The State Research Center of Russian Federation
BUDKER INSTITUTE OF NUCLEAR PHYSICS

V.N. Baier, A.I. Milstein and R.Zh. Shaisultanov

PHOTON SPLITTING IN A VERY STRONG MAGNETIC FIELD

Budker INP 96-18

NOVOSIBIRSK
1996

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V.N. Baier, A.I. Milstein and R.Zh. Shaisultanov

The State Research Center of Russian Federation
"Budker Institute of Nuclear Physics SB RAS"
630090 Novosibirsk, Russia

Abstract

Photon splitting in a very strong magnetic field is analyzed for energy $\omega < 2m$. The amplitude obtained on the base of operator-diagram technique is used. It is shown that in a magnetic field much higher than critical one the splitting amplitude is independent on the field. Our calculation is in a good agreement with previous results of Adler and in a strong contradiction with recent paper of Mentzel et al.

e-mail: baier@inp.nsk.su

Virtual creation and annihilation of electron-positron pairs is known to induce non-linear self-action of an electromagnetic field. Photon splitting in an external field is one of corresponding processes of nonlinear QED. Observation of a photon splitting is still a challenge for experiment.

Theoretical study of this process has a rather long history. Photon splitting in a constant and uniform external field was considered in the beginning of 70th in [1-4], where earlier paper, containing errors, were cited. In [1],[3] the process was considered as a possible mechanism for production of linearly polarized photons in a pulsar field (assuming the field $H \sim H_0$). At low photon energies ($\omega \ll m$, m is the electron mass, we set $\hbar = c = 1$) the splitting process can be analyzed by using the Heisenberg-Euler (HE) effective Lagrangian. In the weak field limit ($H \ll H_0$), where $H_0 = m^2/e = 4.41 \cdot 10^{13}G$ is the critical magnetic field, the first term of expansion of HE effective Lagrangian can be used and the hexagon diagram contributes only. This was done in [1],[2]. The polarization selection rules, especially with allowance for dispersion, were also obtained in [1], see also textbook [5], Sect. 129, 130 where the problem is given in detail. The comprehensive investigation of the process under consideration was carried out by Adler [3]. For $\omega \ll m$ and an arbitrary field strength the matrix element of the process was found as a result of the application of the full HE effective Lagrangian. At the same time, the allowed transition amplitude was calculated for the general case of an arbitrary field strength and photon energy below pair creation threshold ($\omega < 2m$). A Green's function of the electron in an external magnetic field in the Schwinger proper-time representation was used. Although the expression for this amplitude turned out to be very unwieldy for application, the amplitude was calculated also numerically in wide interval of magnetic fields $0 \leq H \leq H_0$ for $\omega = m$ as well as for $\omega \ll m$. In [4] photon splitting was considered in a crossed field $\mathbf{E} \perp \mathbf{H}, E = H$, using likewise the electron Green's function in the proper-time representation. Another form of the photon splitting amplitude in a magnetic field in a general case was obtained in [6] using similar approach for the Green's function calculation. Later, photon splitting in a constant and uniform electromagnetic field for arbitrary values of both field invariants was considered in [7]. The operator diagram technique developed by Katkov, Strakhovenko and one of us [9] was used. As a result, the solution of this technically quite cumbersome problem was substantially simplified. The amplitudes obtained in particular case of zero electric (or magnetic) field were found to be noticeably compact than those obtained in [3]. The results of [7] for $\omega \ll m$ and $H \ll H_0$ agree with those of [1]-[3]. In [7] we performed numerical calculations for the case $\omega \gg m$, because we were interested in another possibility: photon splitting in electric fields of single crystals at high energies [8]. It is worth to note that in all mentioned paper relativistic covariant and gauge invariant formulation of QED was used.

Recently, photon splitting was considered once more [10]. That was motivated by new astrophysics achievements. In this calculation a non-covariant perturbation theory and Landau gauge were used. The results of this paper as well as its subsequent application [11] are in a strong contradiction with all previous results. Matrix element of photon splitting is found in [10] in the form of very cumbersome threefold infinite sum. There is a series of short-comings in [10]: a) a low energy limit and weak field limit are not found from general expression; b) the authors of [10] suppose that their approach is applicable for $H = H_0$, but the result of calculation with the full HE effective Lagrangian for $\omega \ll m$ is not reproduced; c) photon dispersion in a magnetic field is not taken into account; d) the study of cut off influence on summation with respect to Landau level numbers is not sufficient because the expression considered contains strong cancellations. Adler [12] criticized strongly papers [10] and [11] and suggested to make the independent calculation of photon splitting amplitude.

Because of the potential astrophysics implications of the process (see e.g. [13]) we perform the numerical calculation and analysis of the expression for photon splitting amplitude obtained using another formulation of QED in external field [7]. We consider the most interesting region $\omega < 2m$ and arbitrary H.

Let photon with energy ω splits into two photons with energies ω_1 and ω_2. There is only one allowed transition in a magnetic field [1] with respect to photon polarizations: $B \to CC$ in notations of [7] or $\perp \to \|\|$ in notations of [1]. Putting electric field $E = 0$ in general representation for photon splitting amplitude (eqs.(2.16)-(2.18), [7]) we have for allowed transition:

$$T = \frac{(4\pi\alpha)^{3/2}\omega}{2\pi^2} \int_0^\infty dx \frac{\exp\left(-\frac{H_0}{H}x\right)}{x \sinh^2 x} \int_0^x dt_2 \left[\int_0^{t_2} dt_1 G\, \mathrm{e}^{c\Phi} + \sinh^2 t_2\, \mathrm{e}^{c\Phi_0} \right] \tag{1}$$

where

$$c = \frac{H_0}{H}\left(\frac{\omega\sigma}{m}\right)^2, \quad \Phi_0 = \frac{t_2(x-t_2)}{x} - \frac{\cosh x - \cosh(2t_2 - x)}{2\sinh x},$$

$$\begin{aligned}
\Phi = &\left[z_1 z_2 (t_1 - t_2)(t_1 - t_2 + x) - z_1 t_1 (t_1 - x) - z_2 t_2 (t_2 - x)\right]/x - \\
&[(1 - z_1 z_2)\cosh x - z_1 \cosh(2t_1 - x) - z_2 \cosh(2t_2 - x) + \\
&z_1 z_2 \cosh(x + 2t_1 - 2t_2)]/(2\sinh x), \\
G = &\left[1 - (z_1 \cosh(2t_1 - x) + z_2 \cosh(2t_2 - x))\cosh x\right]/x + \\
&2c z_1 z_2 \sinh^2(t_2 - t_1)\left[z_1 \sinh^2 t_1 + z_2 \sinh^2(t_2 - x)\right].
\end{aligned} \tag{2}$$

Here $\alpha = e^2 = 1/137$, $z_{1,2} = \omega_{1,2}/\omega$, $\sigma = \sin\vartheta$, ϑ is the angle between direction of magnetic field and momentum of the initial photon. To derive (1) we rotated the contour of integration over each variable: $x \to -ix$, $t_{1,2} \to -it_{1,2}$. This transformation is valid for any $\omega < 2m$. As a result, the integrand in (1) doesn't contain oscillating trigonometric functions. So, it is convenient for numerical calculation. It is easy to show that the amplitude T is symmetric with respect to interchange of final photons ($\omega_1 \leftrightarrow \omega_2$). By virtue of gauge invariance the amplitude $T \propto \omega\omega_1\omega_2$ if $\omega \to 0$ and also $T \to 0$ if $\omega_1 \to 0$ for any ω. That means that there are very strong compensations in (1) and this circumstance should be taken into account under numerical integration. To overcome the difficulties it is convenient to perform subtraction in the integrand of (1): $\mathrm{e}^{c\Phi} \to \mathrm{e}^{c\Phi} - 1$ for the first term of G (proportional to $1/x$) and $\mathrm{e}^{c\Phi_0} \to \mathrm{e}^{c\Phi_0} - 1$. The sum of the subtracted terms is equal to zero.

For $\omega \ll m$ the main contribution to the amplitude is given by the domain of variables where $c\Phi \ll 1$, $c\Phi_0 \ll 1$. Then expanding the corresponding exponents and keeping linear in c terms one can take the integrals over t_1 and t_2. The result coincides with the photon splitting amplitude found with the use of full HE effective Lagrangian (eq.(22) in [3]).

For $\omega \sim m$ the integrals in (1) are calculated numerically. Without loss of generality one can put $\sigma = 1$. In Fig.1 the dependence of the amplitude T on the final photon energy is shown for $H = H_0/2$ (a) and $H = H_0$ (b) at different energies of the initial photon. The amplitude T is normalized on T_0:

$$T_0 = \frac{13}{315}\frac{(4\pi\alpha)^{3/2}}{\pi^2}\frac{\omega^3}{m^2}\left(\frac{H}{H_0}\right)^3$$

For $\omega/m = 0.1$ the result coincides with a very good accuracy (better than 10^{-3}) with the result obtained from the full HE effective Lagrangian.

The total probability of allowed transition vs $w = \omega/m$ is shown in Fig.2, where

$$W_0 = T_0^2/(960\pi\omega) = 0.116\left(\frac{\omega}{m}\right)^5\left(\frac{H}{H_0}\right)^6 \ \mathrm{cm}^{-1}.$$

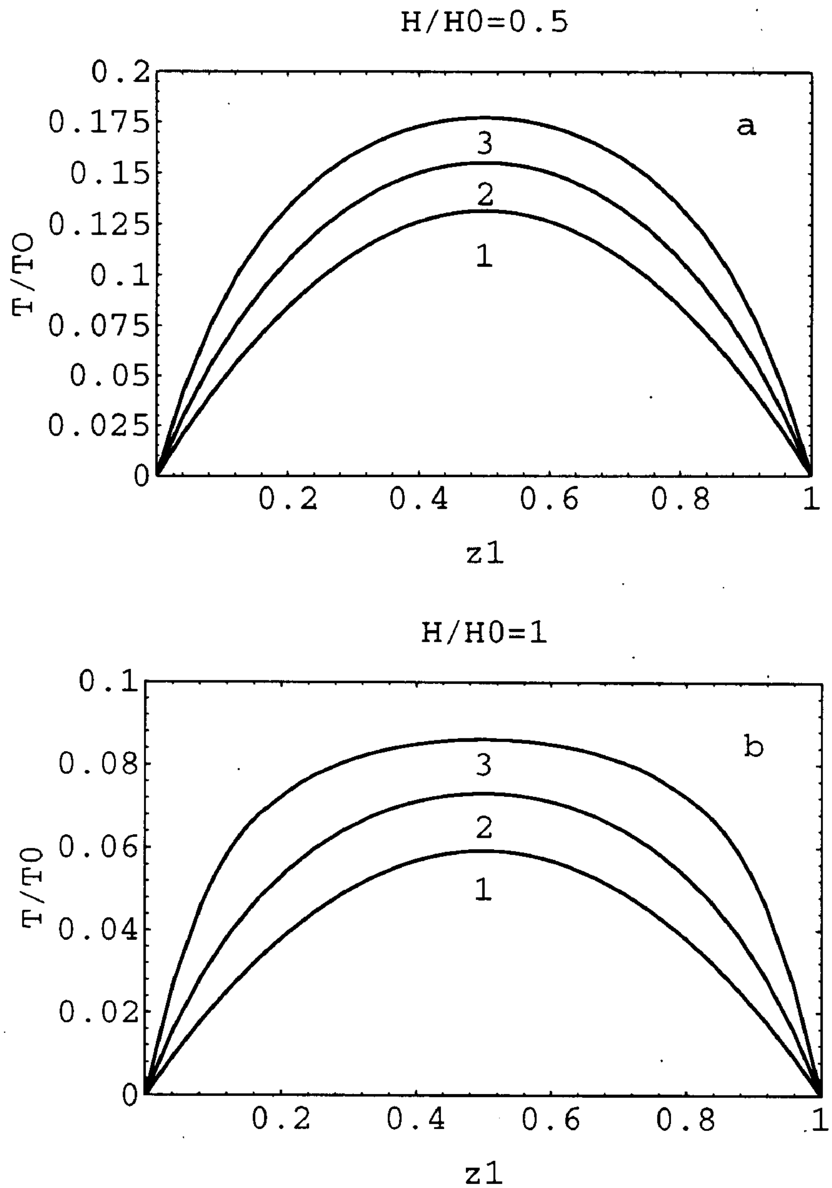

Figure 1: The dependence of photon splitting amplitude on the final photon energy ($z_1 = \omega_1/\omega$) for $H = H_0/2$ (a) and $H = H_0$ (b) at different energies of the initial photon: $\omega/m = 0.1(1)$, $\omega/m = 1.5(2)$ and $\omega/m = 1.9(3)$. The amplitude T is normalized on T_0 given in the text.

Curve (1) corresponds to $H/H_0 = 1$ and curve (2) to $H/H_0 = 1/2$. Although the probability varies by many orders of magnitude in the interval of parameters considered the essential part of the variation is absorbed by W_0. The function W_0 is nothing but the photon splitting probability given by hexagon diagrams at $\omega \ll m$. Therefore, Fig.2 shows the influence of higher order corrections with respect to ω/m and H/H_0. The probability found with the use of full HE effective Lagrangian also proportional to $(\omega/m)^5$. So, the intersection points of the curves with ordinate axis coincide with the probability W_{HE} found in this approximation. One can see from Fig.2 that the probabilities W and W_{HE} are essentially smaller than W_0 at $H \sim H_0$. At the same time, W/W_{HE} grows appreciably at $\omega \to 2m$. Therefore, the exact photon energy-dependence should be taken into account. Our numerical results agree (within a few percent) with obtained that by Adler in paper [3] where the case $\omega = m$ was considered.

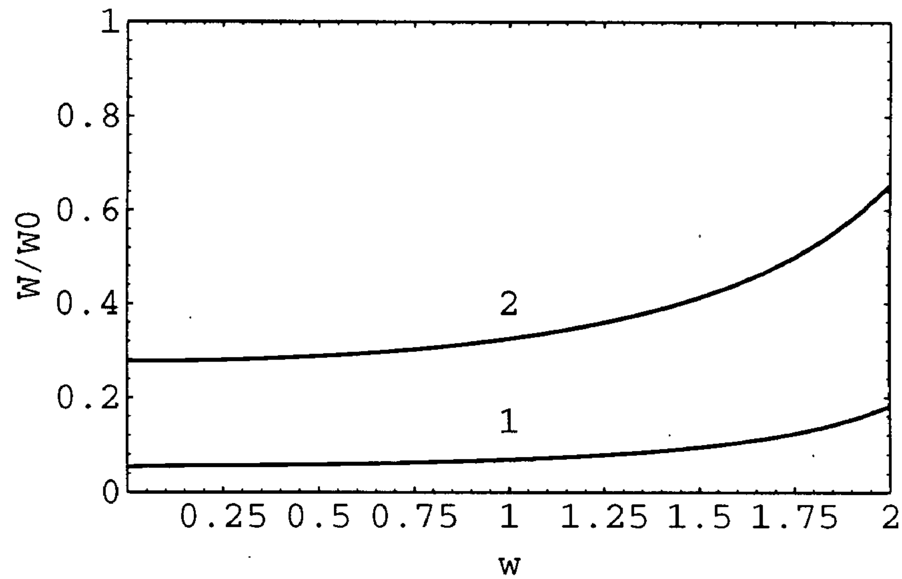

Figure 2: The dependence of total probability W (in term of W_0) on photon energy ($w = \omega/m$) for $H = H_0$ (curve 1) and $H = H_0/2$ (curve 2). The probability W_0 is given in the text.

The behavior of the amplitude T in a very strong magnetic field $H \gg H_0$ is of evident interest from theoretical point of view. In connection with this problem it is necessary to consider the selection rules for photon splitting in a strong field. Using the expression for eigenvalues of the photon polarization operator in a magnetic field found in [9], eqs.(3.33)-(3.35) we obtain that $n_{\|} = 1 + \alpha/6\pi$ and $n_{\perp} =\propto H$ for $H \gg H_0$ and $n_{\perp} > n_{\|}$ for any H. This means ([3, 5]) that there is only one allowed transition, which we considered above, for any field H.

For $H \gg H_0$ we found that the amplitude T is independent of magnetic field and can be evaluated in analytic form. The main contribution to two-fold integral in (1) is given by the domain $x \sim H/H_0$ and $x - t_2 \sim 1$. For threefold integral in (1) the main contribution comes from two domains: $x \sim H/H_0$, $t_1 \sim H/H_0$, $x - t_2 \sim 1$; and $x \sim H/H_0$, $t_1 \sim 1$,

$t_2 \sim H/H_0$. Performing the corresponding expansion, we obtain

$$T(H \gg H_0) = T_1 \frac{24m^4}{\omega^3}\Big[\frac{\omega_1}{\omega_2\sqrt{4m^2-\omega_2^2}}\arctan\left(\frac{\omega_2}{\sqrt{4m^2-\omega_2^2}}\right)$$
$$+\frac{\omega_2}{\omega_1\sqrt{4m^2-\omega_1^2}}\arctan\left(\frac{\omega_1}{\sqrt{4m^2-\omega_1^2}}\right) - \frac{\omega}{4m^2}\Big] \tag{3}$$

where

$$T_1 = \frac{(4\pi\alpha)^{3/2}}{12\pi^2}\frac{\omega^3}{m^2}$$

The amplitude calculated using the full HE effective Lagrangian at $H \gg H_0$ is $T_{HE} = T_1\omega_1\omega_2/\omega^2$. The dependence of the amplitude T in this limit on the final photon energy ($z_1 = \omega_1/\omega$) is shown in Fig.3 for different energies of the initial photon. For strong field and $\omega \to 2m$ one can see in Figs.1 and 3 a tendency of plato formation in the middle of the distribution.

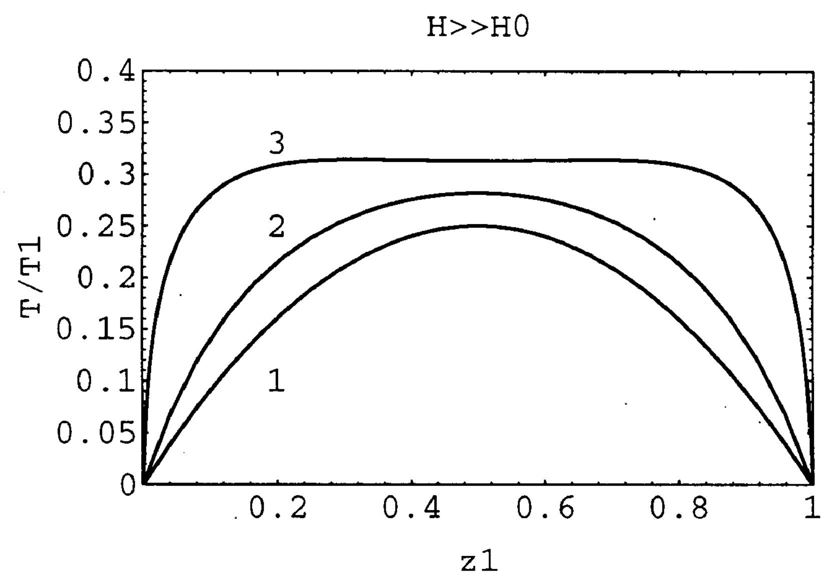

Figure 3: The dependence of the amplitude T on the final photon energy for $H \gg H_0$ for different energies of the initial photon: $\omega/m = 0.1(1)$, $\omega/m = 1.5(2)$ and $\omega/m = 1.99(3)$.

Thus, we performed the calculation of photon splitting amplitude using the exact formula valid for any magnetic field H and $\omega < 2m$. If $\omega \ll m$ then our results coincide with the amplitude obtained from the full HE effective Lagrangian. We obtained that in a very strong field $H \gg H_0$ the amplitude doesn't depend on a magnetic field. We found that the refractive index $n_\perp > n_\parallel$ for any H. So, there is only one allowed transition $\perp \to \parallel\parallel$. Therefore, a photon cascade could develop only if magnetic field changes it's direction (on distances much larger than the formation length of photon splitting). The results of our calculation are in a good agreement with that obtained by Adler [3] and in a strong contradiction with recent paper of Mentzel et al [10].

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V.N. Baier, A.I. Milstein and R.Zh. Shaisultanov

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В.Н. Вайер, Ф.И. Мильштейн, Р.Ж. Шайсултанов

Расщепление фотона в очень сильном магнитном поле

Budker INP 96-18

Ответственный за выпуск С.Г. Попов
Работа поступила 27.03.1996 г.

Сдано в набор 8.04.1996 г.
Подписано в печать 8.04.1996 г.
Формат бумаги 60×90 1/16 Объем 0.8 печ.л., 0.7 уч.-изд.л.
Тираж 150 экз. Бесплатно. Заказ N° 18

Обработано на IBM PC и отпечатано на
ротапринте ГНЦ РФ "ИЯФ им. Г.И. Будкера СО РАН",
Новосибирск, 630090, пр. академика Лаврентьева, 11.