of each diagram in the Feynman gauge. Let us remind that, if a diagram contains a gluon line connecting two parts with strongly different Sudakov variables, then for calculating its asymptotic contribution it is convenient to decompose $g^{\mu\nu}$ in the gluon propagator in the form

$$g^{\mu\nu} = 2 \frac{(p_-^\mu p_+^\nu + p_+^\mu p_-^\nu)}{s} + g_\perp^{\mu\nu}. \quad (42)$$

The asymptotic contribution is given only by the first term in the RHS of Eq.(42), supposing that the Sudakov variables of β -type (α -type) at the μ -vertex are much smaller (much larger) than those at the ν -vertex. Clearly, this trick is used for the gluon lines with momentum q_2 in the diagrams of Fig. 2(a,b,c). It is also used

for the gluon lines with momentum q_1 in the diagrams of Fig. 2(d,e), because the dependence on the gluon momentum is factorized for these diagrams, in the same way as for the soft gluon emission, and after this we have all the consequences for applying the trick.

We could perform all the calculations in a gauge invariant manner, but it occurs that a suitable choice of the gauge makes our task easier. This happens in the axial gauge

$$e_G p_+ = 0 . \quad (43)$$

Let us then express the amplitude (41) in terms of the Sudakov variables and using the gauge condition (43). First of all we introduce the simpler notations

$$k \equiv p_G , \quad e \equiv e_G , \quad q_1 \equiv p_A - p_{A_1} , \quad q_2 \equiv p_{B_1} - p_B ,$$

$$k = q_1 - q_2 = \alpha p_+ + \beta p_- + k_\perp . \quad (44)$$

Moreover, by means of the gauge condition (43) and of the transversality requirement $ek = 0$, we can express the polarization vector in terms of its transverse part:

$$e = e_\perp - p_+ \frac{e_\perp k_\perp}{p_+ k} . \quad (45)$$

Hence all the invariants necessary to calculate the amplitude $\mathcal{A}_{AB}^{A_1 G B_1}$ acquire the following expression in terms of the Sudakov variables:

$$2p_+ k = s\beta , \quad 2p_- k = -\frac{k_\perp^2}{\beta} , \quad 2p_+ p_{A_1} = s(1 - \beta) ,$$

$$2p_- p_{A_1} = \frac{m_A^2 - q_{1\perp}^2}{1 - \beta} , \quad 2p_A k = \frac{m_A^2 \beta^2 - k_\perp^2}{\beta} ,$$

$$2p_{A_1} k = \frac{m_A^2 \beta^2 - (k + \beta q_2)_\perp^2}{\beta(1 - \beta)} , \quad q_1^2 = \frac{q_{1\perp}^2 - m_A^2 \beta^2}{1 - \beta} , \quad q_2^2 = q_{2\perp}^2 . \quad (46)$$

Using now the polarization vector (45), the invariants (46) and the commutation relation (18) for the term of Eq.(41) containing $\langle A_1|T^c T^j|A\rangle$, through a simple calculation of spinor algebra, we arrive at

$$\begin{aligned} \mathcal{A}_{AB}^{A_1 G B_1} = & 2g^3 \bar{u}(p_{A_1}) \left\{ \langle A_1|T^j T^c|A\rangle [L(k_\perp) - L(k_\perp + \beta q_{2\perp})] \right. \\ & \left. - i f_{ijc} \langle A_1|T^i|A\rangle [L(q_{1\perp}) - L(k_\perp + \beta q_{2\perp})] \right\} \not{p}_+ u(p_A) \langle B_1|T^j|B\rangle \frac{1}{q_{2\perp}^2} \epsilon_G^{*c} \delta_{\lambda_B, \lambda_{B_1}}. \end{aligned} \quad (47)$$

Here we have put

$$L(k_\perp) = L_\mu(k_\perp) e_\perp^{*\mu}, \quad L^\mu(k_\perp) = \frac{\gamma_\perp^\mu (m_A \beta^2 - \beta k_\perp) + 2k_\perp^\mu}{m_A^2 \beta^2 - k_\perp^2}. \quad (48)$$

In order to go from Eq.(41) to Eq.(47) we must take into account that in the helicity basis one has

$$\bar{u}(p_{B_1}) \not{p}_- u(p_B) = s \delta_{\lambda_B, \lambda_{B_1}}. \quad (49)$$

The corresponding expression for the amplitude $\mathcal{A}_{A'B'}^{A_1 G B_1}$ can be obtained in the same way and reads

$$\begin{aligned} \mathcal{A}_{A'B'}^{A_1 G B_1} = & 2g^3 \bar{u}(p_{A_1}) \left\{ \langle A_1|T^j T^c|A'\rangle (L(k_\perp + \beta q_\perp) - L(k_\perp + \beta q_{2\perp})) \right. \\ & \left. - i f_{ijc} \langle A_1|T^i|A'\rangle (L(q_{1\perp} - (1 - \beta)q_\perp) - L(k_\perp + \beta q_{2\perp})) \right\} \\ & \times \not{p}_+ u(p_{A'}) \epsilon_G^{*c} \frac{1}{(q_2 - q)_\perp^2} \langle B_1|T^j|B'\rangle \delta_{\lambda_{B'}, \lambda_{B_1}}. \end{aligned} \quad (50)$$

Let us note that this expression follows from Eq.(47) by simple substitutions. Indeed, the amplitude $\mathcal{A}_{A'B'}^{A_1 G B_1}$ must take the same form of $\mathcal{A}_{AB}^{A_1 G B_1}$ when expressed in terms of *primed* Sudakov variables and *primed* vectors p'_\pm , defined by

$$\begin{aligned} p_{A'} = p'_- + \frac{m_A^2}{s} p'_+, \quad p_{B'} = p'_+ + \frac{m_B^2}{s} p'_-, \\ p_i = \beta'_i p'_- + \alpha'_i p'_+ + p_{i\perp'}, \end{aligned} \quad (51)$$

where \perp' means transversal to the $(p_{A'}, p_{B'})$ plane. Since

$$\begin{aligned} p_{A'} &= p_- + \frac{m_A^2 - q_\perp^2}{s} p_+ - q_\perp, \\ p_{B'} &= p_+ + \frac{m_B^2 - q_\perp^2}{s} p_- + q_\perp, \end{aligned} \quad (52)$$

we get the following connection between basis vectors,

$$\begin{aligned} p'_+ &\simeq p_+ - \frac{q_\perp^2}{s} p_- + q_\perp, \\ p'_- &\simeq p_- - \frac{q_\perp^2}{s} p_+ - q_\perp, \end{aligned} \quad (53)$$

and Sudakov variables,

$$\begin{aligned} \beta'_i &\simeq \beta_i - \frac{q_\perp^2}{s} \alpha_i + 2 \frac{q_\perp p_{i\perp}}{s}, \quad \alpha'_i \simeq \alpha_i - \frac{q_\perp^2}{s} \beta_i - 2 \frac{q_\perp p_{i\perp}}{s}, \\ p_{i\perp'} &\simeq p_{i\perp} + (\beta_i - \alpha_i) q_\perp + 2 \frac{p_+ - p_-}{s} \left[(\beta_i - \alpha_i) q_\perp^2 + q_\perp p_{i\perp} \right]. \end{aligned} \quad (54)$$

Keeping only the leading terms, in the kinematical region defined by Eqs. (36), (37) and (40), we find that the amplitude $\mathcal{A}_{A'B'}^{A_1 G B_1}$ can be obtained from the RHS of Eq.(47) by the substitution

$$k_\perp \rightarrow k_\perp + \beta q_\perp, \quad q_{2\perp} = p_{B_1\perp} \rightarrow q_{2\perp} - q_\perp, \quad (55)$$

besides the obvious change $A \rightarrow A'$ and $B \rightarrow B'$.

We are now able to calculate the discontinuity (32). Using Eqs. (47) and (50) and performing the summation over colour and spin states of the intermediate particles, we obtain

$$\begin{aligned} \sum_{A_1, G, B_1} \mathcal{A}_{AB}^{A_1 G B_1} \mathcal{A}_{A'B'}^{*A_1 G B_1} &= \frac{4g^6 \delta_{\lambda_B, \lambda_{B'}}}{q_{2\perp}^2 (q_2 - q_\perp)_\perp^2} \left[\langle A' | T^c T^{j'} T^j T^c | A \rangle R_a \right. \\ &\quad \left. + f_{i'j'c} f_{ijc} \langle A' | T^{i'} T^i | A \rangle R_{na} + i f_{ij'c} \langle A' | T^i T^j T^c | A \rangle R_{int} \right] \end{aligned}$$

$$-if_{ijc}\langle A'|T^cT^jT^i|A\rangle R'_{int}\Big]\langle B'|T^jT^j|B\rangle. \quad (56)$$

Here the first two terms in the square brackets come respectively from the product of *abelian* and *non abelian* parts of the amplitudes (47) and (50); one has

$$R_a = \bar{u}(p_{A'})\not{v}_+ \left[\bar{L}^\mu(k_\perp + \beta q_{2\perp}) - \bar{L}^\mu(k_\perp + \beta q_\perp) \right] (\not{v}_{A_1} + m_A) \\ \times [L_\mu(k_\perp) - L_\mu(k_\perp + \beta q_{2\perp})] \not{v}_+ u(p_A), \quad (57)$$

with

$$\bar{L}(k_\perp) = \frac{(m_A\beta^2 - \beta k_\perp)\gamma_\perp^\mu + 2k_\perp^\mu}{m_A^2\beta^2 - k_\perp^2} \quad (58)$$

and

$$R_{na} = \bar{u}(p_{A'})\not{v}_+ \left[\bar{L}^\mu(k_\perp + \beta q_{2\perp}) - \bar{L}^\mu(q_{1\perp} - (1 - \beta)q_\perp) \right] (\not{v}_{A_1} + m_A) \\ \times [L_\mu(q_{1\perp}) - L_\mu(k_\perp + \beta q_{2\perp})] \not{v}_+ u(p_A). \quad (59)$$

The remaining terms come from the interference of the *abelian* and *non abelian* parts. It is not necessary to write them explicitly, because they do not contribute to the discontinuity which we are interested in. Remind that we need to calculate the helicity conserving part of the discontinuity for the amplitude with colour octet state in the t channel and negative signature. As it was discussed just before writing the relations (18), we must antisymmetrize the colour factor $\langle B'|T^jT^j|B\rangle$ with respect to the inversion of the order of the group generators (antisymmetrization of the colour factor for the particle A leads to the same result).

Bearing that in mind, we first calculate the colour factors in Eq.(56). As for the R_{na} term, using the relations (18) and taking into account that the second of them gives us (in the case of adjoint representation $(T^i)_{lm} = -if^i{}_{lm}$)

$$f_{ijk}f_{ilm}f_{jmn} = \frac{N}{2}f_{knl}, \quad (60)$$

we get

$$\begin{aligned}
& \frac{1}{2} \langle B' | T^k | B \rangle i f_{kj'j} f_{i'j'c} f_{ijc} \langle A' | T^{i'} T^i | A \rangle = \\
& \frac{N}{4} \langle B' | T^k | B \rangle i f_{ki'i} \langle A' | T^{i'} T^i | A \rangle = \\
& - \frac{N^2}{8} \langle B' | T^k | B \rangle \langle A' | T^k | A \rangle .
\end{aligned} \tag{61}$$

The same relations allow to show that the colour factors for the interference terms are zero. For example, the colour factor of the R_{int} term is

$$\begin{aligned}
& \frac{1}{2} i f_{kj'j} \langle B' | T^k | B \rangle i f_{ij'c} \langle A' | T^i T^j T^c | A \rangle = \\
& \frac{1}{2} i f_{kj'j} \langle B' | T^k | B \rangle i f_{ij'c} \left(i f_{ljc} \langle A' | T^i T^l | A \rangle + \langle A' | T^i T^c T^j | A \rangle \right) = \\
& \frac{1}{2} \langle B' | T^k | B \rangle \left(\frac{N}{2} i f_{kli} \langle A' | T^i T^l | A \rangle + \frac{N}{2} i f_{kj'j} \langle A' | T^{j'} T^j | A \rangle \right) = 0 .
\end{aligned} \tag{62}$$

The same result is obtained for R'_{int} . For the calculation of the colour factor of the R_a term we need the relation

$$T^a T^c T^a = \left(C_2 - \frac{N}{2} \right) T^c . \tag{63}$$

Here C_2 is the eigenvalue of the Casimir operator $T^a T^a$ and its expression for quarks is

$$C_2 = \frac{N^2 - 1}{2N} .$$

Eq.(63) can be obtained using the relations (18). At the end, for the colour factor of the R_a term, we find

$$\begin{aligned}
& \frac{i}{2} f_{kj'j} \langle B' | T^k | B \rangle \langle A' | T^c T^{j'} T^j T^c | A \rangle = \\
& - \frac{N}{4} \langle B' | T^k | B \rangle \langle A' | T^c T^k T^c | A \rangle = \frac{1}{8} \langle B' | T^k | B \rangle \langle A' | T^k | A \rangle .
\end{aligned} \tag{64}$$

It remains now to calculate R_a and R_{na} . It is clear from Eqs. (57) and (59) that both quantities can be obtained from the general expression

$$G(r_{\perp}, r'_{\perp}) = \bar{u}(p_{A'}) \not{r}'_{\perp} \bar{L}^{\mu}(r_{\perp}) (\not{r}'_{A_1} + m_A) L_{\mu}(r_{\perp} + r'_{\perp}) \not{r}'_{\perp} u(p_A), \quad (65)$$

where r_{\perp} is linear in k_{\perp} and r'_{\perp} does not depend on k_{\perp} . Taking into account that

$$p_+ \gamma_{\perp} = 0, \quad \not{r}'_{\perp}{}^2 = 0, \quad 2p_+ p_{A_1} = s(1 - \beta),$$

and putting

$$d(k_{\perp}) = \frac{1}{m_A^2 \beta^2 - k_{\perp}^2}, \quad (66)$$

we find that the expression (65) may be written as

$$\begin{aligned} G(r_{\perp}, r'_{\perp}) &= s(1 - \beta) d(r_{\perp}) d(r_{\perp} + r'_{\perp}) \bar{u}(p_{A'}) \\ &\times \left[(-m_A \beta^2 - \beta \not{r}'_{\perp}) \gamma_{\perp}^{\mu} + 2r_{\perp}^{\mu} \right] \left[\gamma_{\perp \mu} (m_A \beta^2 - \beta (\not{r}'_{\perp} + \not{r}'_{\perp})) + 2(r_{\perp} + r'_{\perp})_{\perp \mu} \right] \not{r}'_{\perp} u(p_A) = \\ &s(1 - \beta) d(r_{\perp}) d(r_{\perp} + r'_{\perp}) \bar{u}(p_{A'}) \left[(D - 2) (-m_A^2 \beta^4 + \beta^2 (r_{\perp}^2 + \not{r}'_{\perp} \not{r}'_{\perp}) + m_A \beta^3 \not{r}'_{\perp}) \right. \\ &\quad \left. + 4r_{\perp} (r_{\perp} + r'_{\perp})_{\perp} (1 - \beta) - 2m_A \beta^2 \not{r}'_{\perp} \right] \not{r}'_{\perp} u(p_A). \end{aligned} \quad (67)$$

We have used here the fact that the term $\not{r}'_{\perp} \not{r}'_{\perp}$, being under integration over the azimuthal angles, is equivalent to $r_{\perp} r'_{\perp}$.

The term \not{r}'_{\perp} , after the integration is performed, becomes proportional to \not{r}'_{\perp} , therefore it can be omitted. In fact, because of the relation (see Eq.(37) in Ref. [12])

$$\begin{aligned} \bar{u}(p_{A'}) \not{r}'_{\perp} \not{r}'_{\perp} u(p_A) &= \bar{u}(p_{A'}) (\not{r}'_{A_1} - \not{r}'_{A_1}) \not{r}'_{\perp} u(p_A) = \\ \bar{u}(p_{A'}) (-2m \not{r}'_{\perp} + s) u(p_A) &= s \left(\bar{u}(p_{A'}) u(p_A) - 2m \delta_{\lambda_A, \lambda_{A'}} \right) = -is \sqrt{-t} \delta_{\lambda_A, -\lambda_{A'}}, \end{aligned} \quad (68)$$

it does not contribute to the helicity conserving part which we are interested in.

After some algebra, Eq.(67) turns into

$$G^{(+)}(r_{\perp}, r'_{\perp}) = s^2 (1 - \beta) d(r_{\perp}) d(r_{\perp} + r'_{\perp}) \left[- \left(2(1 - \beta) + (D - 2) \frac{\beta^2}{2} \right) \right]$$

$$\left(r_{\perp}^{\prime 2} + \frac{1}{d(r_{\perp} + r'_{\perp})} + \frac{1}{d(r_{\perp})} \right) + 4m_A^2 \beta^2 (1 - \beta) \quad (69)$$

where the superscript (+) stands for helicity conserving part.

Let us denote by $\Delta^{(3A)}$ the contribution to $\Delta_S^{(3)}$ defined by Eqs.(11) and (13), which comes from the fragmentation region of the particles A and A' . Applying the unitarity condition (32) and Eq.(56) with the colour factors given by formulas (61), (62) and (64), we have

$$\Delta^{(3A)} = \frac{4g^4 t}{s} \int \frac{d\Phi_3(p_A + p_B; p_{A_1}, k, p_{B_1})}{2\pi q_{2\perp}^2 (q_2 - q)_{\perp}^2} \left[-\frac{1}{8} R_a^{(+)} + \frac{N^2}{8} R_{na}^{(+)} \right], \quad (70)$$

where the superscript (+) means, as before, helicity conserving part. The integration runs over the fragmentation region of the particles A and A' , where the phase space element takes the form

$$\frac{d\Phi_3(p_A + p_B; p_{A_1}, k, p_{B_1})}{2\pi} = \frac{1}{4s} \frac{d\beta}{\beta(1-\beta)} \frac{d^{(D-2)} q_{1\perp} d^{(D-2)} q_{2\perp}}{(2\pi)^{2(D-1)}}, \quad (71)$$

with

$$q_1 = p_A - p_{A_1}, \quad q_2 = p_{B_1} - p_B, \quad k = q_1 - q_2.$$

It is convenient to split $\Delta^{(3A)}$ into two parts,

$$\Delta^{(3A)} = \Delta_a^{(3A)} + \Delta_{na}^{(3A)}, \quad (72)$$

which respectively contain the terms $R_a^{(+)}$ and $R_{na}^{(+)}$. Using the phase space element (71), the expressions (57) and (59) for $R_a^{(+)}$ and $R_{na}^{(+)}$ respectively and Eqs.(65), (69) for $G(r_{\perp}, r'_{\perp})$, from Eq.(70) (here and below all vectors are $(D-2)$ -dimensional and orthogonal to the (p_A, p_B) plane) we get

$$\begin{aligned} \Delta_a^{(3A)} = & \frac{g^4 t}{8} \int \frac{d^{(D-2)} q_1}{(2\pi)^{D-1}} \frac{d^{(D-2)} q_2}{(2\pi)^{D-1}} \int_{\beta_0}^1 \frac{d\beta}{\beta} \frac{\beta^2}{q_2^2 (q_2 - q)^2} \left[\left(2(1 - \beta) + \frac{\beta^2}{2} (D - 2) \right) \right. \\ & \left. \times d(k) d(k + \beta q) d(k + \beta q_2) \left(\frac{-q^2}{d(k + \beta q_2)} + \frac{q_2^2}{d(k + \beta q)} + \frac{(q_2 - q)^2}{d(k)} \right) \right] \end{aligned}$$

$$+4m_A^2(1-\beta)(d(k+\beta q_2)-d(k))(d(k+\beta q_2)-d(k+\beta q))], \quad (73)$$

and

$$\begin{aligned} \Delta_{na}^{(3A)} = & \frac{g^4 N^2 t}{8} \int \frac{d^{(D-2)} q_1}{(2\pi)^{D-1}} \frac{d^{(D-2)} q_2}{(2\pi)^{D-1}} \int_{\beta_0}^1 \frac{d\beta}{\beta} \frac{(1-\beta)^2}{q_2^2 (q_2 - q)^2} \left[\left(2(1-\beta) + \frac{\beta^2}{2}(D-2) \right) \right. \\ & \times d(q_1) d(k+\beta q_2) d(q_1 - (1-\beta)q) \left(\frac{q^2}{d(k+\beta q_2)} - \frac{q_2^2}{d(q_1 - (1-\beta)q)} - \frac{(q_2 - q)^2}{d(q_1)} \right) \\ & \left. - \frac{4m_A^2 \beta^2}{1-\beta} (d(k+\beta q_2) - d(q_1)) (d(k+\beta q_2) - d(q_1 - (1-\beta)q)) \right]. \quad (74) \end{aligned}$$

Here β_0 is a not specified, artificially introduced boundary of the fragmentation region of the particles A and A' . Eqs.(72)-(74) give us the contribution of the fragmentation region of the particles A and A' . It should be supplemented by two contributions. One of them comes from the fragmentation region of the particles B and B' which, in analogy to Eq.(72), can be divided into two parts accordingly to the colour factors,

$$\Delta^{(3B)} = \Delta_a^{(3B)} + \Delta_{na}^{(3B)}. \quad (75)$$

Evidently, $\Delta_a^{(3B)}$ and $\Delta_{na}^{(3B)}$ can be respectively obtained from Eqs.(73) and (74) by the substitution

$$\begin{aligned} q_{1,2} & \rightarrow -q_{2,1}, & q & \rightarrow -q, & m_A & \rightarrow m_B, \\ \beta & \rightarrow \alpha \equiv \alpha_G = \frac{-k^2}{s\beta}, & \beta_0 & \rightarrow \alpha_0. \end{aligned} \quad (76)$$

The other contribution, that we call $\Delta^{(3I)}$, comes from the multi-Regge region, defined by Eqs.(36), (37) and (39), which is intermediate between the two fragmentation regions. This contribution was calculated [3] and reads

$$\Delta^{(3I)} = \frac{g^4 N^2 t}{8} \int \frac{d^{(D-2)} q_1}{(2\pi)^{D-1}} \frac{d^{(D-2)} q_2}{(2\pi)^{D-1}} \int_{\gg \frac{|t|}{s}}^{\ll 1} \frac{d\beta}{\beta} \frac{1}{q_2^2 (q_2 - q)^2}$$

$$\times 2 \left[\frac{q^2}{q_1^2(q_1 - q)^2} - \frac{q_2^2}{q_1^2 k^2} - \frac{(q_2 - q)^2}{(q_1 - q)^2 k^2} \right]. \quad (77)$$

One can easily recognize that it is obtained from the expression (74) for $\Delta_{na}^{(3A)}$ by changing the limits of integration over β and going to small β in the integrand. Notice that the abelian part (73) does not contribute in this case. It means that the integrands of Eqs.(73) and (74) are valid in a region wider than that of the fragmentation one, namely not simply for $\beta \sim 1$ but for $|t|/s \ll \beta \leq 1$. This is not unusual because $|t|/s$ is not an artificial bound but a natural one where the applicability should be broken. That allows us not to consider the multi-Regge region separately, but to include it in any of the two fragmentation regions which, then, become overlapping. Hence, the total three particle discontinuity can be obtained as the sum of the contributions of the two fragmentation regions,

$$\Delta_S^{(3)} = \Delta^{(3A)} + \Delta^{(3B)}. \quad (78)$$

Here the first term in the RHS is given by Eqs.(72)-(74), the other one follows from it by the substitution (76), with the parameters β_0 and α_0 satisfying the condition $s\beta_0\alpha_0 = -k^2$. Choosing

$$\beta_0 = \alpha_0 = \sqrt{\frac{-k^2}{s}}, \quad (79)$$

we get $\Delta^{(3B)}$ from $\Delta^{(3A)}$ simply by the substitution $m_A \rightarrow m_B$.

For the abelian part $\Delta_a^{(3A)}$ of the last discontinuity, it is clear from Eq.(73) that β_0 could be put equal to zero because the small β region does not contribute. That is in accordance with our experience in QED, where it is well known that cones of photon emission by two scattered particles do not overlap at high energies.

Performing the integration over q_1 in Eq.(73) with the help of the Feynman

parametrization,

$$\int \frac{d^{(D-2)}r}{(2\pi)^{D-1}} d(r)d(r+l) = \frac{2\Gamma\left(3 - \frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \int_0^1 \frac{dx}{c_{m\beta}(l)}, \quad (80)$$

where

$$c_m(l) = [m_A^2 - l^2 x(1-x)]^{3-\frac{D}{2}}, \quad (81)$$

we find

$$\begin{aligned} \Delta_a^{(3A)} &= \frac{g^4 t \Gamma\left(3 - \frac{D}{2}\right)}{4 (4\pi)^{\frac{D}{2}}} \int \frac{d^{(D-2)}q_2}{(2\pi)^{D-1} q_2^2 (q_2 - q)^2} \int_0^1 \frac{d\beta}{\beta^{(5-D)}} \int_0^1 dx \left[\left(2(1-\beta) + \frac{\beta^2}{2}(D-2) \right) \right. \\ &\quad \times \left. \left(\frac{-q^2}{c_m(q)} + \frac{2q_2^2}{c_m(q_2)} \right) + 4m_A^2(1-\beta) \left(\frac{1}{c_m(0)} + \frac{1}{c_m(q)} - \frac{2}{c_m(q_2)} \right) \right]. \quad (82) \end{aligned}$$

After this step the integration over β can be easily performed and yields

$$\begin{aligned} \Delta_a^{(3A)} &= \frac{g^4 t \Gamma\left(2 - \frac{D}{2}\right)}{4 (4\pi)^{\frac{D}{2}}} \int \frac{d^{(D-2)}q'}{(2\pi)^{D-1} q'^2 (q' - q)^2} \\ &\quad \times \int_0^1 dx \left[\left(\frac{D-4}{4} + \frac{1}{D-3} \right) \left(\frac{q^2}{c_m(q)} - \frac{2q'^2}{c_m(q')} \right) \right. \\ &\quad \left. + \frac{2m_A^2}{D-3} \left(\frac{2}{c_m(q')} - \frac{1}{c_m(q)} - \frac{1}{c_m(0)} \right) \right]. \quad (83) \end{aligned}$$

In order to calculate the non abelian contribution $\Delta_{na}^{(3A)}$ given by Eq.(74) with $\beta_0 = \sqrt{-k^2/s}$, we apply the following trick: we subtract and add this contribution considered for a massless quark, $\Delta_{na}^{(3A)}(m_A = 0)$. Performing the integration over q_1 of the difference

$$\delta_{na}^{(3A)} = \Delta_{na}^{(3A)} - \Delta_{na}^{(3A)}(m_A = 0) \quad (84)$$

we may put $\beta_0 = 0$. With the help of Eq.(80) we arrive at

$$\delta_{na}^{(3A)} = \frac{g^4 N^2 t \Gamma\left(3 - \frac{D}{2}\right)}{4 (4\pi)^{\frac{D}{2}}}$$

$$\begin{aligned}
& \times \int \frac{d^{(D-2)}q_2}{(2\pi)^{D-1}q_2^2(q_2-q)^2} \int_0^1 d\beta \frac{(1-\beta)^2}{\beta} \int_0^1 dx \left\{ \left(2(1-\beta) + \frac{\beta^2}{2}(D-2) \right) \right. \\
& \times \left[q^2 \left(\frac{1}{c_{m\beta}(q(1-\beta))} - \frac{1}{c_0(q(1-\beta))} \right) - 2q_2^2 \left(\frac{1}{c_{m\beta}(q_2(1-\beta))} - \frac{1}{c_0(q_2(1-\beta))} \right) \right] \\
& \left. - \frac{4m_A^2\beta^2}{1-\beta} \left[\frac{1}{c_{m\beta}(q(1-\beta))} - \frac{2}{c_{m\beta}(q_2(1-\beta))} + \frac{1}{c_{m\beta}(0)} \right] \right\}, \quad (85)
\end{aligned}$$

The total contribution of the three particle intermediate state to the discontinuity is then given by the sum

$$\Delta_S^{(3)} = \Delta_a^{(3A)} + \delta_{na}^{(3A)} + \Delta_a^{(3B)} + \delta_{na}^{(3B)} + \Delta_{na}^{(3A)}(m_A = 0) + \Delta_{na}^{(3B)}(m_B = 0). \quad (86)$$

Here $\Delta_a^{(3A)}$ and $\delta_{na}^{(3A)}$ are given by Eqs. (83) and (85) respectively, $\Delta_a^{(3B)}$ and $\delta_{na}^{(3B)}$ follow from these equations by the substitution $m_A \rightarrow m_B$ and $\Delta_{na}^{(3A)}(m_A = 0)$, which is equal to $\Delta_{na}^{(3B)}(m_B = 0)$, comes from Eq.(74) with $\beta_0 = \sqrt{-k^2/s}$.

5. TWO-LOOP CORRECTION TO THE GLUON TRAJECTORY

We are now ready to discuss the formula (12) for the correction $\omega^{(2)}(t)$. It contains the discontinuity Δ_S which has been calculated in this paper and, according to Eq.(13), is given by the sum of the contributions $\Delta_S^{(2)}$ and $\Delta_S^{(3)}$, respectively quoted in Eqs.(21) and (86). Besides that, Eq.(12) involves the leading contribution to the trajectory $\omega^{(1)}(t)$ shown in Eq.(4) and the one-loop corrections to the QQR vertex $\Gamma_{QQ}^{(+)}$, which are given by Eqs.(22)-(24), (27), (28) and (31). Notice that the discontinuity $\Delta_S^{(2)}$ itself is expressed in terms of $\omega^{(1)}(t)$ and QQR vertex. Therefore, the correction $\omega^{(2)}(t)$ looks rather complicated. Fortunately, a series of remarkable cancellations occurs among various terms in it.

First of all, the term $\Delta_a^{(3A)}$ in $\Delta_S^{(3)}$ cancels the term with $a_Q(r, m_A^2)$ coming from $\Delta_S^{(2)}$ in Eq.(21) and $\Gamma_{A'A}^{(+)}\omega^{(1)}(t)$ in Eq.(12). One might easily see that putting the expressions (24) for $a_Q(r, m_A^2)$ and (4) for $\omega^{(1)}(t)$ into Eqs.(21) and (12), and

comparing the result with Eq.(83). Of course, the same cancellation exists between the term $\Delta_a^{(3B)}$ and the terms with $a_Q(r, m_B^2)$. Quite analogously, the term $\delta_{na}^{(3A)}$ (85) in $\Delta_S^{(3)}$ cancels the terms with $\delta_g(r, m_A^2)$ (31). In order to make this cancellation more evident, one only needs to change the variables in Eq.(31):

$$x_1 \rightarrow \beta, \quad x_2 \rightarrow (1 - \beta)x. \quad (87)$$

In turn also the term $\delta_{na}^{(3B)}$ cancels the terms with $\delta_g(r, m_B^2)$.

After these cancellations are performed, one realizes that the correction $\omega^{(2)}(t)$ of Eq.(12) does not depend on the masses of the scattered quarks, as it should be, because the trajectory, by definition, could not depend on the properties of the scattered particles. In fact, Eq.(12), because of Eqs.(86), (21), (22) and (27), gives us

$$\begin{aligned} \omega^{(2)}(t) = & 2\Delta_{na}^{(3A)}(m_A = 0) + \frac{g^2 N t}{(2\pi)^{D-1}} \int \frac{d^{(D-2)}q_1}{q_1^2 (q_1 - q)^2} \left[\omega^{(1)}(q_1^2) \ln \left(\frac{s}{-q_1^2} \right) \right. \\ & \left. + 2a_f(q_1^2) + 2a_g(q_1^2, 0) \right] - \left(\omega^{(1)}(t) \right)^2 \ln \left(\frac{s}{-t} \right) - 2\omega^{(1)}(t) [a_f(t) + a_g(t, 0)], \quad (88) \end{aligned}$$

where $\omega^{(1)}(t)$, $a_f(t)$, $a_g(t, 0)$ and $\Delta_{na}^{(3A)}$ are given by Eqs.(4), (23), (28) and (74) respectively.

Looking at Eq.(88), we observe that not all the cancellations are performed up to this point. Indeed, the trajectory cannot depend on s , therefore the terms with $\ln(s)$ must cancel each other. Let us note that the cancellation of these terms is a consequence of the gluon Reggeization in LLA, which was previously proved [3, 7], and can serve as a check of our calculations, whereas the cancellations discussed above confirm the gluon Reggeization beyond LLA.

In order to explicitly demonstrate the cancellation of the terms with $\ln(s)$ and to present the correction $\omega^{(2)}(t)$ in a more transparent form, we integrate over β in

Eq.(74) with $m_A = 0$. We do that in the following way. We divide the integration region into two parts, one from $\beta_0 = \sqrt{-k^2/s}$ to δ , the other one from δ to 1, with $\delta \ll 1$ (at the end of the calculation we let δ go to zero). In the first region we may put $\beta = 0$ everywhere besides the factor $d\beta/\beta$; thus the corresponding contribution to $\Delta_{na}^{(3A)}(m_A = 0)$ is

$$\frac{g^4 N^2 t}{8} \int \frac{d^{(D-2)}q_1}{(2\pi)^{D-1}} \frac{d^{(D-2)}q_2}{(2\pi)^{D-1}} \frac{1}{q_1^2(q_2 - q)^2} \left[\frac{q^2}{q_2^2(q_1 - q)^2} - \frac{2}{(q_1 - q_2)^2} \right] \ln \left(\frac{s\delta^2}{-(q_1 - q_2)^2} \right). \quad (89)$$

In the second region the change of the variable $q_1 \rightarrow (1 - \beta)q_1$ factorizes the β -dependence, yielding

$$\int_s^1 \frac{d\beta}{\beta} (1-\beta)^{D-4} \left(2(1-\beta) + \frac{\beta^2}{2}(D-2) \right) = 2 \ln \left(\frac{1}{\delta} \right) + 2\psi(1) - 2\psi(D-3) - \frac{3}{2(D-3)}. \quad (90)$$

To obtain this result we have used the decomposition

$$\int_s^1 \frac{d\beta}{\beta} f(\beta) = \int_s^1 \frac{d\beta}{\beta} (f(\beta) - f(0)) + \int_s^1 \frac{d\beta}{\beta} f(0), \quad (91)$$

and the integral

$$\int_0^1 \frac{d\beta}{\beta} [(1-\beta)^{D-4} - 1] = \psi(1) - \psi(D-3). \quad (92)$$

Given Eqs.(89) and (90), from Eq.(74) we get

$$\Delta_{na}^{(3A)}(m_A = 0) = \frac{g^4 N^2 t}{8} \int \frac{d^{(D-2)}q_1}{(2\pi)^{D-1}} \frac{d^{(D-2)}q_2}{(2\pi)^{D-1}} \frac{1}{q_1^2(q_2 - q)^2} \left[\frac{q^2}{q_2^2(q_1 - q)^2} - \frac{2}{(q_1 - q_2)^2} \right] \left[\ln \left(\frac{s}{-(q_1 - q_2)^2} \right) + 2\psi(1) - 2\psi(D-3) - \frac{3}{2(D-3)} \right]. \quad (93)$$

We now substitute this expression, together with the expressions (4) for $\omega^{(1)}(t)$, (23) and (28) for $a_f(t)$ and $a_g(t, 0)$ respectively, in Eq.(88). Making use of the equality

$$\int \frac{d^{(D-2)}r}{(2\pi)^{D-1} r^2 (r - r')^2} = -\frac{4\Gamma\left(2 - \frac{D}{2}\right) \Gamma^2\left(\frac{D}{2} - 1\right)}{(4\pi)^{\frac{D}{2}} \Gamma(D-3) (-r'^2)^{3 - \frac{D}{2}}} \quad (94)$$

in order to represent the contribution of $a_g(t, 0)$ in integral form, we arrive at the final result

$$\begin{aligned}
\omega^{(2)}(t) = & \frac{g^4 N^2 t}{4} \int \frac{d^{(D-2)} q_1}{(2\pi)^{D-1} q_1^2} \left\{ \int \frac{d^{(D-2)} q_2}{(2\pi)^{D-1} q_2^2} \left[\frac{q^2}{(q_1 - q)^2 (q_2 - q)^2} \ln \left(\frac{q^2}{(q_1 - q_2)^2} \right) \right. \right. \\
& + \frac{2}{(q_1 + q_2 - q)^2} \ln \left(\frac{q_1^2}{(q_1 - q)^2} \right) + \left. \left(-\frac{q^2}{(q_1 - q)^2 (q_2 - q)^2} + \frac{2}{(q_1 + q_2 - q)^2} \right) \right. \\
& \times \left(2\psi(D-3) + \psi\left(3 - \frac{D}{2}\right) - 2\psi\left(\frac{D}{2} - 2\right) - \psi(1) \right. \\
& \left. \left. + \frac{1}{(D-3)} \left(\frac{1}{4(D-1)} - \frac{2}{D-4} - \frac{1}{4} \right) \right) \right] - \frac{8}{N} \frac{\Gamma\left(2 - \frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}}} \frac{1}{(q_1 - q)^2} \\
& \times \sum_f \int_0^1 dx x(1-x) \left[\frac{2}{(m_f^2 - q_1^2 x(1-x))^{2-\frac{D}{2}}} - \frac{1}{(m_f^2 - q^2 x(1-x))^{2-\frac{D}{2}}} \right] \left. \right\}. \quad (95)
\end{aligned}$$

This equation gives the two-loop correction to the gluon trajectory in a closed form. Summation is performed over quark flavours and $t = q^2$. Remind that all vectors here are $(D-2)$ -dimensional and spacelike, $r^2 = -\bar{r}^2$.

6. Summary

We have calculated the two-loop correction $\omega^{(2)}(t)$ to the trajectory of the Reggeized gluon in QCD. To find this correction we used the scattering process of massive quarks at large energies \sqrt{s} and fixed momentum transfer $\sqrt{-t}$. The correction is given by Eq.(12) in terms of the helicity conserving part of s-channel discontinuity Δ_S of the amplitude with colour octet state and negative signature in the t channel. Furthermore, it depends on the leading contribution $\omega^{(1)}(t)$ to the trajectory [3] (see. Eq.(4)) and the one-loop correction to the helicity conserving part of the quark-quark-Reggeon vertex $\Gamma_{QQ}^{(+)}(t)$ [12] (see. Eqs. (22)-(24), (27), (28) and (31)).

The discontinuity Δ_S , obtained in the two-loop approximation, consists of two

contributions, $\Delta_S^{(2)}$ (see. Eq.(21)) and $\Delta_S^{(3)}$ (see Eqs.(86), (83),(85) and (74)), which come from two and three particle intermediate states in the unitary condition.

We have obtained the final result (95) for the two-loop contribution $\omega^{(2)}(t)$ to the trajectory as a consequence of a series of remarkable cancellations. These cancellations exhibit the independence of $\omega^{(2)}(t)$ on any specific property of the scattered quarks and confirm the gluon Reggeization beyond the leading logarithmic approximation.

Eq.(95) gives us the correction $\omega^{(2)}(t)$ for the space-time dimension D . Of course, our main interest relies in the physical case $D = 4$. Unfortunately, in this case the correction shows up divergences which are both ultraviolet and infrared. Indeed, the former ones are not difficult to deal with. They are due to simple first order poles and can be removed by expressing the bare coupling constant g in terms of the renormalized one in the total expression for the trajectory:

$$\omega(t) = \omega^{(1)}(t) + \omega^{(2)}(t) + \dots$$

In the \overline{MS} scheme one has

$$g = g_\mu \mu^{2-\frac{D}{2}} \left\{ 1 + \left(\frac{11}{3}N - \frac{2}{3}n_f \right) \frac{g_\mu^2}{(4\pi)^2} \left[\frac{1}{D-4} - \frac{1}{2} \ln(4\pi) - \frac{1}{2} \psi(1) \right] + \dots \right\}, \quad (96)$$

where g_μ is the renormalized coupling constant at the normalization point μ .

The infrared divergences, on the contrary, are much more severe. They could not be cancelled inside the trajectory $\omega(t)$ because the gluon is a colour object, whereas we may expect their cancellation for the scattering of colourless objects only. So, we should check the cancellation of the infrared divergences when the trajectory would be put into the Bethe-Salpeter type equation for the amplitude

with vacuum quantum numbers in the t channel. We hope to do that in subsequent papers.

Acknowledgement: One of us (V.F.) thanks the Dipartimento di Fisica della Università della Calabria and the Istituto Nazionale di Fisica Nucleare - Gruppo collegato di Cosenza for their warm hospitality while part of this work was done. He thanks also the International Science Foundation for financial support.

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Figure Captions

Fig. 1: Two particle contribution to the two-loop s-channel discontinuity. One of the amplitudes is in the Born approximation, the other one in the one-loop approximation.

Fig. 2: Feynman diagrams for gluon emission.

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РЕДЖЕЗАЦИЯ АМПЛИТУДЫ РАССЕЯНИЯ КВАРКА НА КВАРКЕ
В КХД

ИЯФ 95-49

Ответственный за выпуск С.Г.Попов

Работа поступила 14 июня 1995 г.

Сданов в набор

Подписано в печать

Формат 60x90 1/16. Объем 2,0 печ.л., 1,6 уч.-изд.л.

Тираж 250 экз. Бесплатно. Заказ № 49

Новосибирск, 630090, Ротапринт ИЯФ СО РАН