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Abstract

We calculate in the two-loop approximation s-channel discontinuity of gluon-gluon scattering amplitude with gluon quantum numbers in t channel and negative signature in the Regge kinematical region. Assuming that the asymptotic behaviour of the amplitude in this region is given by the Reggeized gluon contribution and using this discontinuity we find the gluon trajectory in the two-loop approximation. Coincidence of the result obtained in this way with the two-loop correction to the trajectory extracted from quark-quark scattering amplitude confirms the gluon Reggeization beyond the leading logarithmic approximation.

1 Introduction

Perturbative QCD combined with the operator product expansion and improved by the renormalization group is successfully used since a long time for study of hard processes[1]. The applicability of the perturbation theory for description of these processes is secured by smallness of the strong coupling constant $\alpha_s(Q^2)$, where Q is the hard scale (typical virtuality). For semihard processes [2] the c.m.s. energy $\sqrt{s}$ of colliding particles is much larger than Q, so that a new important parameter appears: $x = Q^2/s$. At very high energy this parameter becomes so small that one needs to sum up terms of the type $\alpha_s^n[\ln(1/x)]^m$. In the leading logarithmic approximation (LLA), which means for the scattering channel considered here summation of the terms with $m = n$, this problem was solved many years ago [3]. Now the results of LLA are widely known and used for description of experimental data. However, LLA has two serious shortcomings.

Firstly, unitarity constraints for scattering amplitudes with vacuum quantum numbers in t -channel don't work in this approximation and, as a result, the Froissart bound $\sigma_{tot} < c(\ln\ s)^2$ is violated. The total cross section σ_{tot} increases with increasing of energy as power of s being calculated in LLA:

$$\sigma_{tot} \sim \frac{s^{\omega_0}}{\sqrt{\ln\ s}}, \tag{1}$$

with the exponent

$$\omega_0 = \frac{g^2}{\pi^2} N \ln\ 2 \tag{2}$$

for the gauge group $SU(N)$ ($N = 3$ for QCD). Here g is the gauge coupling constant ($\alpha_s = g^2/4\pi$). In terms of parton distributions this means their sharp power increase with decreasing x. The behaviour (1) contradicts the unitarity and should be modified at asymptotically large energies. This problem is extremely important from a theoretical point of view and became the subject of many papers (see, for example, [4]). From a practical point of view it seems more important to improve the second shortcoming. The matter is, that dependence of α_s on virtuality is beyond the accuracy of LLA. Therefore numerical results of LLA can be change strongly by changing a scale of the virtuality. It diminishes the predictive power of LLA, which is used now for calculation of structure functions in the small x region (see, for example, [5]).

The scale of the virtuality in the coupling constant argument can be fixed, the uncertainties of LLA predictions can be removed, and the region of applicability of these predictions can be determined by radiative corrections. Therefore the problem of calculating of radiative corrections to LLA is very important now.

For solving this problem the key point can be [6] the gluon Reggeization, which was proved [3],[7] in LLA. The gluon trajectory

$$j(t) = 1 + \omega(t) \tag{3}$$

in the one-loop approximation is given by [3]:

$$\omega(t) = \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}} \frac{N}{2} \int \frac{d^{D-2}k_\perp}{k_\perp^2 (k-q)_\perp^2}, \tag{4}$$

where D is the space-time dimension ($D \neq 4$ is introduced to regularize Feynman integrals), q is the momentum transfer and $t = q^2 \approx q_\perp^2$. The integration in (4) is performed over the $(D-2)$ dimensional subspace, which is orthogonal to the initial particle momentum plane.

The problem of calculating next-to-leading corrections can be turned into calculating corrections to the kernel of the Bethe-Salpeter type equation for the t-channel partial amplitude with the vacuum quantum numbers [6]. This kernel is expressed through the gluon trajectory and the Reggeon-Reggeon-gluon vertex. Corrections to the vertex were calculated already [8]-[10], therefore, the calculation of the two-loop correction $\omega^{(2)}(t)$ to the gluon trajectory appears to be the most urgent problem.

In this paper we present results and details of the calculation of the two-loop correction to the gluon trajectory for the case of QCD with massive quarks. The paper is organized as follows. In the Section 2 we discuss the method of calculations. In Section 3 we calculate the contribution of the

two-particle intermediate state to s -channel discontinuity of the gluon-gluon scattering amplitude. An analogous calculations for the contributions of the three-gluon and quark-antiquark-gluon intermediate states are given in the Sections 4 and 5 respectively. The final result for the two-loop correction to the gluon Regge trajectory $\omega^{(2)}(t)$ is presented in Section 6. Section 7 contains a brief summary.

2 Method of calculation

For getting the correction $\omega^{(2)}(t)$ to the gluon trajectory it is sufficient to calculate the two-loop contribution to the s-channel discontinuity $\left[\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'}\right]_s$ of the amplitude $\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'}$ of the elastic gluon-gluon scattering $A + B \to A' + B'$ with gluon quantum numbers in the t -channel and negative signature. Assuming that an asymptotic behaviour of the amplitude in the kinematical region ($s = (p_A + p_B)^2 \to \infty$, $t = (p_{A'} - p_A)^2$ -fixed) is given by the Reggeized gluon contribution, this amplitude has the factorized form:

$$\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'} = \Gamma^c_{A'A} \frac{s}{t} \left[\left(\frac{s}{-t}\right)^{\omega(t)} + \left(\frac{-s}{-t}\right)^{\omega(t)}\right] \Gamma^c_{B'B}, \tag{5}$$

where $\Gamma^c_{A'A}$ are the vertices of gluon-gluon-Reggeon interaction (GGR vertices) and $\omega(t)$ is the gluon Regge trajectory. In the helicity basis the GGR vertices can be presented in the following form [9],[11]

$$\Gamma^c_{G'G} = g\langle G'|T^c|G\rangle \left[\delta_{\lambda_{G'},\lambda_G}\left(1 + \Gamma^{(+)}_{GG}(t)\right) + \delta_{\lambda_{G'},-\lambda_G}\Gamma^{(-)}_{GG}(t)\right]. \tag{6}$$

Here $\langle G'|T^c|G\rangle = T^c_{G'G}$ are matrix elements of the $SU(N)$ colour group generators in the adjoint representation and $\Gamma^{(\pm)}_{GG}(t)$ are the radiative corrections to the helicity conserving LLA vertex [3]. In the following we will use one-loop approximation for these corrections.

It is convenient to extract from the amplitude the part conserving helicities of each of colliding particles ($\lambda_{A'} = \lambda_A, \lambda_{B'} = \lambda_B$) and to take its average over these helicities. Let us define the value Δ_s by the following relation:

$$g^2 \langle A'|T^i|A\rangle\langle B'|T^i|B\rangle \left(-2\pi i \frac{s}{t}\right) \Delta_s =$$

$$= \frac{1}{(D-2)^2} \sum_{\lambda_A,\lambda_B} \left[\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'(two-loop)}\right]_s \Big|_{\substack{\lambda_{A'}=\lambda_A \\ \lambda_{B'}=\lambda_B}}. \tag{7}$$

The factor $\frac{1}{(D-2)^2}$ appears here on account of taking the average, because the number of helicity states in the D-dimensional space-time is equal $(D-2)$. Calculating RHS of Eq.(7) with use of the representations (5) for the amplitude and (6) for the vertices we obtain:

$$\Delta_s = \ln\left(\frac{s}{-t}\right)\left(\omega^{(1)}(t)\right)^2 + 2\omega^{(1)}(t)\Gamma_{GG}^{(+)}(t) + \omega^{(2)}(t). \qquad (8)$$

So, the two-loop correction to the trajectory $\omega^{(2)}(t)$ is expressed through the two-loop discontinuity Δ_s and the one-loop contributions to the trajectory $\omega^{(1)}(t)$ and to the GGR vertex $\Gamma_{GG}^{(+)}(t)$. The gluon trajectory in the one-loop approximation [3] $\omega^{(1)}(t)$ is presented by Eq.(4). The corrections $\Gamma_{GG}^{(\pm)}(t)$ depend on the definition of δ-symbols in Eq.(6). These symbols have a literal sense only in the physical case $D = 4$ and only for a suitable choice of relative phases in spin wave functions. It appears convenient to use the definitions:

$$\delta_{\lambda',\lambda} = -e_\lambda^\alpha(p_G)e^{*\alpha'}_{\lambda'}(p_{G'})g_{\alpha\alpha'}^{\perp\perp'},$$

$$\delta_{\lambda',-\lambda} = e_\lambda^\alpha(p_G)e^{*\alpha'}_{\lambda'}(p_{G'})\left(-g_{\alpha\alpha'}^{\perp\perp'} + (D-2)\frac{q_\alpha^\perp q_{\alpha'}^{\perp'}}{q^2}\right). \qquad (9)$$

Here $e_\lambda^\alpha(p_G), e_{\lambda'}^{\alpha'}(p_{G'})$ are polarization vectors of gluons G and G' with helicities λ and λ' correspondingly, $q = (p_G - p_{G'})$ is the momentum transfer $(q^2 = t)$ and $\perp$ and $\perp'$ mean orthogonal components to the (p_A, p_B) and $(p_{A'}, p_{B'})$ planes respectively:

$$g_{\alpha\alpha'}^{\perp\perp'} = P_{\alpha\mu}P'_{\alpha'\nu}g^{\mu\nu}, \quad q_\alpha^\perp = P_{\alpha\mu}q^\mu, \quad q_{\alpha'}^{\perp'} = P'_{\alpha'\nu}q^\nu,$$

$$P^{\mu\nu} = g^{\mu\nu} - \frac{p_A^\mu p_B^\nu + p_B^\mu p_A^\nu}{(p_A p_B)}, \quad P'^{\mu\nu} = g^{\mu\nu} - \frac{p_{A'}^\mu p_{B'}^\nu + p_{B'}^\mu p_{A'}^\nu}{(p_{A'} p_{B'})}. \qquad (10)$$

The tensors P and P' are the projectors on the subspaces orthogonal to the (p_A, p_B) and $(p_{A'}, p_{B'})$ planes. For $D \neq 4$ the definitions (9) differ from those used in Refs. [9],[11], causing a difference of our vertices $\Gamma_{GG}^{(\pm)}(t)$ in comparison with [9],[11]. Besides that, Refs. [9],[11] contain misprints in final expressions, therefore, we present here recalculated vertices $\Gamma_{GG}^{(\pm)}(t)$. They can be written as the sum of gluon and quark contributions:

$$\Gamma_{GG}^{(\pm)}(t) = \Gamma_{GG}^{(\pm)(gluon)}(t) + \Gamma_{GG}^{(\pm)(quark)}(t) \,. \qquad (11)$$

With the choice (9) the helicity conserving terms which we are interested in are given by

$$\Gamma_{GG}^{(+)(gluon)}(t) = \frac{g^2 N}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma\left(2-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}-1\right)}{(-t)^{2-\frac{D}{2}}\Gamma(D-2)}\left\{(D-3)\left[\psi\left(3-\frac{D}{2}\right)\right.\right.$$

$$\left.\left. -2\psi\left(\frac{D}{2}-2\right) + \psi(1)\right] - \frac{9(D-2)^2+8}{4(D-1)(D-2)}\right\}, \qquad (12)$$

where $\psi(x)$ is the logarithmic derivative of the gamma function:

$$\psi(x) = \frac{\Gamma'(x)}{\Gamma(x)}, \qquad (13)$$

and

$$\Gamma_{GG}^{(+)(quark)}(t) = \frac{g^2}{(4\pi)^{\frac{D}{2}}}\Gamma\left(2-\frac{D}{2}\right)\sum_f\left\{-2\int_0^1 \frac{dx\,x(1-x)}{[m_f^2 - x(1-x)t]^{2-\frac{D}{2}}}\right.$$

$$+\int_0^1\int_0^1 \frac{dx_1 dx_2\theta(1-x_1-x_2)}{\left[m_f^2 - tx_1x_2\right]^{3-\frac{D}{2}}}\left[(2-x_1-x_2)(m_f^2 + (3-D)tx_1x_2)\right.$$

$$\left.\left. +2\frac{D-4}{D-2}tx_1x_2(1-x_1-x_2)\right] - \frac{2m_f^{D-4}}{3}\right\}, \qquad (14)$$

where the summation is taken over the quark flavours, m_f is the mass of the quark flavour f. For the sake of completeness we present here the helicity non conserving terms also:

$$\Gamma_{GG}^{(-)(gluon)}(t) = \frac{2g^2 N}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma\left(3-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}-1\right)}{(-t)^{2-\frac{D}{2}}\Gamma(D)};$$

$$\Gamma_{GG}^{(-)(quark)}(t) =$$

$$\frac{4g^2}{(4\pi)^{\frac{D}{2}}}\frac{\Gamma\left(3-\frac{D}{2}\right)t}{D-2}\sum_f\int_0^1\int_0^1 \frac{dx_1dx_2\theta(1-x_1-x_2)}{\left[m_f^2 - tx_1x_2\right]^{3-\frac{D}{2}}}x_1x_2(1-x_1-x_2)\,. \qquad (15)$$

From Eq.(8) we have:

$$\omega^{(2)}(t) = \Delta_s - \left(\omega^{(1)}(t)\right)^2 \ln\left(\frac{s}{-t}\right) - 2\Gamma^{(+)}_{GG}(t)\omega^{(1)}(t). \qquad (16)$$

Consequently, we will know the gluon trajectory with two-loop accuracy after calculating of Δ_s . This discontinuity can be found from s -channel unitarity condition. In the approximation accepted here only two-gluon, three-gluon and quark-antiquark-gluon intermediate states can contribute to the unitarity relation, therefore

$$\Delta_s = \Delta_s^{(2g)} + \Delta_s^{(3g)} + \Delta_s^{(Q\bar{Q}g)}. \qquad (17)$$

Details of calculation of these three contributions to Δ_s are presented in the next three Sections.

3 Two-gluon intermediate state

The contribution of the two-gluon intermediate state to the discontinuity can be obtained from the relation

$$\left[\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'}\right]_s^{(2g)} = \frac{i}{2!}\int d\Phi_2(p_A + p_B; p_{G_1}, p_{G_2}) P_8^{(-)} \sum \mathcal{A}_{AB}^{G_1G_2} \mathcal{A}^{*G_1G_2}_{A'B'}, \qquad (18)$$

where the summation is performed over colour and spin states of intermediate gluons, $P_8^{(-)}$ is the projector on the colour octet state and negative signature in the t -channel, $d\Phi_2$ is the two-particle phase space element . For $n-$ particle state

$$d\Phi_n(P; p_1, ..., p_n) = (2\pi)^D \delta^{(D)}\left(P - \sum_{i=1}^n p_i\right) \prod_{i=1}^n \frac{d^{(D-1)}p_i}{2E_i(2\pi)^{D-1}}. \qquad (19)$$

The two-loop contribution to the discontinuity can be presented as

$$\left[\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'}\right]_s^{(2g)} \text{(two-loop)} =$$

$$2i \int_{(\theta_1 \le \frac{\pi}{2})} d\Phi_2 P_8^{(-)} \sum \left(\mathcal{A}^{*G_1G_2}_{A'B'}\right)^{(Born)} Re\left(\mathcal{A}_{AB}^{G_1G_2}\right)^{(one-loop)}. \qquad (20)$$

Due to intermediate gluon identity the integration in Eq.(20) is performed over only half of the phase space volume, where the angle θ_1 between momenta of gluons A and G_1 is smaller than $\pi/2$. Seeing that in essential region of the integration

$$|p_{G_1\perp}|, |p_{G_2\perp}| \sim |q_\perp|, \qquad (21)$$

we can take the amplitudes in the r.h.s. of Eq.(20) in Regge asymptotic form. Moreover, for the discontinuity $\left[\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'}\right]_s^{(2g)}$ without helicity change only helicity conserving parts of the amplitudes $\left(\mathcal{A}_{A'B'}^{G_1G_2}\right)^{(Born)}$ and $\left(\mathcal{A}_{AB}^{G_1G_2}\right)^{(one-loop)}$ with the octet colour state and negative signature in the corresponding t channels contribute in the unitary condition. The reason is that the Born amplitude is real and in the Regge region it conserves the helicities of each of scattered particles; besides that, it contains in this region only the colour octet state with negative signature in the t channel, as well as the real part of the one- loop amplitude. Consequently, we can use the representation (5) for them. It gives us the following equality:

$$P_8^{(-)} \sum \left(\mathcal{A}^{*G_1G_2}_{A'B'}\right)^{(Born)} Re\left(\mathcal{A}_{AB}^{G_1G_2}\right)^{(one-loop)}\Big|_{\lambda_{A'}=\lambda_A, \lambda_{B'}=\lambda_B} =$$

$$-g^4 N \langle A'|T^c|A\rangle\langle B'|T^c|B\rangle \frac{s^2}{t_1 t_1'} \times \left[\omega^{(1)}(t_1)\ln\left(\frac{s}{-t_1}\right) + 2\Gamma^{(+)}_{GG}(t_1)\right], \qquad (22)$$

where $t_1 = (p_{G_1} - p_A)^2$, $t_1' = (p_{G_1} - p_{A'})^2$. Taking into account, that with the required accuracy the phase space element can be written as

$$d\Phi_2(p_A + p_B; p_{G_1}, p_{G_2}) = \frac{1}{2s}\frac{d^{(D-2)}q_{1\perp}}{(2\pi)^{D-2}}, \qquad (23)$$

with $q_1 = p_{G_1} - p_A$, and expressing t_1, t_1' through the integration variables,

$$t_1 = q_{1\perp}^2, \quad t_1' = (q_1 - q)_\perp^2, \qquad (24)$$

we obtain the contribution of the two-gluon intermediate state to Δ_s in the following form:

$$\Delta_s^{(2g)} = \frac{g^2 N t}{(2\pi)^{D-1}} \int \frac{d^{(D-2)}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q})^2}\left[\omega^{(1)}(-\vec{q}_1^{\,2})\ln\left(\frac{s}{\vec{q}_1^{\,2}}\right) + 2\Gamma^{(+)}_{GG}(-\vec{q}_1^{\,2})\right]. \qquad (25)$$

Seeing that the transverse momenta are spacelike, we passed to integration over $(D-2)-$dimensional Euclidian vectors and omitted the sign of the transversality.

4 Three-gluon intermediate state

The contribution of the three-gluon intermediate state to the discontinuity $\left[\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'}\right]_s$ of the elastic gluon-gluon scattering amplitude is given by

$$\left[\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'}\right]_s^{(3g)} =$$

$$= \frac{i}{3!}\int d\Phi_3\,(p_A+p_B;p_{G_1},p_{G_2},p_{G_3})P_8^{(-)}\sum \mathcal{A}_{AB}^{G_1G_3G_2}\mathcal{A}^{*\,G_1G_3G_2}_{A'B'}, \qquad (26)$$

where the summation is performed over colour and spin states of the intermediate gluons. Since we are interested in the part of the amplitude conserving helicities of each of the colliding particles and take an average over these helicities, with account of the gluon identity we can throw away the factor $\frac{1}{3!}$ in the Eq.(26) and integrate over such part of the phase spase volume, where two of the intermediate gluons, for definiteness the gluons G_1 and G_3, have emission angles with respect to momentum of the gluon A, θ_1 and θ_3 correspondingly, smaller than $\pi/2$. The gluon G_2, consequently, has the emission angle larger than $\pi/2$, because of momentum conservation. Therefore, we have:

$$\left[\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'}\right]_s^{(3g)} = i\int_{(\theta_1,\theta_3\le\frac{\pi}{2})} d\Phi_3 P_8^{(-)}\sum \mathcal{A}_{AB}^{G_1G_3G_2}\mathcal{A}^{*\,G_1G_3G_2}_{A'B'}. \qquad (27)$$

In the integration region , which can give a contribution of the required order, intermediate gluon momenta are limited[12] :

$$|k_{i\perp}| \sim |q_\perp|. \qquad (28)$$

Here and below k_i is the momentum of an intermediate gluon G_i , $k_i \equiv p_{G_i}$.

The amplitude $\mathcal{A}_{AB}^{G_1G_3G_2}$ in the essential region of the integration was calculated in Ref.[12] and has the following gauge invariant form

$$\mathcal{A}_{AB}^{G_1G_3G_2} = 8g^3 s\delta_{\lambda_B,\lambda_{G_2}}\left[\mathcal{A}_1 T^{c_2}_{i_2B}T^{i_3}_{c_2c_1}T^{c_1}_{i_1A} + (1\leftrightarrow 3)\right], \qquad (29)$$

where $\mathcal{A}_1$ is defined by the convolution

$$\mathcal{A}_1 = a^{\mu\nu\rho}e_{A\mu}e^*_{1\nu}e^*_{3\rho}, \qquad (30)$$

in which e_A,e_1 and e_3 are polarization vectors of the gluons A, G_1 and G_3 correspondingly and

$$a^{\mu\nu\rho} = \frac{p_B^\mu}{s}\left[\frac{p_B^\nu p_B^\rho}{\beta_3 s^2} - \frac{p_B^\nu k_1^\rho - k_3^\nu p_B^\rho}{s_{13}s} + \frac{p_A^\nu p_B^\rho}{\beta_3 s t_1} - \frac{p_A^\nu k_1^\rho}{t_2}\left(\frac{1}{t_1}+\frac{1}{s_{13}}\right) + \right.$$
$$\left.\frac{k_3^\nu p_A^\rho}{t_2 s_{13}} + \frac{p_A^\nu p_A^\rho}{t_2 t_1}\right] + k_1^\mu\left[\frac{p_B^\nu p_B^\rho}{\beta_3 s^2 t_1} - \frac{p_B^\nu k_1^\rho - k_3^\nu p_B^\rho}{s t_2}\left(\frac{1}{t_1}+\frac{1}{s_{13}}\right) + \right.$$
$$\left.\frac{p_B^\nu p_A^\rho}{s t_2 t_1}\right] - k_3^\mu\left[\frac{p_B^\nu k_1^\rho - k_3^\nu p_B^\rho}{s_{13} s t_2} - \frac{p_A^\nu p_B^\rho}{s t_2 t_1}\right] - \frac{g^{\nu\rho}}{2}\left[\frac{(t_2\beta_3 - t_3)p_B}{s_{13} s t_2} + \right.$$
$$\left.\left(\frac{1}{t_1}+\frac{1}{s_{13}}\right)\frac{\beta_3 k_1}{t_2} - \frac{\beta_1 k_3}{t_2 s_{13}}\right]^\mu - \frac{g^{\mu\nu}}{2}\left[\left(\frac{t_2}{t_1\beta_3} - \frac{t_3}{t_1}\right)\frac{p_B}{s t_2} + \frac{\beta_1 p_A}{t_2 t_1} - \right.$$
$$\left.\left(\frac{1}{t_1}+\frac{1}{s_{13}}\right)\frac{k_1}{t_2}\right]^\rho - \frac{g^{\mu\rho}}{2}\left[-\frac{p_B}{s t_2} + \frac{\beta_3 p_A}{t_2 t_1} + \frac{k_3}{t_2 s_{13}}\right]^\nu. \qquad (31)$$

In the last formula we used the notations:

$$s = (p_A+p_B)^2,\ \ \beta_{1,3} = \frac{(k_{1,3}+p_B)^2}{s},\ \ t_{1,3} = (k_{1,3}-p_A)^2,$$

$$t_2 = (k_2 - p_B)^2,\ \ s_{13} = (k_1+k_3)^2, \qquad (32)$$

which are connected by relations

$$\beta_1 + \beta_3 = 1 + \frac{t_2}{s},\ \ s_{13} = t_2 - t_1 - t_3. \qquad (33)$$

The amplitude $\mathcal{A}_{A'B'}^{G_1G_3G_2}$ can be obtained from $\mathcal{A}_{AB}^{G_1G_3G_2}$ by evident substitution

$$A \to A',\ \ B \to B'. \qquad (34)$$

Using the form (29) one can easily perform in Eq. (27) the summation over helicities of the gluon G_2 as well as over colour states of all intermediate gluons, projecting the result on the octet colour state with negative signature in the t -channel. Taking $\lambda_B = \lambda_{B'}$ and averaging over λ_B we get:

$$\frac{1}{D-2}\sum_{\lambda_B}\left[\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'}\right]_s^{(3g)}\Bigg|_{\lambda_B=\lambda_{B'}} =$$

$$-\,iT^c_{A'A}T^c_{B'B}16g^6N^2s^2\int_{(\theta_1,\theta_3\le\frac{\pi}{2})} d\Phi_3\sum_{\lambda_1,\lambda_3}\mathcal{A}_1\mathcal{A}_1'^{\,*}, \qquad (35)$$

where $\mathcal{A}_1'$ can be obtained from $\mathcal{A}_1$ by the substitution (34). Since the amplitude $\mathcal{A}_1$ entering in Eq.(35) is gauge invariant we can choose the most convenient gauge for the gluons A, G_1 and G_3. We use the following gauge condition:

$$(e_i p_B) = 0, \quad i = A, 1, 3. \tag{36}$$

Together with the relation $(k_i e_i) = 0$ it gives that the polarization vectors can be presented as

$$e_i = -\frac{2(k_{i\perp} e_{i\perp})}{s\beta_i} p_B + e_{i\perp}, \tag{37}$$

where $k_A \equiv p_A, \beta_A \equiv 1$. It is easily to see that

$$\sum_{\lambda_i} e^{*\mu}_{i\perp} e^{\nu}_{i\perp} = -P^{\mu\nu}, \tag{38}$$

where the projector $P^{\mu\nu}$ is defined by Eqs.(10). Using the form (30), (31) for the amplitude $\mathcal{A}_1$, the representation (37) for the polarization vectors and Sudakov decomposition for the momenta of the gluons G_1, G_3,

$$k_i = \beta_i p_A + \alpha_i p_B + k_{i\perp}, \quad s\alpha_i\beta_i = -k_{i\perp}^2 \tag{39}$$

with account of their on-mass-shellness, we obtain with the required accuracy

$$\mathcal{A}_1 = -\frac{1}{2t_2}\Big[-\beta_1\beta_3(e^*_{1\perp}e^*_{3\perp})(R_\perp e_{A\perp}) + \beta_1(e_{A\perp}e^*_{1\perp})(R_\perp e^*_{3\perp}) + \beta_3(e_{A\perp}e^*_{3\perp})\times$$

$$\times (R_\perp e^*_{1\perp})\Big], \quad R_\perp = \left(-\frac{k_1}{t_1\beta_1} + \frac{(k_3 - \beta_3(k_1+k_3))}{\beta_1\beta_3 s_{13}}\right)_\perp. \tag{40}$$

The analogous expression for $\mathcal{A}_1'$ takes the form

$$\mathcal{A}_1' = -\frac{1}{2t_2'}\Big[-\beta_1\beta_3(e^*_{1\perp}e^*_{3\perp})(R'_\perp e_{A'\perp}) + \beta_1(e_{A'\perp}e^*_{1\perp})(R'_\perp e^*_{3\perp}) + \beta_3(e_{A'\perp}e^*_{3\perp})\times$$

$$\times (R'_\perp e^*_{1\perp})\Big], \quad R'_\perp = \left(\frac{((1-\beta_3)p_{A'} - k_1)}{t_1'\beta_1} + \frac{(k_3 - \beta_3(k_1+k_3))}{\beta_1\beta_3 s_{13}}\right)_\perp, \tag{41}$$

where the invariants t_1', t_2' are obtained from t_1, t_2 (32) by the substitution (34). Performing in the product $\mathcal{A}_1\mathcal{A}_1'^*$ the summation over helicities of the gluons G_1, G_3 with the help of (38) and averaging over $\lambda_A = \lambda_{A'}$ using the relation

$$\sum_\lambda e^{*\mu}_{\lambda\perp}(p_A) e^{\nu}_{\lambda\perp}(p_{A'}) = -P^{\mu\nu}, \tag{42}$$

which is valid in our approximation, we find

$$\frac{1}{D-2}\sum_{\lambda_1,\lambda_3,\lambda_A} \mathcal{A}_1\mathcal{A}_1'^*\Big|_{\lambda_{A'}=\lambda_A} = \frac{-1}{4t_2t_2'}\left((1-\beta_3)^2 + \beta_3^2 + (1-\beta_3)^2\beta_3^2\right)(R_\perp R'_\perp). \tag{43}$$

Putting (43) in (35), calculating the product $(R_\perp R'_\perp)$ and taking into account, that the phase space element in the essential region of integration can be written as

$$d\Phi_3 = d^{D-2}k_{1\perp} d^{D-2}k_{3\perp} \frac{d\beta_3}{\beta_1\beta_3}\frac{1}{4s(2\pi)^{(2D-3)}}, \tag{44}$$

with $\beta_1 = 1 - \beta_3$, we get for the three-gluon contribution to the discontinuity Δ_s the following expression:

$$\Delta_s^{(3g)} = \frac{g^4N^2t}{2(2\pi)^{2(D-1)}}\int d^{D-2}k_{1\perp} d^{D-2}k_{3\perp}\int\frac{d\beta_3}{\beta_3}\left((1-\beta_3)^3(1+\beta_3^2) + \right.$$

$$\left. +(1-\beta_3)\beta_3^2\right)\frac{1}{t_2t_2'}\left[\frac{t}{(\beta_1t_1)(\beta_1t_1')} + \frac{t_2}{(\beta_1t_1)(\beta_1\beta_3s_{13})} + \frac{t_2'}{(\beta_1t_1')(\beta_1\beta_3s_{13})}\right]. \tag{45}$$

The upper and lower limits of the integral over β_3 are defined by conditions $\theta_1 \le \frac{\pi}{2}$ and $\theta_3 \le \frac{\pi}{2}$ respectively. With the accuracy required the upper limit in can be put equal 1, and lower limit is equal $\sqrt{\vec{k}^2_{3\perp}/s}$.

In terms of the integration variables the invariants in Eq.(45) have the following form:

$$t_1 = \frac{k_{1\perp}^2}{\beta_1}, \quad t_1' = \frac{(k_1 - (1-\beta_3)q)_\perp^2}{\beta_1}, \quad t_2 = (k_1+k_3)_\perp^2,$$

$$t_2' = (k_1 + k_3 - q)_\perp^2, \quad s_{13} = -\frac{(k_3 - \beta_3(k_1+k_3))_\perp^2}{\beta_1\beta_3}. \tag{46}$$

Using this form, making a change of the variables

$$q_{1\perp} = k_{1\perp}, \quad q_{2\perp} = (k_1+k_3)_\perp \tag{47}$$

and throwing away the subscript of β_3, one can write Eq.(45) as

$$\Delta_s^{(3g)} = \frac{g^4N^2t}{2(2\pi)^{2(D-1)}}\int\frac{d^{D-2}q_{1\perp}d^{D-2}q_{2\perp}}{q_{2\perp}^2(q_2-q)_\perp^2}\int_{\beta_0}^{1}\frac{d\beta}{\beta}\left((1-\beta)^3(1+\beta^2) + \right.$$

$$+(1-\beta)\beta^2)\left(\frac{q_\perp^2}{q_{1\perp}^2(q_1-(1-\beta)q)_\perp^2}-\right.$$

$$\left.-\frac{q_{2\perp}^2}{q_{1\perp}^2(q_1-(1-\beta)q_2)_\perp^2}-\frac{(q_2-q)_\perp^2}{(q_1-(1-\beta)q)_\perp^2(q_1-(1-\beta)q_2)_\perp^2}\right), \quad (48)$$

where $\beta_0=\sqrt{-(q_2-q_1)_\perp^2/s}$. The integration over the Sudakov variable β in Eq.(48) can be performed in the following way. The integration region can be divided into two parts:

$$\int_{\beta_0}^{1}=\int_{\beta_0}^{\delta}+\int_{\delta}^{1}, \quad (49)$$

where

$$\beta_0<<\delta\ll 1. \quad (50)$$

Calculating the first integral one can neglect β where it is possible, and get the following expression for the corresponding contribution to $\Delta_s^{(3g)}$

$$\left(\Delta_s^{(3g)}\right)_1=\frac{g^4N^2t}{2(2\pi)^{2(D-1)}}\int\frac{d^{D-2}q_{1\perp}d^{D-2}q_{2\perp}}{q_{2\perp}^2(q_2-q)_\perp^2}\ln(\frac{\delta}{\beta_0})$$

$$\times\left(\frac{q_\perp^2}{q_{1\perp}^2(q_1-q)_\perp^2}-\frac{q_{2\perp}^2}{q_{1\perp}^2(q_1-q_2)_\perp^2}-\frac{(q_2-q)_\perp^2}{(q_1-q)_\perp^2(q_1-q_2)_\perp^2}\right). \quad (51)$$

To calculate a contribution of the second integral in the RHS of Eq.(49) to $\Delta_s^{(3g)}$ we change the order of integrations in Eq.(48) so that the integration over β becomes last. After that, by virtue of convergency of the integral over orthogonal momenta , we can make the substitution $q_1\to(1-\beta)q_1$ in this integral, leading to its factorization:

$$\left(\Delta_s^{(3g)}\right)_2=\frac{g^4N^2t}{2(2\pi)^{2(D-1)}}\int\frac{d^{D-2}q_{1\perp}d^{D-2}q_{2\perp}}{q_{2\perp}^2(q_2-q)_\perp^2}\times$$

$$\times\left(\frac{q_\perp^2}{q_{1\perp}^2(q_1-q)_\perp^2}-\frac{q_{2\perp}^2}{q_{1\perp}^2(q_1-q_2)_\perp^2}-\frac{(q_2-q)_\perp^2}{(q_1-q)_\perp^2(q_1-q_2)_\perp^2}\right)\times$$

$$\times\int_{\delta}^{1}\frac{d\beta}{\beta}(1-\beta)^{D-5}\left((1-\beta)^2(1+\beta^2)+\beta^2\right). \quad (52)$$

In the limit $\delta\to 0$ we have

$$\int_{\delta}^{1}\frac{d\beta}{\beta}(1-\beta)^{D-5}\left((1-\beta)^2(1+\beta^2)+\beta^2\right)=\ln\left(\frac{1}{\delta}\right)+\psi(1)-\psi(D-2)$$

$$+\frac{1}{(D-3)(D-4)}+\frac{1}{(D-1)(D-2)}. \quad (53)$$

For the total contribution of the three-gluon intermediate state to the discontinuity $\Delta_s^{(3g)}=\left(\Delta_s^{(3g)}\right)_1+\left(\Delta_s^{(3g)}\right)_2$ we obtain using Eqs. (51-53)

$$\Delta_s^{(3g)}=\frac{g^4N^2t}{2(2\pi)^{2(D-1)}}\int\frac{d^{D-2}q_1d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2-\vec{q})^2}\left(\frac{1}{2}\ln\left(\frac{s}{(\vec{q}_1-\vec{q}_2)^2}\right)+\psi(1)-\psi(D-2)\right.$$

$$\left.+\frac{1}{(D-3)(D-4)}+\frac{1}{(D-1)(D-2)}\right)\times$$

$$\times\left(\frac{-\vec{q}^{\,2}}{\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2}+\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}(\vec{q}_1-\vec{q}_2)^2}+\frac{(\vec{q}_2-\vec{q})^2}{(\vec{q}_1-\vec{q})^2(\vec{q}_1-\vec{q}_2)^2}\right), \quad (54)$$

where the integration over q_1 and q_2 is carried out in $(D-2)$ -dimensional Euclidian space.

5 Quark-antiquark-gluon intermediate state

The contribution of the $Q\bar{Q}g$ - intermediate state to the discontinuity $\left[\left(\mathcal{A}_8^{(-)}\right)_{AB}^{A'B'}\right]_s$ has the same form as one of the three-gluon intermediate state (26) with evident changes. The essential kinematical region consists of two separated parts. In both of them transverse momenta of produced particles are restricted:

$$|p_{Q\perp}|,\ |p_{\bar{Q}\perp}|\sim|q_\perp|, \quad (55)$$

where p_Q and $p_{\bar{Q}}$ are the momenta of the quark and antiquark correspondingly, but in the first part the produced quark-antiquark pair moves in the same direction as the gluon A (fragmentation region of the gluon A), while in the second - along B (fragmentation region of the gluon B). Both regions give equal contributions to Δ_s, therefore we will consider only the first of them and duplicate its contribution. The production amplitude in this region takes the following form:

$$\mathcal{A}_{AB}^{Q\bar{Q}G} = 2g^3 T_{i_2B}^d \delta_{\lambda_2,\lambda_B} \bar{u}(p_Q) \left[\frac{1}{t_2 s_{Q\bar{Q}}} \left(\hat{e}_A s - \hat{p}_B \left(2(p_Q + p_{\bar{Q}}) e_A \right) \right) T_{Ac}^d t^c - \right.$$
$$\left. - \frac{1}{t_2 t_Q} \hat{e}_A (\hat{p}_Q - \hat{p}_A + m) \hat{p}_B (t^A t^d) - \frac{1}{t_2 t_{\bar{Q}}} \hat{p}_B (\hat{p}_A - \hat{p}_{\bar{Q}} + m) \hat{e}_A (t^d t^A) \right] v(p_{\bar{Q}}), \tag{56}$$

where the i_2 and λ_2 are the colour index and the helicity of the intermediate gluon, t^i is the colour group generator in the quark representation, m is the quark mass and we use invariants:

$$t_Q = -2(p_Q p_A), \quad t_{\bar{Q}} = -2(p_{\bar{Q}} p_A), \quad s_{Q\bar{Q}} = (p_Q + p_{\bar{Q}})^2, t_2 = (k_2 - p_B)^2. \tag{57}$$

In the last expression k_2 is the momentum of the intermediate gluon.

Summing the product $\mathcal{A}_{AB}^{Q\bar{Q}G} \times \mathcal{A}^{*\,Q\bar{Q}G}_{A'B'}$ over polarizations of the intermediate particles and projecting on the octet colour state with negative signature in the t -channel, we obtain for the contribution of $Q\bar{Q}G-$ intermediate state to the helicity conserving part of the discontinuity

$$\left[\left(\mathcal{A}_8^{(-)} \right)_{AB}^{A'B'} \right]_s^{(Q\bar{Q}g)} \Bigg|_{\substack{\lambda_{A'}=\lambda_A \\ \lambda_{B'}=\lambda_B}} = T_{A'A}^c T_{B'B}^c g^2 \left(-2\pi i \frac{s}{t} \right) \frac{2g^4 N s t}{(D-2)} \int \frac{d\Phi_{(Q\bar{Q}g)}}{2\pi} \times$$
$$\times \frac{\beta_Q \beta_{\bar{Q}}}{q_{2\perp}^2 (q_2 - q)_\perp^2} \left((D-2) m^2 F - 2 \left(\frac{D-2}{2} - 2\beta_Q (1 - \beta_Q) \right) (S'_\perp S_\perp) \right). \tag{58}$$

In this formula we have used the following notations

$$F = \left(\frac{1}{(m^2 - (q_1 - (1-\beta_Q) q_2)_\perp^2)} - \frac{1}{(m^2 - q_{1\perp}^2)} \right) \times$$
$$\times \left(\frac{1}{(m^2 - (q_1 - (1-\beta_Q) q_2)_\perp^2)} - \frac{1}{(m^2 - (q_1 - (1-\beta_Q) q)_\perp^2)} \right);$$
$$S_\perp = \left(- \frac{q_{1\perp}}{(m^2 - q_{1\perp}^2)} + \frac{(q_1 - (1-\beta_Q) q_2)_\perp}{(m^2 - (q_1 - (1-\beta_Q) q_2)_\perp^2)} \right);$$
$$S'_\perp = \left(- \frac{(q_1 - (1-\beta_Q) q)_\perp}{(m^2 - (q_1 - (1-\beta_Q) q)_\perp^2)} + \frac{(q_1 - (1-\beta_Q) q_2)_\perp}{(m^2 - (q_1 - (1-\beta_Q) q_2)_\perp^2)} \right);$$
$$\beta_Q = \frac{2(p_B p_Q)}{s}, \quad \beta_{\bar{Q}} = \frac{2(p_B p_{\bar{Q}})}{s}, \quad q_1 = p_{\bar{Q}}, \quad q_2 = p_Q + p_{\bar{Q}}. \tag{59}$$

The phase space element in the gluon A fragmentation region has the form (44):

$$d\Phi_{(Q\bar{Q}g)} = \frac{2\pi}{4(2\pi)^{2(D-1)}} \frac{1}{s} d^{D-2} q_{1\perp} d^{D-2} q_{2\perp} \frac{d\beta_Q}{\beta_Q \beta_{\bar{Q}}}, \quad \beta_{\bar{Q}} = 1 - \beta_Q; \tag{60}$$

Seeing that regions of small β_Q and $\beta_{\bar{Q}}$ give negligible contributions in (58) (in contrast with the case of the three-gluon intermediate state, where regions of small β_1 and β_3 give contributions, logarithmical growing with s), we can put the upper and lower limits of the integration over β_Q in (58) equal to 1 and 0 respectively. Due to this circumstance $\Delta_s^{(Q\bar{Q}g)}$ doesn't contain a dependence on s at all and is a function of the variable $t = (p_{A'} - p_A)^2$ only.

Comparing Eq.(58) with account of (59)-(60) with Eq. (7) we can present the quark-antiquark-gluon intermediate state contribution to Δ_s as

$$\Delta_s^{(Q\bar{Q}g)} = - \frac{g^2 N t}{(2\pi)^{D-1}} \int \frac{d^{D-2} q_2}{\vec{q}_2^{\,2} (\vec{q}_2 - \vec{q})^2} \left(a^{(Q\bar{Q}g)}(\vec{q}^{\,2}) - 2 a^{(Q\bar{Q}g)}(\vec{q}_2^{\,2}) \right), \tag{61}$$

where

$$a^{(Q\bar{Q}g)}(\vec{q}^{\,2}) = \frac{g^2}{2(2\pi)^{D-1}(D-2)} \sum_f \left[\int_0^1 d\beta \left(\vec{q}^{\,2} \beta^2 \left(\frac{D-2}{2} - 2\beta(1-\beta) \right) - \right. \right.$$
$$\left. \left. -4m_f^2 \beta(1-\beta) \right) \int \frac{d^{D-2} q_1}{\left(\vec{q}_1^{\,2} + m_f^2 \right) \left((\vec{q}_1 - \beta \vec{q})^2 + m_f^2 \right)} + \frac{2m_f^2}{3} \int \frac{d^{D-2} q_1}{\left(\vec{q}_1^{\,2} + m_f^2 \right)^2} \right]. \tag{62}$$

The summation here is performed over quark flavours.

After integration over $\vec{q}_1$ the expression for $a^{(Q\bar{Q}g)}(\vec{q}^{\,2})$ takes the form

$$a^{(Q\bar{Q}g)}(\vec{q}^{\,2}) = - \frac{g^2 \Gamma\left(2 - \frac{D}{2}\right)}{2(4\pi)^{\frac{D}{2}}} \sum_f \left[\frac{2}{3} \left(\frac{D-4}{D-2} \right) (m_f^2)^{\frac{D}{2}-2} + \right.$$
$$+ \int_0^1 \int_0^1 \frac{dx_1 dx_2 \theta(1 - x_1 - x_2)}{\left(m_f^2 + \vec{q}^{\,2} x_1 x_2 \right)^{2-\frac{D}{2}}} \left(\frac{D-4}{D-2} \right) \times$$
$$\left. \times \frac{\vec{q}^{\,2}(x_1 + x_2)\left(\frac{D-2}{2} - 2(x_1 + x_2)(1 - x_1 - x_2) \right) - 4m_f^2 (1 - x_1 - x_2)}{\left(m_f^2 + \vec{q}^{\,2} x_1 x_2 \right)} \right]. \tag{63}$$

6 Two-loop correction to the gluon trajectory

With help of Eqs.(17) and (25) the two-loop contribution to the gluon trajectory (16) can be written as

$$\omega^{(2)}(t) = \Delta_s{}^{(3g)} + \Delta_s{}^{(Q\bar{Q}g)} - \left(\omega^{(1)}(t)\right)^2 \ln\left(\frac{s}{-t}\right) - 2\Gamma^{(+)}_{GG}(t)\omega^{(1)}(t)$$

$$+ \frac{g^2 N t}{(2\pi)^{D-1}} \int \frac{d^{(D-2)}q_1}{\vec{q}_1^{\,2}(\vec{q}_1 - \vec{q})^2} \left[\omega^{(1)}(-\vec{q}_1^{\,2}) \ln\left(\frac{s}{\vec{q}_1^{\,2}}\right) + 2\Gamma^{(+)}_{GG}(-\vec{q}_1^{\,2})\right]. \qquad (64)$$

Here the discontinuities $\Delta_s{}^{(3g)}$ and $\Delta_s{}^{(Q\bar{Q}g)}$are given by Eqs. (54) and (61)-(63) correspondingly, one–loop contributions to the trajectory $\omega^{(1)}$ and to the GGR-vertex $\Gamma^{(+)}_{GG}$ - respectively by Eqs. (4) and (11)-(14). So, the expression looks rather complicated; but there are a lot of remarkable cancellations between various terms in it. Let us demonstrate these cancellations.

Firstly, let us consider the contribution to the r.h.s. of Eq. (64), connected with quarks. It is given by the discontinuity $\Delta_s{}^{(Q\bar{Q}g)}$ (61)-(63) and the terms containing the part $\Gamma^{(+)(quark)}_{GG}$ (14) of the vertex $\Gamma^{(+)}_{GG}$ (11). Using the representation (4) for $\omega^{(1)}$, we can present this contribution in the following form:

$$\omega^{(Q)}(t)^{(two-loop)} = -\frac{N g^2 t}{(2\pi)^{D-1}} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}(\vec{q}_2 - \vec{q})^2} \left[f^{(Q)}(\vec{q}^{\,2}) - 2f^{(Q)}(\vec{q}_2^{\,2})\right], \qquad (65)$$

where

$$f^{(Q)}(\vec{q}^{\,2}) = a^{(Q\bar{Q}g)}(\vec{q}^{\,2}) + \Gamma^{(+)(quark)}_{GG}(-\vec{q}^{\,2}). \qquad (66)$$

It appears, that the sum of the term $a^{(Q\bar{Q}g)}(\vec{q}^{\,2})$ (63) and such terms in $\Gamma^{(+)(quark)}_{GG}(-\vec{q}^{\,2})$, which are presented in Eq.(14) by the double integral and by the term without integration, is zero. Indeed, let us present this sum as $\frac{g^2}{2(4\pi)^{D/2}}B(\vec{q}^{\,2})$; then

$$B(\vec{q}^{\,2}) = -4\Gamma\left(2 - \frac{D}{2}\right) \sum_f \left[\frac{(3D-8)}{6(D-2)} (m_f^2)^{\frac{D}{2}-2} + \int_0^1\int_0^1 \frac{dx_1 dx_2 \theta(1-x_1-x_2)}{\left(m_f^2 + \vec{q}^{\,2}x_1x_2\right)^{2-\frac{D}{2}}} \times\right.$$

$$\times \left((3-D)\left(\frac{2-x_1-x_2}{2}\right) + \left(\frac{D-4}{D-2}\right)\left(\frac{x_1+x_2}{2}\right) + \left(\frac{D-4}{D-2}\right) \times\right.$$

$$\left.\left.\times \frac{\vec{q}^{\,2}\left(\frac{(D-2)(x_1+x_2)}{8} - \frac{x_1^2(1-x_1)}{2} - \frac{x_2^2(1-x_2)}{2}\right) + \frac{(D-3)m_f^2(2-x_1-x_2)}{2}}{\left(m_f^2 + \vec{q}^{\,2}x_1x_2\right)}\right)\right]. \qquad (67)$$

Due to symmetry of the integration region of the double integral in the RHS of the Eq.(67) with respect to exchange $x_1 \leftrightarrow x_2$ we can rewrite the expression for $B(\vec{q}^{\,2})$:

$$B(\vec{q}^{\,2}) = -4\frac{\Gamma\left(2-\frac{D}{2}\right)}{(D-2)} \sum_f \left[\left(\frac{D}{2} - \frac{4}{3}\right)(m_f^2)^{\frac{D}{2}-2} + \int_0^1\int_0^1 dx_1 dx_2 \theta(1-x_1-x_2) \times\right.$$

$$\times \left(\frac{(D-4)x_2}{\left(m_f^2 + \vec{q}^{\,2}x_1x_2\right)^{2-\frac{D}{2}}} + \frac{\left(\frac{D}{2}-1\right)(6-2D)(1-x_1)}{\left(m_f^2 + \vec{q}^{\,2}x_1x_2\right)^{2-\frac{D}{2}}} + \right.$$

$$\left.\left.\frac{\left(\frac{D}{2}-2\right)\vec{q}^{\,2}x_1\left(\frac{D-2}{2} - 2x_1(1-x_1)\right) - m_f^2\left(\frac{D}{2}-2\right)(6-2D)(1-x_1)}{\left(m_f^2 + \vec{q}^{\,2}x_1x_2\right)^{3-\frac{D}{2}}}\right)\right]. \qquad (68)$$

Using the identity

$$\frac{(D-4)x_2}{\left(m_f^2 + \vec{q}^{\,2}x_1x_2\right)^{2-\frac{D}{2}}} = \frac{\partial}{\partial x_2} \frac{(D-4)x_2^2}{\left(m_f^2 + \vec{q}^{\,2}x_1x_2\right)^{2-\frac{D}{2}}} - \frac{\left(\frac{D}{2}-1\right)(D-4)x_2}{\left(m_f^2 + \vec{q}^{\,2}x_1x_2\right)^{2-\frac{D}{2}}} +$$

$$\frac{m_f^2\left(\frac{D}{2}-2\right)(D-4)x_2}{\left(m_f^2 + \vec{q}^{\,2}x_1x_2\right)^{3-\frac{D}{2}}}, \qquad (69)$$

for the first term in the double integral in (68) and replacing x_2 by x_1 in the numerators of the last two terms of the r.h.s. of this identity due to the symmetry of the integral discussed above, we can present $B(\vec{q}^{\,2})$ in the following form:

$$B(\vec{q}^{\,2}) = -4\frac{\Gamma\left(2-\frac{D}{2}\right)}{(D-2)} \sum_f \left[\left(\frac{D}{2} - \frac{4}{3}\right)(m_f^2)^{\frac{D}{2}-2} + \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \times\right.$$

$$\left.\times \frac{\partial}{\partial x_2} \frac{((6-2D)(1-x_1) + (D-4)(x_2-x_1))\,x_2 - 2x_1(1-x_1) + \frac{D-2}{2}}{\left(m_f^2 + \vec{q}^{\,2}x_1x_2\right)^{2-\frac{D}{2}}}\right]. \qquad (70)$$

Now the integration is performed easily, leading to $B(\vec{q}^2)=0$. Consequently, only the first term in $\Gamma_{GG}^{(+)(quark)}(-\vec{q}^2)$ (14) does contribute to $f^{(Q)}(\vec{q}^2)$ (66), and therefore for the quark contribution to $\omega^{(2)}(t)$ we obtain from (65):

$$\omega^{(Q)}(t)^{(two-loop)} = \frac{g^4 t}{4}\int \frac{d^{D-2}q_1}{(2\pi)^{D-1}}\frac{1}{\vec{q}_1^2(\vec{q}_1-\vec{q})^2}\frac{8\Gamma\left(2-\frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}}}\times$$

$$\times N\sum_f \int_0^1 dx x(1-x)\left[\frac{1}{\left(m_f^2+\vec{q}^2x(1-x)\right)^{2-\frac{D}{2}}}-\frac{2}{\left(m_f^2+\vec{q}_1^2x(1-x)\right)^{2-\frac{D}{2}}}\right]. \tag{71}$$

The remaining contributions to $\omega^{(2)}(t)$ (64) are pure gluon ones. Evidently, there must be cancellations between them, because a dependence on s must vanish in (64). Using the representation (4) for $\omega^{(1)}$, we can perform the cancellations explicitly and obtain:

$$\omega^{(g)}(t)^{(two-loop)} = \frac{g^4N^2t}{4(2\pi)^{2(D-1)}}\int\frac{d^{D-2}q_1d^{D-2}q_2}{\vec{q}_1^2\vec{q}_2^2}\times$$

$$\times\left[\frac{-\vec{q}^2}{(\vec{q}_1-\vec{q})^2(\vec{q}_2-\vec{q})^2}\ln\left(\frac{\vec{q}^2}{(\vec{q}_1-\vec{q}_2)^2}\right)+\frac{2}{(\vec{q}_1+\vec{q}_2-\vec{q})^2}\ln\left(\frac{(\vec{q}_1-\vec{q})^2}{\vec{q}_1^2}\right)+\right.$$

$$+\left(\frac{-\vec{q}^2}{(\vec{q}_1-\vec{q})^2(\vec{q}_2-\vec{q})^2}+\frac{2}{(\vec{q}_1+\vec{q}_2-\vec{q})^2}\right)\left(\psi(1)-2\psi(D-3)-\right.$$

$$\left.\left.-\psi\left(3-\frac{D}{2}\right)+2\psi\left(\frac{D}{2}-2\right)+\frac{2}{(D-3)(D-4)}+\frac{(D-2)}{4(D-1)(D-3)}\right)\right]. \tag{72}$$

Remind, that $\psi(x)$ is the logarithmic derivative of the Gamma-function.

The total correction to the gluon Regge trajectory is given by sum of (72) and (71).

7 Summary

We have obtained the two-loop correction $\omega^{(2)}(t)$ to the trajectory $\omega(t)$ of the Reggeized gluon in QCD. It is given by the sum of quark $\omega^{(Q)}(t)^{(two-loop)}$ (71) and gluon $\omega^{(g)}(t)^{(two-loop)}$ (72) contributions. The correction was calculated assuming that an asymptotic behaviour of the gluon-gluon scattering amplitude in the kinematical region of asymptotically large energies $\sqrt{s}$ and restricted momentum transfer $\sqrt{-t}$ is given by the Reggeized gluon contribution (5). In this case the correction can be presented by Eq.(16) in terms of the helicity conserving part of s-channel discontinuity Δ_S of the amplitude with colour octet state and negative signature in the t channel, the leading contribution $\omega^{(1)}(t)$ to the trajectory (4) and the one-loop correction to the helicity conserving part of the gluon-gluon-Reggeon vertex $\Gamma_{GG}^{(+)}(t)$ (11)-(14). We have calculated the discontinuity Δ_S using $s-$channel unitarity condition. In the two-loop approximation it can be written (17) as sum of three contributions: $\Delta_S^{(2g)}$ (25), $\Delta_S^{(3g)}$ (54) and $\Delta_S^{(QQg)}$ (61),(63), which come correspondingly from two-gluon, three-gluon and quark-antiquark-gluon intermediate states in the unitary condition. The final results (71) ,(72) are obtained after series of remarkable cancellations.

By definition, Regge trajectory has to be process- independent. Therefore, the correction $\omega^{(2)}(t)$ can not depend on properties of colliding particles and can be obtained from an amplitude of any process, if the asymptotic behaviour of this amplitude is given by the Reggeized gluon contribution. Recently [13] the two-loop correction to the gluon trajectory was calculated using the quark-quark scattering amplitude. Our result for the trajectory coincides with the result of Ref.[13]. The coincidences of these results gives us a strong confirmation of the gluon Reggeization in QCD beyond the leading logarithmic approximation .

We used dimensional regularization, keeping the space-time dimension $D\neq 4$. In the physical case $D=4$ the two-loop correction to the gluon trajectory contains ultraviolet as well as infrared divergences. The first of them are trivial. To eliminate them, it is enough to express the trajectory

$$\omega(t)=\omega^{(1)}(t)+\omega^{(2)}(t)+\cdots. \tag{73}$$

in terms of the renormalized coupling g_μ instead of the bare coupling constant g . For example, in the $\overline{MS}$ -scheme one has

$$g=g_\mu\mu^{2-\frac{D}{2}}\left\{1+\left(\frac{11}{3}N-\frac{2}{3}n_f\right)\frac{g_\mu^2}{(4\pi)^2}\left[\frac{1}{D-4}-\frac{1}{2}\ln(4\pi)-\frac{1}{2}\psi(1)\right]+\cdots\right\}, \tag{74}$$

where g_μ is the renormalized coupling constant at the renormalization point μ.

Contrary, the infrared divergences are inherent for the gluon trajectory (let us remind, that they are present already in the leading contribution $\omega^{(1)}(t)$ to the trajectory (4)). The reason is that a gluon is a colour object, whereas we can expect the infrared divergency cancellation for the scattering of colourless objects only. The cancellation should occur after substitution of the trajectory in the equation for the t -channel partial amplitude with vacuum quantum numbers. We hope to deal with this problem in subsequent papers.

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