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GRAVITATIONAL INTERACTION OF SPINNING BODIES,  
CENTER-OF-MASS COORDINATE AND  
RADIATION OF COMPACT BINARY SYSTEMS

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**Gravitational interaction of spinning bodies,  
center-of-mass coordinate and radiation  
of compact binary systems**

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**Abstract**

Spin-orbit and spin-spin effects in the gravitational interaction are treated in a close analogy with the fine and hyperfine interactions in atoms. The proper definition of the center-of-mass coordinate is discussed. The technique developed is applied then to the gravitational radiation of compact binary stars. Our result for the spin-orbit correction differs from that obtained by other authors. New effects possible for the motion of a spinning particle in a gravitational field are pointed out. The corresponding corrections, nonlinear in spin, are in principle of the same order of magnitude as the ordinary spin-spin interaction.

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**1.** It is expected that in few years the gravitational radiation from coalescing binary stars will be observed by laser interferometer systems LIGO and VIRGO. Its successful detection depends crucially on the accurate theoretical prediction of the exact form of the signal. In this way the observed effect becomes sensitive to the relativistic corrections of the  $c^{-2}$ ,  $c^{-3}$  and  $c^{-4}$  orders to the motion of a binary system and to the radiation intensity. In particular, the spin-orbit interaction becomes essential, and for two extreme Kerr black holes even the spin-spin one [1].

Some years ago it was noticed that the general relativity can accommodate in a natural way a specific gravitational magnetic moment coupling [2] (see also [3]). The starting point of the present work was the observation that the spin self-interaction arising in this way is of the same order of magnitude as the spin-spin interaction, and therefore in principle its existence can be checked in the gravitational-wave experiments.

However, in the course of the investigation, when trying to rederive previous calculations related to the spin effects in the gravitational radiation of binary stars, we came to the results which differ from those of Refs. [1, 4] as concerns the spin-orbit contributions. The origin of this discrepancy can be traced back to, what is to our belief, a long-standing confusion concerning the definition of the center of mass in the case when spin is taken into account. The problem is quite instructive and amusing by itself, and on the other hand, the spin-orbit correction is the leading one among spin effects. That is why we would like to start our discussion with this subject.

**2.** The spin-orbit and spin-spin interactions in the two-body problem can be immediately obtained in fact from the well-known results for the limiting case when one of the bodies (say, 2) is very heavy (see, e.g., book [5]). In this limit we have the usual spin-orbit interaction [6]

$$V_{1ls}^1 = \frac{3}{2} \frac{k}{c^2 r^3} \frac{m_2}{m_1} \vec{l} \vec{s}_1, \quad (1)$$

the interaction of the orbital angular momentum  $\vec{l}$  with the spin  $\vec{s}_2$  of the central body [7]

$$V_{2ls}^1 = 2 \frac{k}{c^2 r^3} \vec{l} \vec{s}_2, \quad (2)$$

and the spin-spin interaction [8]

$$V_{ss} = \frac{k}{c^2 r^3} [3(\vec{s}_1 \vec{n})(\vec{s}_2 \vec{n}) - \vec{s}_1 \vec{s}_2]. \quad (3)$$

Simple symmetry arguments dictate now the form of the spin-orbit interaction for the two-body problem:

$$V_{1ls} = \frac{3}{2} \frac{k}{c^2 r^3} \vec{l} \left( \frac{m_2}{m_1} \vec{s}_1 + \frac{m_1}{m_2} \vec{s}_2 \right), \quad (4)$$

$$V_{2ls} = 2 \frac{k}{c^2 r^3} \vec{l} (\vec{s}_1 + \vec{s}_2). \quad (5)$$

As to the spin-spin interaction, it is of the same form (3).

However, due to the mentioned discrepancy concerning the spin-orbit corrections to the gravitational radiation, it turns out expedient to derive explicitly the interactions discussed. This is only an elementary generalization of the solution of the corresponding problem for the case of a heavy central body, as given in book [5] (§106, Problem 4). We will start with the two-body Lagrangian including  $c^{-2}$  corrections:

$$L = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} + \frac{k m_1 m_2}{r} + \frac{m_1 v_1^4 + m_2 v_2^4}{8 c^2} + \frac{k m_1 m_2}{2 c^2 r} [3(v_1^2 + v_2^2) - 7(\vec{v}_1 \vec{v}_2) - (\vec{v}_1 \vec{n})(\vec{v}_2 \vec{n})] - \frac{k^2 m_1 m_2 (m_1 + m_2)}{2 c^2 r^2}. \quad (6)$$

Here  $\vec{r} = \vec{r}_1 - \vec{r}_2$ ;  $\vec{n} = \vec{r}/r$ ;  $m_i$ ,  $\vec{r}_i$ ,  $\vec{v}_i$  are the mass, coordinate and velocity, respectively, of the  $i$ th particle,  $i = 1, 2$ .

Let us take the term with  $v_1^2$  in the second line of Eq. (6). We write the velocities of individual elements of the top 1 (with mass  $dm_1$ ) in the form

$$\vec{v}_1 + \vec{\omega}_1 \times \vec{\rho}_1,$$

where  $\vec{v}_1$  is the velocity of the orbital motion,  $\vec{\omega}_1$  is the angular velocity. The radius-vector  $\vec{\rho}_1$  of the element  $dm_1$  is counted off the center of mass of the top 1, so that the integral over the volume of the top

$$\int \vec{\rho}_1 dm_1 = 0. \quad (7)$$

Due to (7) the first-order term of the expansion in  $\rho_1/r$  of the interaction discussed vanishes. As to the second-order term, with the obvious definition

$$\vec{\omega}_1 \int \rho_{1m} \rho_{1n} dm_1 = \frac{1}{2} \vec{s}_1 \delta_{mn}$$

of the spin  $\vec{s}_1$  of the top 1, it generates

$$\frac{3}{2} \frac{k}{c^2 r^3} \frac{m_2}{m_1} \vec{s}_1 [\vec{r} \times \vec{p}_1]$$

in the spin-orbit potential. Treating in this way the next term, that with  $v_2^2$ , in (6), we completely restore the spin-orbit potential (4) in the center-of-mass system for the binary, where  $\vec{p}_1 = -\vec{p}_2 = \vec{p}$ . The similar procedure applied to the terms with  $-7(\vec{v}_1 \vec{v}_2) -$

$(\vec{v}_1\vec{n})(\vec{v}_2\vec{n})$  in (6) leads to the next spin-orbit contribution (5), as well as to the spin-spin potential (3).

It should be mentioned that the above expressions for the spin-orbit and spin-spin interaction in the two-body problem were obtained previously in Refs. [9, 10] from the analysis of the scattering amplitude for spin-1/2 particles in the one-graviton-exchange approximation. (As to the spin-independent relativistic corrections, some terms of this type are missing from their expression.)

An amusing fact is that the obtained spin-orbit and spin-spin interactions are exact analogues (up to an obvious change of notations) of the corresponding well-known terms in the hydrogen atom. We mean the fine and hyperfine structure, the last interaction being induced by the coupling of the nuclear spin with the electron orbital angular momentum and spin. (Of course, in our classical approach we cannot reproduce the contact Fermi spin-spin interaction with  $\delta(\vec{r})$ ).

However, the expression for the spin-orbit correction to the acceleration, presented in Refs. [1, 4], differs from that which can be derived from our formulae  $V_{1ls}$  and  $V_{2ls}$ . The discrepancy is due to the difference in the definitions of the center-of-mass coordinate of a rotating star. The coordinate  $\vec{x}_i$  advocated and used in Ref. [1] is related to our  $\vec{r}_i$  as follows:

$$\vec{r}_i = \vec{x}_i + \frac{1}{2m_i}\vec{v}_i \times \vec{s}_i \quad (8)$$

(from now on we put  $c = 1$  in our explicit formulae). The shift by itself is of course a matter of convention, but it is in fact our definition which *just by construction* (see Eq. (7) and the arguments leading to it) corresponds to the true center-of-mass coordinate of a rotating star.

Still, what is the meaning of the vector  $\vec{x}$  and why is it irrelevant to the problem under consideration? The answer can be conveniently formulated with the following example. For the free Dirac particle with the Hamiltonian

$$H = \vec{\alpha}\vec{p} + \beta m$$

the operator whose expectation value equals to  $\vec{r}$ , is not  $\vec{r}$  itself, but [11]

$$\vec{x} = \vec{r} + \frac{i\beta\vec{\alpha}}{2E_p} - \frac{i\beta(\vec{\alpha}\vec{p})\vec{p} + [\vec{\Sigma} \times \vec{p}] E_p}{2E_p^2(E_p + m)}; \quad E_p = \sqrt{p^2 + m^2}; \quad \vec{\Sigma} = \frac{1}{2i}[\vec{\alpha} \times \vec{\alpha}]. \quad (9)$$

To lowest nonvanishing order in  $c^{-2}$  expression (9) reduces to

$$\vec{x} = \vec{r} - \frac{1}{2m}\vec{v} \times \vec{s}; \quad \vec{s} = \frac{\vec{\sigma}}{2} \quad (10)$$

which might prompt indeed substitution (8). However, the transition from the exact Dirac equation in an external field to its approximate form containing only the first-order correction in  $c^{-2}$  is performed by means of the Foldy-Wouthuysen (FW) transformation. And under the same FW transformation the relativistic operator  $\vec{x}$  (its form for an interacting particle is more complicated than (9)) goes over into mere  $\vec{r}$ . In other words, in

the arising Hamiltonian the coordinate of spinning electron has the same meaning  $\vec{r}$  as in the completely nonrelativistic case. Nobody makes substitution (8) when treating the spin-orbit interaction in the hydrogen atom.

**3.** Let us consider now the fully covariant equation of motion for a spinning particle in an external field

$$\frac{D}{D\tau} \left( m u^\mu + \frac{DS^{\mu\nu}}{D\tau} u_\nu \right) = -\frac{1}{2} R_{\rho\sigma}^{\mu\nu} u_\nu S^{\rho\sigma} + e F^{\mu\nu} u_\nu \quad (11)$$

derived by Papapetrou [12]. Here  $D/D\tau$  means the covariant derivative with respect to the proper time;  $u^\mu = dx^\mu/dt$  is the four-velocity;  $S^{\mu\nu}$  is the antisymmetric tensor of spin;  $R_{\rho\sigma}^{\mu\nu}$  is the Riemann tensor. We have included as well into this equation the interaction with an external electromagnetic field  $F^{\mu\nu}$ . A close analogy between the two terms, electromagnetic and gravitational, in the rhs of Eq. (11) was emphasized in Ref. [2].

We will use the common definition of the relativistic spin. According to it, the only nonvanishing components of the tensor of spin (and the vector of spin) in the particle rest frame are the space ones. Transition to an arbitrary frame is performed by a boost. This definition guarantees automatically the constraint for spin

$$S^{\mu\nu} u_\nu = 0. \quad (12)$$

Due to this constraint,

$$\frac{DS^{\mu\nu}}{D\tau} u_\nu = -S^{\mu\nu} \frac{Du_\nu}{D\tau}. \quad (13)$$

So, if the electromagnetic field is switched off and terms nonlinear in spin neglected, the second term in the lhs of Eq. (11) should be deleted. Clearly,  $-\frac{1}{2} R_{\rho\sigma}^{\mu\nu} u_\nu S^{\rho\sigma}$  is nothing else but a covariant expression for the force due to the spin-orbit interaction. In the field created by a heavy mass  $M$  this term reduces to first order in  $c^{-2}$  to

$$-3 \frac{kM}{r^3} (\vec{v} \times \vec{s} - (\vec{n}\vec{v}) \vec{n} \times \vec{s} - 2\vec{n}(\vec{n}[\vec{v} \times \vec{s}])) \quad (14)$$

which coincides with the corresponding force from Ref. [1]. However, the force extracted from potential (1) is different:

$$-3 \frac{kM}{r^3} \left( \vec{v} \times \vec{s} - \frac{3}{2} (\vec{n}\vec{v}) \vec{n} \times \vec{s} - \frac{3}{2} \vec{n}(\vec{n}[\vec{v} \times \vec{s}]) \right). \quad (15)$$

This discrepancy was pointed out long ago in Ref. [13] where the force (15) was derived from the scattering amplitude for the Dirac particle. The explanation suggested in Ref. [13] for the disagreement is that expression (14) refers to an extended body and (15) to a point particle. It does not look satisfactory. For instance, is the proton in a gravitational field a point particle or extended body? Obviously and as long as we do not go into details of its structure and as long as we do not consider its internal excitations, an extended body can be treated as a point particle.

To make the problem even more acute, let us consider another limit, that of vanishing gravitational field. In this case Eq. (11) describes a particle with spin, but without magnetic moment. Still, its spin interacts with an external electromagnetic field, which to lowest order in  $c^{-2}$  should be described by the Thomas interaction

$$V_t = \frac{e}{2m} \vec{s} [\vec{E} \times \vec{v}]. \quad (16)$$

This expression can be easily recovered from the well-known results for the spin precession (see, e.g., book [14]) at the vanishing  $g$ -factor,  $g = 0$ . When the electric field  $\vec{E}$  is that of a point charge  $-Ze$ , the force corresponding to interaction (16) is

$$\frac{Ze^2}{r^3} \left( \vec{v} \times \vec{s} - \frac{3}{2} (\vec{n}\vec{v}) \vec{n} \times \vec{s} - \frac{3}{2} \vec{n} (\vec{n} [\vec{v} \times \vec{s}]) \right). \quad (17)$$

However, the force obtained in this case from the second term in the lhs of Eq. (11) to lowest order in  $c^{-2}$ , is different:

$$\frac{Ze^2}{r^3} (\vec{v} \times \vec{s} - (\vec{n}\vec{v}) \vec{n} \times \vec{s} - 2\vec{n} (\vec{n} [\vec{v} \times \vec{s}])). \quad (18)$$

The reason of both discrepancies is clear now:  $\vec{r}$  entering expressions (14), (18) is just the relativistic coordinate of Eq. (11), it is nothing else but  $\vec{x}$  in the notations of relations (9). Therefore, the transition from the fully relativistic Eq. (11) to the  $c^{-2}$  approximations to it (14), (18), should be accompanied indeed by substitution (10). This substitution should be performed of course both in the acceleration entering the Newton equation of motion, and in the (formally) nonrelativistic force. In this way correct Eqs. (15), (17) are restored.

**4.** The above consideration of the Papapetrou equation (11) is instructive in one more respect. It was pointed out that this equation describes a particle with spin, but without magnetic moment. The magnetic moment interaction is well-known to be taken into account by the following term in the relativistic Hamiltonian:

$$V_{mm} = \frac{eg}{4m} S^{\mu\nu} F_{\mu\nu}. \quad (19)$$

In Ref. [2] it was demonstrated that expression (19) has a close gravitational analogue

$$V_{gm} = -\frac{\kappa}{8m} S^{\mu\nu} S^{\rho\sigma} R_{\mu\nu\rho\sigma} \quad (20)$$

which can be called gravitational magnetic moment interaction. In particular, this coupling arises in a natural way in wave equations, and the value  $\kappa = 1$  for the constant in it is as preferable from the point of view of the high-energy behaviour as  $g = 2$  is in the electromagnetic case [2]. Both interactions, (19) and (20), should include in fact some additional terms treated in detail in Refs. [15] (for usual magnetic moment) and [3]. Being certainly of higher order in  $v/c$ , those terms can be omitted in our treatment of binary stars.

For the field created by a heavy mass  $M$  interaction (20) reduces in lowest, first order in  $c^{-2}$  to the quadrupole form:

$$V_s^1 = \frac{3kM}{2r^3} Q_{mn}^s n_m n_n \quad (21)$$

where the effective quadrupole moment

$$Q_{mn}^s = \frac{1}{m} (s_m s_n - \frac{1}{3} \delta_{mn} s^2)$$

resembles by its spin dependence the well-known expression from quantum mechanics. For the two-body problem under discussion expression (21) generalizes to the following self-interaction of spins:

$$V_s = \kappa \frac{k}{2r^3} \left( \frac{m_2}{m_1} s_{1m} s_{1n} + \frac{m_1}{m_2} s_{2m} s_{2n} \right) (3n_m n_n - \delta_{mn}). \quad (22)$$

resembling the usual spin-spin interaction (3).

Let us compare now the effective quadrupole interaction (21) or (22) with the usual quadrupole one. At  $\kappa \sim 1$  interaction (22) is of the same order of magnitude as the spin-spin one (3). Even in the most favourable case when they can become essential, that of two extreme Kerr black holes, both interactions are of the  $c^{-4}$  order. The star rotation velocity is here  $\sim c$ , but its radius is close to the gravitational one  $r_g \sim c^{-2}$ , so that each spin  $s \sim c^{-1}$  [1]. (The same argument demonstrates that the spin-orbit interaction is of the  $c^{-3}$  order [1].) As to the usual quadrupole interaction, it is suppressed by the small value of the quadrupole deformation and, according to Ref. [16], can also manifest itself in the case of two extreme Kerr black holes only.

**5.** We are going over at last to the spin effects in the gravitational radiation of binary stars. In fact, the only essentially new correction to the energy loss obtained by us is that due to the spin self-interaction and originating mainly from interaction (22). Our final result for the contribution due to the spin-spin interaction (3) coincides with that presented in Refs. [1, 4]. As to the spin-orbit correction, our result for it can be in fact obtained from the expression given in [1, 4] by merely going back to the simple-minded definition of the center-of-mass coordinate advocated by us above. However, in parallel with calculating the correction due to the spin self-interaction (22), we will present corresponding contributions induced by the spin-orbit and spin-spin couplings (4), (5), (3). It serves as an independent check of the results presented previously in Refs. [1, 4], and we confirm in this way their result, but for the spin-spin interaction only.

We start with the well-known expression for the metric perturbation  $h_{mn}$  at large distance  $R$  from the source (see, e.g., [5], §110):

$$\psi_{mn}(R, t) = -\frac{4k}{R} \int d\vec{r} \tau_{mn}(\vec{r}, t - R + \vec{r}\vec{n}); \quad \psi_{mn} = h_{mn} + \frac{1}{2} \delta_{mn} h_{\rho\rho}; \quad \vec{n} = \frac{\vec{R}}{R}. \quad (23)$$

The source  $\tau_{mn}$  includes not only the energy-momentum tensor of matter, but generally speaking corresponding nonlinearities of the gravitational field itself. It is conserved in



the sense

$$\partial_\mu \tau_{\mu\nu} = 0. \quad (24)$$

As usual, we will be interested in the part of the perturbation  $\psi_{mn}$  which is orthogonal to  $\vec{n}$  and trace-free (otherwise Eq. (23) would look slightly more complicated). It should be mentioned here that both expression (23) and the  $c^{-2}$  Lagrangian (6) are valid under the same gauge conditions

$$\partial_m h_{m0} - \frac{1}{2} \partial_0 h_{mm} = 0, \quad \partial_\mu h_{\mu n} - \frac{1}{2} \partial_n h_{\mu\mu} = 0.$$

One can easily check it by inspecting the corresponding derivations in book [5] (§§106, 110).

Neglecting the retardation  $\vec{r}\vec{n}$  in expression (23) we reduce it to the quadrupole formula

$$\psi_{mn}^0 = -\frac{2k}{R} \partial_0^2 \int d\vec{r} r_m r_n \tau_{00}. \quad (25)$$

To lowest order in  $v/c$   $\tau_{00}$  reduces to rest masses, and the integral gives the usual quadrupole moment

$$Q_{mn} = \mu (r_m r_n - \frac{1}{3} \delta_{mn} r^2); \quad \mu = \frac{m_1 m_2}{m_1 + m_2}.$$

Here new terms in the quadrupole radiation intensity are generated by the spin-dependent corrections to the orbit radius  $r$  and to the equations of motion used to evaluate time derivatives of  $\vec{r}$ . In all our discussions of gravitational radiation we restrict to the case of circular orbits which is most interesting from the physical point of view [1], and much more simple as concerns calculations. In this way we get the following relative corrections to the usual quadrupole formula:

$$\frac{I_{ls}^{q1}}{I^q} = -\frac{1}{m_1 m_2 r^2} \vec{l} (6 \vec{s} + \frac{9}{2} \vec{\xi}); \quad (26)$$

$$\frac{I_{ss}^{q1}}{I^q} = \frac{9}{2 m_1 m_2 r^2} (3 s_{1t} s_{2t} - \vec{s}_1 \vec{s}_2); \quad (27)$$

$$\frac{I_s^{q1}}{I^q} = \frac{9\kappa}{4 m_1 m_2 r^2} \left[ \frac{m_2}{m_1} (3 s_{1t}^2 - s_1^2) + \frac{m_1}{m_2} (3 s_{2t}^2 - s_2^2) \right]. \quad (28)$$

Here

$$I^q = \frac{32k^4 m_1^2 m_2^2 m}{5r^5}$$

is the unperturbed quadrupole intensity,  $m = m_1 + m_2$ ; subscripts  $ls$ ,  $ss$ ,  $s$  refer to the spin-orbit, spin-spin and spin-self-interaction contributions respectively;

$$\vec{s} = \vec{s}_1 + \vec{s}_2; \quad \vec{\xi} = \frac{m_2}{m_1} \vec{s}_1 + \frac{m_1}{m_2} \vec{s}_2.$$

The expressions for  $I_{ss}^{q1}$  and  $I_s^{q1}$  have been averaged over the period of rotation. That is why they contain the spin components  $s_t$  orthogonal to the orbit plane.

The next correction to the quadrupole radiation originates from the terms of the relative order  $c^{-2}$  in  $\tau_{00}$ . The only spin-dependent contribution here is of the  $ls$  type. The same procedure which has generated the spin interactions from Lagrangian (6) allows one to extract from

$$\frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2}$$

the following correction to the quadrupole moment

$$\delta Q_{mn}^1 = \frac{\mu}{m} r_m \epsilon_{nrs} v_r \xi_s. \quad (29)$$

Since this expression will be anyway contracted with the symmetric  $Q_{mn}$ , there is no need to symmetrize it explicitly. Certainly, correction (29) makes a spin-orbit type contribution to the radiation intensity only. But we will postpone its calculation for the time being.

Let us go over now to the retardation effects in radiation. The first-order correction in formula (23) looks as follows:

$$\psi_{ab}^1 = -\frac{4k}{R} \partial_0 \int d\vec{r} z \tau_{ab}. \quad (30)$$

We have made explicit here that the wave propagates along the  $z$  axis,  $a, b = 1, 2$ . Simple transformations based on the continuity equation (24) (see [5], §110) lead to the following identity:

$$\int d\vec{r} r_k \tau_{mn} = \frac{1}{6} \partial_0^2 \int d\vec{r} r_k r_m r_n \tau_{00} + \frac{2}{3} \partial_0 \int d\vec{r} r_n (r_k \tau_{0m} - r_m \tau_{0k}). \quad (31)$$

The first, totally symmetric term in this expression generates the octupole radiation. Being spin-independent, it is not of interest to us.

In the second structure we restrict to the term in  $\tau_{0m}$  which is of lowest order in  $v/c$ , and obtain in this way

$$\int d\vec{r} r_n (r_k \tau_{0m} - r_m \tau_{0k}) = m_1 r_{1n} (r_{1m} v_{1k} - r_{1k} v_{1m}) + m_2 r_{2n} (r_{2m} v_{2k} - r_{2k} v_{2m}). \quad (32)$$

With the previous trick we single out in this tensor the spin-dependent terms and arrive at the expression which can be presented as

$$J_{kmn} = \epsilon_{nkl} J_{lm}.$$

The second-rank tensor here is

$$J_{mn} = -\frac{m_1 - m_2}{2m} (r_m l_n + r_n l_m) + \frac{3}{4} \mu (r_m \zeta_n + r_n \zeta_m - \frac{2}{3} r \vec{\zeta}) \quad (33)$$

where

$$\vec{\zeta} = \frac{\vec{s}_1}{m_1} - \frac{\vec{s}_2}{m_2}.$$

It is a close analogue of the magnetic quadrupole moment in electrodynamics, one can single out in it in an obvious way the contributions of the convection and spin currents.

The intensity of this gravimagnetic quadrupole radiation can be conveniently calculated via the following transformation of the initial structure  $n_k e_{mn} J_{kmn}$ :

$$n_k e_{mn} J_{kmn} = \epsilon_{lnk} e_{mn} n_k J_{lm} = \tilde{e}_{lm} J_{lm}. \quad (34)$$

If we choose the independent components of the polarization tensor as

$$e_{mn} = \left( \frac{e_{11} - e_{22}}{2}, \frac{e_{12} + e_{21}}{2} \right),$$

then the dual polarization is

$$\tilde{e}_{lm} = \epsilon_{lnk} e_{mn} n_k = \left( -\frac{e_{12} + e_{21}}{2}, \frac{e_{11} - e_{22}}{2} \right). \quad (35)$$

Formally the sum over independent dual polarizations  $\tilde{e}$  in  $(\tilde{e}_{lm} J_{lm})^2$  looks exactly the same as that over common polarizations  $e$  when calculating the usual quadrupole radiation. In the present case the intensity equals [1, 17]

$$I^{gmq} = \frac{16}{45} k^4 J_{mn}^{(3)} J_{mn}^{(3)} \quad (36)$$

where the superscript at  $J$  denotes the third time derivative. The calculation of these derivatives is simplified in the present case by the fact that to our accuracy both  $\vec{l}$  and  $\vec{s}_i$  can be considered as constant in time. The spin-dependent corrections arising in this way are

$$\frac{I_{ls}^{gmq}}{I^q} = -\frac{1}{12 m_1 m_2 r^2} \vec{l} (\vec{s} - \vec{\xi}); \quad (37)$$

$$\frac{I_{ss}^{gmq}}{I^q} = \frac{1}{48 m_1 m_2 r^2} (s_{1t} s_{2t} - 7 \vec{s}_1 \vec{s}_2). \quad (38)$$

Even a spin-self-interaction correction (somehow missed in Ref. [1]) arises here:

$$\frac{I_s^{gmq}}{I^q} = -\frac{1}{96 m_1 m_2 r^2} \left[ \frac{m_2}{m_1} (s_{1t}^2 - 7 s_1^2) + \frac{m_1}{m_2} (s_{2t}^2 - 7 s_2^2) \right]. \quad (39)$$

Let us consider at last the second-order retardation correction in formula (23)

$$\psi_{ab}^2 = -\frac{4k}{R} \frac{1}{2} \partial_0^2 \int d\vec{r}' z^2 \tau_{ab}. \quad (40)$$

A spin contribution can be produced in it only by velocity-dependent terms in  $r_k r_l \tau_{mn}$ , in other words only by the energy-momentum tensor of matter. These terms are of the type  $v_m v_n r_k r_l$  and generate the following structure

$$2 \frac{\mu}{m} r_k v_m \epsilon_{lnr} \dot{\xi}_r.$$

The  $(k, l)$  and  $(m, n)$  symmetrizations are implied here. The irreducible part of the third-rank tensor  $r_k v_m \dot{\xi}_r$  can be omitted at once since it does not interfere in the total intensity with the second-rank tensor  $Q_{mn}$ . Then, we omit also the structures of the

type  $(r_k v_m - r_m v_k) \xi_r$  since both orbital angular momentum and spin can be considered constant in time to our accuracy. The resulting structure transforms as follows:

$$\begin{aligned} \frac{2}{3} \frac{\mu}{m} n_k n_l e_{mn} \epsilon_{lnr} v_m (r_k \xi_r - r_r \xi_k) &= \frac{2}{3} \frac{\mu}{m} n_k n_l e_{mn} \epsilon_{lnr} v_m \epsilon_{krs} \epsilon_{ijs} r_i \xi_j = \\ &= -\frac{2}{3} \frac{\mu}{m} e_{mn} v_m \epsilon_{nrs} r_r \xi_s. \end{aligned}$$

In other words, this correction to the quadrupole moment is

$$\delta Q_{mn}^2 = -\frac{2}{3} \frac{\mu}{m} v_m \epsilon_{nrs} r_r \xi_s. \quad (41)$$

Adding up expressions (29) and (41) and neglecting again the term  $(r_m v_r - r_r v_m) \xi_s$ , we obtain the following total spin-dependent correction to the quadrupole moment:

$$\delta Q_{mn} = \frac{1}{3} \frac{\mu}{m} v_m \epsilon_{nrs} r_r \xi_s. \quad (42)$$

The corresponding relative correction to the radiation intensity constitutes

$$\frac{I_{ls}^{q2}}{I^q} = \frac{2}{3 m_1 m_2 r^2} \vec{l} \vec{\xi}. \quad (43)$$

Now, expressions (26), (37) and (43), taken together, give the following total spin-orbit correction:

$$\frac{I_{ls}}{I^q} = -\frac{\vec{l} (73 \vec{s} + 45 \vec{\xi})}{12 m_1 m_2 r^2}. \quad (44)$$

It can be easily checked that the corresponding result of Refs. [1, 4] would be reconciled with this one under the proper definition of the center-of-mass coordinate.

Adding up expressions (27) and (38), we obtain the result of Refs. [1, 4] for the spin-spin correction:

$$\frac{I_{ss}}{I^q} = \frac{1}{48 m_1 m_2 r^2} (649 s_{1t} s_{2t} - 223 \vec{s}_1 \vec{s}_2). \quad (45)$$

And at last, the total spin-self-interaction correction, generated by (28) and (39), is

$$\begin{aligned} \frac{I_s}{I^q} &= \frac{1}{4 m_1 m_2 r^2} \times \\ &\times \left[ \left( 27 \kappa - \frac{1}{24} \right) \left( \frac{m_2}{m_1} s_{1t}^2 + \frac{m_1}{m_2} s_{2t}^2 \right) - \left( 9 \kappa - \frac{7}{24} \right) \left( \frac{m_2}{m_1} s_1^2 + \frac{m_1}{m_2} s_2^2 \right) \right]. \end{aligned} \quad (46)$$

As has been mentioned already, at  $\kappa \sim 1$  this new correction is quite comparable to the spin-spin one.

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