Emission of Two Hard Photons in Large Angle Bhabha Scattering

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Abstract

The closed expressions for the differential cross section of the large angle Bhabha e^+e^- scattering which explicitly takes into account the leading and next to leading contributions due to the emission of two hard photons is presented. Both collinear and semi–collinear kinematical regions are considered.

1 Introduction

The large angle Bhabha process is well suited for the determination of the luminosity $\mathcal{L}$ at e^+e^- colliders of the intermediate energy range $\sqrt{s} = 2\varepsilon \sim$ 1GeV [1,2]. As far as 0.1% accuracy is needed in the determination of $\mathcal{L}$ the corresponding requirement

$$\left|\frac{\Delta\sigma}{\sigma}\right| \leq 10^{-3} \qquad (1)$$

on the Bhabha cross section theoretical description appears. $\Delta\sigma$ is the unknown uncertainty in the cross section due to higher order radiative corrections. A great attention was paid to this process during the last decades [3]. The Born cross section with the weak interactions taken into account as well as the radiative corrections to it, including the emission of a single virtual photon, soft and hard real one, where studied in details [4]. Both contributions, the one reinforced by "the large logarithmic multiplier" $L = \ln s/m^2$ (where $s = (p_+ + p_-)^2 = 4\varepsilon^2$ is the square of total centre–of–mass (CM) energy, m is the electron mass), and the one without L are to be kept in frames (1): $\alpha L/\pi$, α/π. As for the corrections in the second order of the perturbation theory, they are necessary in the leading and the next to leading approximations and take the following orders respectively:

$$(\frac{\alpha}{\pi})^2 L^2, \quad (\frac{\alpha}{\pi})^2 L. \qquad (2)$$

The total two-loop ($\sim (\alpha/\pi)^2$) correction may be constructed from: 1) the two-loop corrections arising from emission of two virtual photons; 2) the one-loop corrections to the single real (soft and hard) photon emission; 3) the ones arising from cross sections of emission of two real photons; 4) the virtual and real e^+e^- pair production. As for the corrections in third order of the perturbation theory, only the leading ones proportional to $(\alpha L/\pi)^3$ are to be taken into account. Their calculation can be performed by means of the electron structure functions method [4].

In this paper we consider the emission of two real hard photons:

$$e^+(p_+) + e^-(p_-) \to e^+(q_+) + e^-(q_-) + \gamma(k_1) + \gamma(k_2). \qquad (3)$$

The relevant contribution to the "experimental" cross section has the following form

$$\sigma_{exp} = \frac{1}{2}\int d\sigma \Theta_+ \Theta_-, \qquad (4)$$

where Θ_+ and Θ_- are the experimental restrictions providing the simultaneous detection of both scattered electron and positron. At first that means that their energy fractions should be larger then a certain (small) quantity $\varepsilon_{th}/\varepsilon$, ε_{th} is the energy threshold of the detectors. The second condition restricts their angles in respect to the beam axes, they should be larger then a certain finite value θ_{min}:

$$\pi - \theta_{min} > \theta_{e^+}, \theta_{e^+} > \theta_{min}, \qquad \theta_{e^-} = \widehat{q_- p_-}, \qquad \theta_{e^+} = \widehat{q_+ p_-}, \qquad (5)$$

where $\theta_{e^+}, \theta_{e^+}$ are the polar angles of the scattered leptons in respect to the beam axes (p_-). The main ($\sim (\alpha L/\pi)^2$) contribution to the total cross section (5) arises from the collinear region: when both emitted photons fly within narrow cones along the charged particle momenta (they may go along the same particle). So we will distinguish 16 kinematical regions:

$$\widehat{ak_1} \text{ and } \widehat{ak_2} < \theta_0, \qquad \widehat{ak_1} \text{ and } \widehat{bk_2} < \theta_0, \qquad (6)$$
$$\frac{m}{\varepsilon} \ll \theta_0 \ll 1, \qquad a \neq b, \quad a, b = p_-, p_+, q_-, q_+ .$$

The squared matrix element modules summed over the spin states in the regions (6) have the form of the Born ones multiplied by the so called collinear factors. The contribution to the cross section of the each region has also the form of $2 \to 2$ Bhabha cross sections in the Born approximation multiplied by factors of the form

$$d\sigma_i^{coll} = d\sigma_{0i}\left[a_i(x_j, y_j)\ln^2\left(\frac{\varepsilon^2\theta_0^2}{m^2}\right) + b_i(x_j, y_j)\ln\left(\frac{\varepsilon^2\theta_0^2}{m^2}\right)\right], \qquad (7)$$

where $x_j = \omega_j/\varepsilon$, $y_1 = q_-^0/\varepsilon$, $y_2 = q_+^0/\varepsilon$ are the energy fractions of the photons and the scattered electron and positron. The dependence on the auxiliary parameter θ_0 will cancel in the sum of the contributions of the collinear and semi-collinear regions. The last region corresponds to the kinematics, when only one of the photons is emitted inside the narrow cone $\theta_1 < \theta_0$ along one of the charged particle momenta and the second photon is emitted outside of any such a cone along charged particles ($\theta_2 > \theta_0$):

$$d\sigma_i^{sc} = \frac{\alpha}{\pi}\ln\left(\frac{4\varepsilon^2}{m^2}\right) d\sigma_{0i}^{\gamma}(k_2), \qquad (8)$$

where $d\sigma_{0i}^{\gamma}$ has the known form of the single hard bremsstrahlung cross section in the Born approximation [5].

We show below explicitly that the result of the integration over the single hard photon emission in eq. (8) in the kinematical region $\theta_2^i > \theta_0$ (θ_2^i is the emission angle of the second hard photon in respect to the direction of one of the four charged particles) has the following form

$$\int d\sigma_{0i}^{\gamma}(k_2) = -2\ln(\theta_0^2)a_i(x, y)d\sigma_0^i + d\tilde{\sigma}^i. \qquad (9)$$

The collinear factors in the double bremsstrahlung process were firstly considered in papers of the CALKUL collaboration [6]. Unfortunately they have rather complicate form, which is less convenient for further analytical integration in comparison with the expressions given below. Calculation of the collinear factors may be considered as a generalization of the quasi-real electron method [7] for the case of multiple bremsstrahlung. Another generalization is needed for the calculations of the cross section of the process $e^+e^- \to e^+e^-e^+e^-$. We will consider it in a separate paper.

It is interesting to note that the collinear factors for the kinematical region of the two hard photons emission along the projectile and the scattered electron are found to be the same as for the electron–proton scattering process considered in paper [8].

There are 40 Feynman diagrams which describe the double bremsstrahlung process in e^+e^- collisions. The expression for the differential cross section in terms of helicity amplitudes was computed about ten years ago [6,9]. It has a very complicated form. We note that the contribution from the kinematical region in which the angles (in the CM system) between any two final particles are large compared with m/ε has the magnitude of the order

$$\frac{\alpha^2 r_0^2 m^2}{\pi^2 \varepsilon^2} \sim 10^{-36}\text{cm}^2, \qquad (10)$$

(r_0 is the classical electron radius). So, the corresponding events will have poor statistics at the colliders with the luminosity $\mathcal{L} \sim 10^{31} - 10^{32}\mathrm{cm}^{-2}\mathrm{s}^{-1}$. More probable are the cases of double bremsstrahlug imitating the processes $e^+e^- \to e^+e^-$ or $e^+e^- \to e^+e^-\gamma$. That corresponds to the emission of one or two photons along charged particles momenta.

2 Kinematics in the collinear region

It is convenient to introduce in the collinear region new variables and transform the phase volume of the final state in the following way (here and further we would work in the CM system)

$$\int \mathrm{d}\Gamma = \int \frac{\mathrm{d}^3q_-\mathrm{d}^3q_+\mathrm{d}^3k_1\mathrm{d}^3k_2}{16\varepsilon_-\varepsilon_+\omega_1\omega_2(2\pi)^8}\delta^4(\eta_1p_- + \eta_2p_+ - \lambda_1q_- - \lambda_2q_+)$$

$$= \frac{m^4\pi^2}{4(2\pi)^6}\int\limits_\Delta^1 \mathrm{d}x_1 \int\limits_\Delta^1 \mathrm{d}x_2x_1x_2 \int\limits_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}\int\limits_0^{z_0}\mathrm{d}z_1\int\limits_0^{z_0}\mathrm{d}z_2\int \mathrm{d}\Gamma_q, \qquad (11)$$

$$\int \mathrm{d}\Gamma_q = \int \frac{\mathrm{d}^3q_-\mathrm{d}^3q_+}{4\varepsilon_-\varepsilon_+(2\pi)^2}\delta^4(\eta_1p_- + \eta_2p_+ - \lambda_1q_- - \lambda_2q_+),$$

$$z_{1,2} = \left(\frac{\theta_{1,2}\varepsilon}{m}\right)^2, \qquad \phi = \widehat{k_{1\perp}k_{2\perp}}, \qquad x_i = \frac{\omega_i}{\varepsilon}, \qquad z_0 \gg 1, \qquad \Delta = \frac{\varepsilon_{th}}{\varepsilon}$$

where ε_{th} is the detector threshold energy resolution; θ_i $(i = 1, 2)$ is the polar angle of the photon emission in respect to the momentum of the charged particle which emitted the photon; $\eta_\pm$, $\lambda_\pm$ depend on the specific emission kinematics, they are given in Table 1.

Table 1. η_i and λ_i for the different collinear kinematics.

	p_-p_-	q_-q_-	p_+p_+	q_+q_+	p_-p_+	q_-q_+	p_-q_-	p_+q_+	p_-q_+	p_+q_-
η_1	y	1	1	1	$1-x_1$	1	$1-x_1$	1	$1-x_1$	1
η_2	1	1	y	1	$1-x_2$	1	1	$1-x_1$	1	$1-x_1$
λ_1	1	$\frac{1}{y}$	1	1	1	$\frac{1}{1-x_1}$	$1+\frac{x_2}{y_1}$	1	1	$1+\frac{x_2}{y_1}$
λ_2	1	1	1	$\frac{1}{y}$	1	$\frac{1}{1-x_2}$	1	$1+\frac{x_2}{y_2}$	$1+\frac{x_2}{y_1}$	1

The columns of the table correspond to a certain choice of the kinematics i the following way: p_-p_- means the emission of both photons along the projectile electron, p_+q_- means that the first of the photons goes along the projectile positron and the second — along the scattered electron, etc. The

contributions from 6 remaining kinematical regions (when the photons in the last 6 columns are interchanged) could be found by the simple substitution $x_1 \leftrightarrow x_2$. We will use the momentum conservation law

$$\eta_1p_- + \eta_2p_+ = \lambda_1q_- + \lambda_1q_-\,, \qquad (12)$$

and the following relations coming from it

$$\eta_1 + \eta_2 = \lambda_1y_1 + \lambda_2y_2, \qquad \lambda_1y_1\sin\theta_- = \lambda_2y_2\sin\theta_+, \qquad (13)$$
$$\eta_1 - \eta_2 = \lambda_1y_1\cos\theta_- + \lambda_2y_2\cos\theta_+, \quad \theta_- = \widehat{q_-p_-},$$
$$\theta_+ = \widehat{q_+p_-}, \quad y_{1,2} = \frac{q^0_{1,2}}{\varepsilon}.$$

One can find from eq. (12) (taking η_i, λ_i, $c = \cos\theta_-$ as the known quantities) that

$$\sin\theta_+ = \sin\theta_-\frac{2\eta_1\eta_2}{\eta_1^2 + \eta_2^2 + (\eta_2^2 - \eta_1^2)c}, \qquad (14)$$
$$\lambda_1y_1 = \frac{2\eta_1\eta_2}{\eta_1 + \eta_2 + (\eta_2 - \eta_1)c}, \qquad \lambda_2y_2 = \frac{\eta_1^2 + \eta_2^2 + (\eta_2^2 - \eta_1^2)c}{\eta_1 + \eta_2 + (\eta_2 - \eta_1)c}.$$

The invariant cross section we put in the form

$$\mathrm{d}\sigma_0(\eta\lambda) = \frac{2\pi\alpha^2}{s^2}\frac{s^4 + t^4 + u^4}{s^2t^2}\mathrm{d}t = \frac{4\pi\alpha^2}{s}\left(\frac{s^2 + t^2 + st}{st}\right)\frac{\mathrm{d}t}{s}, \qquad (15)$$
$$s = 4\varepsilon^2\eta_1\eta_2, \quad s + t + u = 0, \quad t = -s\frac{\eta_1(1 - c)}{\eta_1 + \eta_2 + (\eta_2 - \eta_1)c}.$$

So, we can express the invariant cross section in terms of the electron scattering angle θ_-:

$$\frac{\mathrm{d}\sigma_0(\eta\lambda)}{\mathrm{d}c} = \frac{2\pi\alpha^2}{\varepsilon^2}\frac{\eta_1^2 + \eta_2^2 + 2c(\eta_2^2 - \eta_1^2 + (\eta_1^2 + \eta_2^2 - \eta_1\eta_2)c^2}{[\eta_1 + \eta_2 + (\eta_2 - \eta_1)c]^4\eta_1^2(1 - c)^2}\,. \qquad (16)$$

Each contribution from 16 ones to the cross section of process (3) can be expressed in terms of the corresponding Born cross section of type (16) multiplied by its collinear factor:

$$\mathrm{d}\sigma^{\gamma\gamma} = \Sigma\mathrm{d}K(\eta\lambda)\mathrm{d}\sigma_0(\eta\lambda)\frac{\eta_1\eta_2}{\lambda_1^2\lambda_2^2}, \qquad (17)$$

$$\mathrm{d}K(\eta\lambda) = \frac{1}{16}\left(\frac{\alpha}{\pi}\right)^2\mathrm{d}x_1\mathrm{d}x_2x_1x_2\frac{1}{2!}\int\limits_0^{z_0}\mathrm{d}^2z\int\limits_0^{2\pi}\frac{\mathrm{d}\phi}{2\pi}m^4K(\eta\lambda),$$

$$\mathcal{K}(p_-p_-) = \frac{2}{y}\mathcal{A}(A_1, A_2, A, x_1, x_2),\ \mathcal{K}(p_+p_+) = \frac{2}{y}\mathcal{A}(C_1, C_2, C, x_1, x_2) \quad (18)$$

$$\mathcal{K}(q_-q_-) = 2y\mathcal{A}(B_1, B_2, B, \frac{-x_1}{y}, \frac{-x_2}{y}),$$

$$\mathcal{K}(q_+q_+) = 2y\mathcal{A}(D_1, D_2, D, \frac{-x_1}{y}, \frac{-x_2}{y}),$$

$$\mathcal{A}(A_1, A_2, A, x_1, x_2) = -\frac{yA_2}{A^2A_1} - \frac{yA_1}{A^2A_2}\frac{1+y^2}{x_1x_2A_1A_2} + \frac{(1-x_1)^3 + y(1-x_2)}{AA_1x_1x_2}$$
$$+\frac{(1-x_2)^3 + y(1-x_1)}{AA_2x_1x_2} + \frac{2m^2(y^2+(1-x_1)^2)}{AA_1^2x_2} + \frac{2m^2(y^2+(1-x_2)^2)}{AA_2^2x_1},$$

$$\mathcal{K}(p_-p_+) = 2\left[\frac{1+(1-x_1)^2}{A_1x_1(1-x_1)} + \frac{2m^2}{A_1^2}\right]\left[\frac{1+(1-x_2)^2}{C_2x_2(1-x_2)} + \frac{2m^2}{C_2^2}\right], \quad (19)$$

$$\mathcal{K}(q_-q_+) = 2\left[\frac{1+(1-x_1)^2}{B_1x_1(1-x_1)} - \frac{2m^2}{B_1^2}\right]\left[\frac{1+(1-x_2)^2}{D_2x_2} - \frac{2m^2}{D_2^2}\right],$$

$$\mathcal{K}(p_-q_+) = -2\left[\frac{1+(1-x_1)^2}{A_1x_1(1-x_1)} + \frac{2m^2}{A_1^2}\right]\left[\frac{y_2^2+(y_2+x_2)^2}{D_2x_2(y_2+x_2)} - \frac{2m^2}{D_2^2}\right],$$

$$\mathcal{K}(p_+q_-) = -2\left[\frac{1+(1-x_1)^2}{C_1x_1(1-x_1)} + \frac{2m^2}{C_1^2}\right]\left[\frac{y_1^2+(y_1+x_2)^2}{B_2x_2(y_1+x_2)} - \frac{2m^2}{B_2^2}\right],$$

$$\mathcal{K}(p_+q_+) = -2\left[\frac{1+(1-x_1)^2}{C_1x_1(1-x_1)} + \frac{2m^2}{C_1^2}\right]\left[\frac{y_2^2+(y_2+x_2)^2}{D_2x_2(y_2+x_2)} - \frac{2m^2}{D_2^2}\right],$$

$$\mathcal{K}(p_-q_-) = -2\left[\frac{1+(1-x_1)^2}{A_1x_1(1-x_1)} + \frac{2m^2}{A_1^2}\right]\left[\frac{y_1^2+(y_1+x_2)^2}{B_2x_2(y_1+x_2)} - \frac{2m^2}{B_2^2}\right].$$

Expressions (19) could be in principle reproduced from the results of paper [6] by exception a more simple form of $\mathcal{K}(q_-q_+)$; as for eq. (18) it has an evident advantage in comparison to the corresponding formulae given in paper [6]. Let us note that the remaining factors $\mathcal{K}(p,q)$ could be obtained from the ones given in eq. (19) using the relations of the following type

$$\mathcal{K}(p_-q_-)(x_1, x_2, A_1, B_2) = \mathcal{K}(q_-p_-)(x_2, x_1, A_2, B_1). \quad (20)$$

Note also that the terms of the form

$$\frac{m^4}{B_2^2C_1^2} \quad (21)$$

do not give the logarithmically reinforced contributions, we will omit them below. The denominators of the propagators entering eq. (18, 19) are listed

here:

$$A_i = (p_- - k_i)^2 - m^2, \qquad A = (p_- - k_1 - k_2)^2 - m^2,$$
$$B_i = (q_+ + k_i)^2 - m^2, \qquad B = (q_+ + k_1 + k_2)^2 - m^2, \quad (22)$$
$$C_i = (-p_+ + k_i)^2 - m^2, \qquad C = (-p_+ + k_1 + k_2)^2 - m^2,$$
$$D_i = (-q_+ + k_i)^2 - m^2, \qquad D = (-q_+ + k_1 + k_2)^2 - m^2,$$

For the further integration it is useful to rewrite the denominators in terms of the photons energy fractions $x_{1,2}$ and their angles. In the case of the emission of both photons along p_- we would have

$$\frac{A}{m^2} = -x_1(1+z_1) - x_2(1+z_2) + x_1x_2(z_1+z_2) + 2x_1x_2\sqrt{z_1z_2}\cos\phi, \quad (23)$$
$$\frac{A_i}{m^2} = -x_i(1+z_i),$$

where $z_i = (\varepsilon\theta_i/m)^2$ and ϕ is the azimuthal angle between the planes containing the space vector pairs p_-, k_1 and p_-, k_2. In the same way one can obtain in the case $k_1, k_2 \| q_-$:

$$\frac{B}{m^2} = \frac{x_1}{y_1}(1+y_1^2z_1) + \frac{x_2}{y_1}(1+y_1^2z_2) + x_1x_2(z_1+z_2) + 2x_1x_2\sqrt{z_1z_2}\cos\phi, \quad (24)$$
$$\frac{B_i}{m^2} = \frac{x_i}{y_1}(1+y_1^2z_i).$$

Then we perform the elementary azimuthal angle integration and the integration over z_1, z_2 with the logarithmical accuracy using the procedure suggested in paper [8]

$$\overline{a} = m^4\int_0^{z_0}dz_1\int_0^{z_0}dz_2\int_0^{2\pi}\frac{d\phi}{2\pi}a, \qquad L_0 = \ln z_0 \equiv L + l, \qquad l = \ln\theta_0^2. \quad (25)$$

The list of the relevant integrals is given in Appendix A. In this way one obtains the differential cross section in the collinear region:

$$d\sigma_c = \frac{\alpha^4L}{4\pi^2s}\frac{d^3q_+d^3q_-}{q_+^0q_-^0}\frac{dx_1dx_2}{x_1x_2}(1+\mathcal{P}_{1,2})\{\frac{1}{yz_1^2}[\frac{1}{2}(L+2l)k_1k_5 \quad (26)$$
$$+(y^2+z_1^4)\ln\frac{x_2z_1^2}{x_1y} + x_1x_2(y-x_1x_2) - 2z_1k_5][B_{p_-p_-}\delta_{p_-p_-} + B_{p_+p_+}\delta_{p_+p_+}]$$
$$+\frac{1}{yz_1^2}[\frac{1}{2}(L+2l+4\ln y)k_1k_5 + (y^2+z_1^4)\ln\frac{x_1z_1^2}{x_2y} + x_1x_2(y-x_1x_2) - 2z_1k_1]$$
$$\times[B_{q_-q_-}\delta_{q_-q_-} + B_{q_+q_+}\delta_{q_+q_+}] + B_{p_-p_+}\delta_{p_-p_+}[(L+2l)\frac{k_1k_2}{z_1z_2} - 2\frac{k_1}{z_1}$$
$$-2\frac{k_2}{z_2}] + B_{q_-q_+}\delta_{q_-q_+}[(L+2l+2\ln(z_1z_2))\frac{k_1k_2}{z_1z_2} - 2\frac{k_1}{z_1} - 2\frac{k_2}{z_2}]$$

$$
\begin{aligned}
&+[B_{p_-q_-}\delta_{p_-q_-} + B_{p_+q_-}\delta_{p_+q_-}][(L+2l+2\ln y_1)\tfrac{k_1k_3}{z_1y_1t_1} - 2\tfrac{k_1}{z_1} \\
&-2\tfrac{k_3}{y_1t_1}] + [B_{p_+q_+}\delta_{p_+q_+} + B_{p_-q_+}\delta_{p_-q_+}] \times \\
&\times[(L+2l+2\ln y_2)\tfrac{k_1k_4}{z_1y_2t_2} - 2\tfrac{k_1}{z_1} - 2\tfrac{k_4}{y_2t_2}]\}.
\end{aligned}
$$

Here we use symbol $\mathcal{P}_{1,2}f(x_1,x_2) = f(x_2,x_1)$ for interchange operator; $x_{1,2} = \frac{\omega_{1,2}}{\varepsilon}$ are the energy fractions of the photons, $y = 1 - x_1 - x_2$ energy fractions of th2e scattered electron and positron are respectively y_1, y_2. We use also the notations:

$$
\begin{gathered}
l = \ln\theta_0^2, \quad t_1 = y_1 + x_2, \quad t_2 = y_2 + x_2, \qquad (27) \\
z_1 = 1 - x_1, \quad z_2 = 1 - x_2, \quad k_1 = 1 + z_1^2, \\
k_2 = 1 + z_2^2, \quad k_3 = y_1^2 + t_1^2, \quad k_4 = y_2^2 + t_2^2, \quad k_5 = y^2 + z_1^2,
\end{gathered}
$$

where θ_0 is the collinear parameter. The symbol $\delta_{p,q}$ corresponds to the specific conservation law of the kinematical situation defined by the pair p, q (see Table 1). Besides, we imply that the first photon is emitted along momentum p and the second — along momentum q ($p, q = p_-, p_+, q_-, q_+$). For instance, $\delta_{q_-q_+} \equiv \delta^4(p_+ + p_- - q_-/(1-x_1) - q_+/(1-x_2))$. And, finally, we define

$$
\begin{aligned}
B_{q_+q_+} = B_{p_-q_+} = B_{p_-p_-} &= \left(\tfrac{s}{t} + \tfrac{t}{s} + 1\right)^2, \qquad (28) \\
B_{p_+p_+} = B_{q_-q_-} &= \left(\tfrac{ys}{t} + \tfrac{t}{ys} + 1\right)^2, \\
B_{p_+q_-} &= \left(\tfrac{sy_1(1-x_1)}{tt_1} + \tfrac{tt_1}{sy_1(1-x_1)} + 1\right)^2, \\
B_{p_-p_+} &= \left(\tfrac{s(1-x_2)}{t} + \tfrac{t}{s(1-x_2)} + 1\right)^2, \\
B_{q_-q_+} = B_{p_+q_+} &= \left(\tfrac{s(1-x_1)}{t} + \tfrac{t}{s(1-x_1)} + 1\right)^2, \\
B_{p_-q_-} &= \left(\tfrac{sy_1}{tt_1} + \tfrac{tt_1}{sy_1} + 1\right)^2, \\
B_{q_-q_+} = B_{p_+q_+} &= \left(\tfrac{s(1-x_1)}{t} + \tfrac{t}{s(1-x_1)} + 1\right)^2,
\end{aligned}
$$

3 Contribution of the semi–collinear region

We will suggest for definiteness that the photon with momentum k_2 moves inside a narrow cone along the direction of motion of one of the charged particles, while the other photon moves in any direction outside that cone. This choice allows us to omit the statistical factor $1/2!$. The quasireal electron

method [7] may be used to obtain the cross section:

$$
\begin{aligned}
d\sigma_{sc} \;=\; &\frac{\alpha^4}{32s\pi^4}\frac{d^3q_-d^3q_+d^3k_1}{q_-^0q_+^0k_1^0}V\frac{d^3k_2}{k_2^0}\{\frac{\mathcal{K}_{p_-}}{p_-k_2}\delta_{p_-}R_{p_-} \qquad (29) \\
+ \; &\frac{\mathcal{K}_{p_+}}{p_+k_2}\delta_{p_+}R_{p_+} + \frac{\mathcal{K}_{q_-}}{q_-k_2}\delta_{q_-}R_{q_-} + \frac{\mathcal{K}_{q_+}}{q_+k_2}\delta_{q_+}R_{q_+}\}.
\end{aligned}
$$

We omitted in eq. (29) the terms of kind $m^2/(p_-k_2)^2$ because their contribution does not contain the large logarithm L. The quantities entering eq. (29) are presented below:

$$
\begin{aligned}
V \;&= \frac{s}{k_1p_+\cdot k_1p_-} + \frac{s'}{k_1q_+\cdot k_1q_-} - \frac{t'}{k_1p_+\cdot k_1q_+} - \frac{t}{k_1p_-\cdot k_1q_-} \qquad (30) \\
&+\frac{u'}{k_1p_+\cdot k_1q_-} + \frac{u}{k_1q_+\cdot k_1p_-}.
\end{aligned}
$$

$\mathcal{K}_i$ are the single photon emission collinear factors:

$$
\mathcal{K}_{p_-} = \mathcal{K}_{p_+} = \frac{1+(1-x_2)^2}{x_2(1-x_2)}, \quad \mathcal{K}_{q_-} = \frac{y_1+(y_1+x_2)^2}{x_2(y_1+x_2)}, \qquad (31)
$$
$$
\mathcal{K}_{q_+} = \frac{y_2+(y_2+x_2)^2}{x_2(y_2+x_2)}.
$$

The quantities R_i could be taken from paper [10], they are the known accompanying radiation factors:

$$
\begin{aligned}
R_{p_-} &= R[s(1-x_2), t', u', s', t(1-x_2), u(1-x_2)], \qquad (32) \\
R_{p_+} &= R[s(1-x_2), t'(1-x_2), u'(1-x_2), s', t, u], \\
R_{q_-} &= R[s, t', u'\frac{t_1}{y_1}, s'\frac{t_1}{y_1}, t\frac{t_1}{y_1}, u], \\
R_{q_+} &= R[s, t'\frac{t_2}{y_2}, u', s'\frac{t_2}{y_2}, t, u\frac{t_2}{y_2}],
\end{aligned}
$$

where function R has the form

$$
R[s,t',u',s',t,u] = \frac{1}{ss'tt'}[ss'(s^2+s'^2) + tt'(t^2+t'^2) + uu'(u^2+u'^2)], \qquad (33)
$$
$$
s = (p_+ + p_-)^2, \quad s' = (q_+ + q_-)^2, \quad t = (p_- - q_-)^2,
$$
$$
t' = (p_+ - q_+)^2, \quad u = (p_- - q_+)^2, \quad u' = (p_+ - q_-)^2.
$$

And finally we defined

$$
\delta_{p_-} = \delta^4(p_-(1-x_2) + p_+ - q_+ - q_- - k_1), \qquad (34)
$$

$$
\begin{aligned}
\delta_{p+} &= \delta^4(p_- + p_+(1-x_2) - q_+ - q_- - k_1), \\
\delta_{q-} &= \delta^4(p_- + p_+ - q_+ - q_-\frac{y_1 + x_2}{y_1} - k_1), \\
\delta_{q+} &= \delta^4(p_- + p_+ - q_+\frac{y_2 + x_2}{y_2} - q_- - k_1).
\end{aligned}
$$

Performing the integration over the angular variables of the collinear photon we obtain

$$
\mathrm{d}\sigma_{sc} = \frac{\alpha^4 L}{16 s\pi^4}\frac{\mathrm{d}^3q_-\mathrm{d}^3q_+\mathrm{d}^3k_1}{q_-^0 q_+^0 k_1^0}\mathrm{d}x_2 V\{\mathcal{K}_{p-}[R_{p-}\delta_{p-} + R_{p+}\delta_{p+}] \quad (35)
$$
$$
+\frac{1}{y_2}\mathcal{K}_{q+}R_{q+}\delta_{q+} + \frac{1}{y_1}\mathcal{K}_{q-}R_{q-}\delta_{q-}\}.
$$

To see that the sum of the cross sections (26) and (36)

$$
\mathrm{d}\sigma^{\gamma\gamma} = \mathrm{d}\sigma_c + \int \mathrm{d}O_1(\frac{\mathrm{d}\sigma_{sc}}{\mathrm{d}O_1}) \quad (36)
$$

does not depend on the auxiliary parameter θ_0 it is convenient to represent the terms entering eq. (36) in the form:

$$
VR_{p-}\delta_{p-} = \frac{1}{k_1p_-}v_{p-p-}\delta_{p-p-} + \frac{1}{k_1p_+}v_{p-p+}\delta_{p-p+} + \frac{1}{k_1q_-}v_{p-q-}\delta_{p-q-} \quad (37)
$$
$$
+\frac{1}{k_1q_+}v_{p-q+}\delta_{p-q+} + [VR_{p-}\delta_{p-}]^f,
$$
$$
[VR_{p-}\delta_{p-}]^f \equiv VR_{p-}\delta_{p-} - \sum_i \frac{1}{k_1q_i}v_{p-q_i}\delta_{p-q_i}, \qquad q_i = p_-, p_+, q_-, q_+,
$$

and the similar expressions for the other terms from eq. (36). Integrating $[VR_{p-}\delta_{p-}]^f$ over the angular variables we can integrate over the whole phase volume for k_1, i.e. we will obtain a finite contribution in the limit $\theta_0 \to 0$. Using the explicit expressions for the quantities

$$
v_{p_iq_j} = (VR_{p_i}k_1q_j)|_{k_1q_j\to 0}, \quad (38)
$$

which are listed in Appendix B, we can see the cancelation of the terms $L \cdot l$ from eq. (26) with the terms

$$
L\frac{k_1^0q_i^0}{2\pi}\int\frac{O_1}{k_1q_i} \sim -L\cdot l, \quad (39)
$$

which appears from 16 regions in the semi–collinear kinematics.

In conclusion we note that the leading ($\sim (\alpha L/\pi)^2$) due to the emission of two hard photons contributions to the inclusive on the scattered electron cross section, which could be obtained from eq. (36) by integration over photon energy fractions, is the relevant ingredient of the Drell–Yan representation (other ingredients are the virtual corrections):

$$
\mathrm{d}\sigma(p_-, p_+; q_-, q_+) = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 D(z_1,\beta)D(z_2,\beta)\int_0^1\frac{\mathrm{d}z_3}{z_3}\int_0^1\frac{\mathrm{d}z_4}{z_4} \quad (40)
$$
$$
\times\mathrm{d}\sigma_0(p_-z_1, p_+z_2; q_-\frac{z_3}{y_1}, q_+\frac{z_4}{y_2})D(\frac{y_1}{z_3},\beta)D(\frac{y_2}{z_4},\beta),
$$

where

$$
D(x,\beta) = \delta(1-x) + \frac{\alpha L}{2\pi}P^{(1)}(x) + \frac{1}{2}(\frac{\alpha L}{2\pi})^2 P^{(2)}(x) + \ldots \quad (41)
$$
$$
P^{(1)}(x) = \Theta(1-x-\eta)\frac{1+x^2}{1-x} + \delta(1-x)(2\ln\eta + \frac{3}{2})|_{\eta\to 0},
$$
$$
P^{(2)}(x) = \int_x^1 \frac{\mathrm{d}y}{y}P^{(1)}(y)P^{(1)}(\frac{y}{x}).
$$

And the cross section $\mathrm{d}\sigma_0$ in the Born approximation is given above (see eq. (16)).

The results of the numerical integration of the differential cross section (eq. (36)) in the experimentally accessible region

$$
\Delta < x_1, x_2 < 1, \qquad \theta_0 < \theta_-, \theta_+ < \pi - \theta_0 \quad (42)
$$

as a function of Δ, θ_0, $\sqrt{s}$.

We are grateful to S. Eidelman, G. Fedotovich, P. Franzini, G. Pancheri and L. Trentadue for fruitful discussions. We are also grateful to INFN, Parma, Frascati. The work was partially supported by INTAS grant 93–1867.

Appendix A

We present here the list of integrals (see eq. (22 - 25)).

$$
\frac{\overline{A_2}}{A^2A_1} = \frac{L_0}{x_1x_2(1-x_1)^2}[\frac{1}{2}L_0 + \ln\frac{x_2(1-x_1)^2}{x_1 y} - 1 + \frac{x_1x_2}{y}], \quad (43)
$$

$$\overline{\frac{1}{AA_1}} = \frac{L_0}{x_1x_2(1-x_1)}\left[\frac{1}{2}L_0 + \ln\frac{x_2(1-x_1)^2}{x_1 y}\right], \quad \overline{\frac{m^2}{AA_1^2}} = -\frac{L_0}{x_1^2 x_2(1-x_1)}$$

$$\overline{\frac{1}{A_1A_2}} = \frac{L_0^2}{x_1x_2}, \quad \overline{\frac{1}{A_1B_2}} = -\frac{y_1 L_0}{x_1x_2}(L_0 + 2\ln y_1).$$

The remaining integrals could be obtained using simple substitutions (see eq. (22 – 25)).

Appendix B

We put here the total list of the quantities $v_{p_i q_j}$ (eq. (38)):

$$\begin{aligned}
v_{p-p-} &= \frac{4(y^2+(1-x_1)^2)}{yx_1(1-x_2)}B_{p-p-}, & v_{p+p+} &= \frac{4(y^2+(1-x_2)^2)}{yx_1(1-x_2)}B_{p+p+},\\
v_{q-q-} &= \frac{4(1+(1-x_1)^2)}{x_1(1-x_1)}B_{q-q-}, & v_{q+q+} &= \frac{4y(1+(1-x_1)^2)}{x_1(1-x_1)}B_{q+q+},\\
v_{p+q+} &= \frac{4(y_2^2+(y_2+x_1)^2)}{x_1(y_2+x_1)}B_{p+q+}, & v_{p-q+} &= \frac{4(y_2^2+(y_2+x_1)^2)}{x_1(y_2+x_1)}B_{p-q+},\\
v_{p-q-} &= \frac{4(y_1^2+(y_1+x_1)^2)}{x_1(y_1+x_1)}B_{p-q-}, & v_{p+q-} &= \frac{4(y_1^2+(y_1+x_1)^2)}{x_1(y_1+x_1)}B_{p+q-},\\
v_{q+q-} &= \frac{4(1+(1-x_1)^2)}{x_1}B_{q-q+}, & v_{p+p-} &= \frac{4(1+(1-x_1)^2)}{x_1(1-x_1)}B_{p-p+}, \quad (44)\\
v_{p-p+} &= \frac{4(1+(1-x_1)^2)}{x_1(1-x_1)}B_{q-q+}, & v_{q-q+} &= \frac{4(1+(1-x_1)^2)}{x_1}B_{p-p+},\\
v_{q-p-} &= \frac{4(1+(1-x_1)^2)}{x_1(1-x_1)}B_{p-q-}, & v_{q+p+} &= \frac{4(1+(1-x_1)^2)}{x_1(1-x_1)}B_{p+q+},\\
v_{q+p-} &= \frac{4(1+(1-x_1)^2)}{x_1(1-x_1)}B_{p-q+}, & v_{q-p+} &= \frac{4(1+(1-x_1)^2)}{x_1(1-x_1)}B_{p+q-}.
\end{aligned}$$

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Ответственный за выпуск С.Г. Попов
Работа поступила 25.10.1995 г.

Сдано в набор 10.11.1995 г.
Подписано в печать 10.11.1995 г.
Формат бумаги 60×90 1/16 Объем 0.8 печ.л., 0.7 уч.-изд.л.
Тираж 220 экз. Бесплатно. Заказ N° 93

Обработано на IBM PC и отпечатано на
ротапринте ГНЦ РФ "ИЯФ им. Г.И. Будкера СО РАН",
Новосибирск, 630090, пр. академика Лаврентьева, 11.