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Abstract

The problem of driver optimization is considered in the case of nonlinear plasma response. The influence of plasma fields on the driver (composed of several short bunches) is taken into account in a qualitative way by artificial "annihilation" of defocused particles. It is shown that, in the optimum driver, the bunches are equidistant to a high accuracy, but their repetition frequency is slightly different from the plasma frequency. This difference is explained by several effects: a Doppler-like shift associated with neutralization of the driver current by plasma electron current, a weak non-periodicity of the driver, and a relativistic decrease in plasma frequency at large wave amplitudes. The correction to the driver modulation frequency is shown to be important for efficiency of the wake-field accelerator.

The concept of plasma-based accelerators has become an area of growing interest in recent years (see, e. g., reviews [1,2] and references therein). It offers the attractive opportunity of achieving very high accelerating gradients that exceed the limits of conventional accelerators. These promising acceleration schemes involve excitation of large amplitude plasma waves by either laser pulses or relativistic charged particle beams. The wave with a phase velocity that is close to the speed of light can then accelerate particles to superhigh energies in future linear colliders.

Several experiments with laser driven plasma waves have already been initiated. Thus far the maximum measured energy gain reported is 45 MeV. The acceleration occurs in a length of 0.5 mm, which corresponds to the accelerating gradient of 90 GeV/m. Plasma Wakefield Acceleration (PWFA) experiments, in which particle beams are used to drive the wave, are not as numerous. Yet they are of comparable interest to the laser experiments for both physical and engineering reasons. After having presented a successful proof-of-principal demonstration [3–6], such experiments need to be developed to the stage where the accelerating gradients, which will be in the GeV/m range, are achieved over a macroscopic distance. The latter requires a better quality driving beam than those generally produced by linacs. A possible solution is to use a modulated beam from an electron-positron storage ring, as proposed in [7–9]. The driver composed of several (N) particle bunches is shown to provide the accelerating electric field E_z of the order of wave-breaking limit even with the bunches of moderate density [10]: $|E_z| \sim E_0 = \sqrt{4\pi n_0 mc^2}$ for $n_b \sim n_0/N$, where n_0 is the plasma density, n_b is the characteristic driver density, c is the light velocity, and m is the electron mass.

The present paper is a part of theoretical studies related to this proposal. These studies should include, in particular, the analysis of plasma wave nonlinearity and the question of whether the beam can maintain its structure in a plasma over a sufficiently long distance without being destroyed by the excited plasma waves. Here we consider the problem of bunch sequence optimization for the case of nonlinear plasma response ($|E_z| \sim E_0$). We also abandon the assumption of "rigid" driver used in previous studies [7,8,10] and, in a qualitative way, take into account the influence of plasma fields on the driver.

There are three different spatial scales involved in this problem. The shortest scale is the wavelength of the excited wakefield c/ω_p, where $\omega_p = \sqrt{4\pi n_0 e^2/m}$ is the plasma electron frequency. Next in ordering is the focusing–defocusing length associated with the radial force, $F_r \sim |eE_z|$, acting on the driving beam. This force tends to change the beam radius a over the distance

$$L_F = c\sqrt{\frac{\gamma m a}{|F_r|}} \sim \frac{c}{\omega_p}\sqrt{\gamma}, \tag{1}$$

where $\gamma \gg 1$ is the relativistic factor of the driver, and we assume $a \sim c/\omega_p$. And the largest of the three is the driver deceleration length $L_d \sim \gamma c/\omega_p$.

The rigid beam approximation is valid only for the shortest scale, c/ω_p. On the focusing scalelength, radial dynamics of the beam becomes important. Depending on the sign of the radial force, the particles are either confined or pushed out and lost relatively quickly compared to the energy loss rate, so that the particle loss can be treated as instant as compared with driver deceleration. For the particles experiencing an inward radial force, we assume the rigid beam approximation to be valid until the driver longitudinal dynamics comes into play. This is true, at least, for the special case when the beam angular distribution, the radial beam density profile, and the focusing force are initially self-consistent. If the transversal driver pressure and the inward forces are not balanced from the outset, the rigid beam approximation is applicable only after the establishment of an equilibrium driver radius (which occurs at focusing scalelength, if ever). Thus, at the scalelength between L_F and L_d, the driver can be also treated as rigid.

The above reasons make it possible to optimize the driver shape with the account of a fully nonlinear plasma response but without looking into details of beam dynamics. The optimized driver should transfer its energy to the plasma wave and propagate over a long distance in plasma. This implies that all driver particles should be correctly positioned to be decelerated and focused by the plasma wave. To construct the optimized driver, we use the modified version of NOVOCODE code [10], into which the procedure of artificial "annihilation" of defocused beam particles was introduced. The equations we solve are:

$$\mathrm{rot}\,\vec{H} = \frac{4\pi}{c}\left(\vec{j}_b - en\vec{v}\right) + \frac{1}{c}\frac{\partial \vec{E}}{\partial t},$$

$$\mathrm{rot}\,\vec{E} = -\frac{1}{c}\frac{\partial \vec{H}}{\partial t}, \qquad \frac{\partial n}{\partial t} + \mathrm{div}\,(n\vec{v}) = 0, \tag{2}$$

$$\frac{\partial\vec{p}}{\partial t} + (\vec{v}\nabla)\vec{p} = -e\vec{E} - \frac{e}{c}\left[\vec{v}\times\vec{H}\right].$$

Here n, $\vec{v}$, and $\vec{p}$ are the density, the velocity, and the momentum of plasma electrons; $\vec{j}_b$ is the driver current, q is the driver particle charge ($\vec{j}_b = qn_bc\vec{z}$); other notation is conventional. Since the electron-fluid approximation is not valid when wave-breaking occurs, we do not examine overdense bunch sequences (with $n_bN \gtrsim n_0$).

The driver density is taken in the form

$$n_b(r, ct-z) = n_\omega(ct-z)\ \exp\left(-\frac{r^2}{2a^2}\right), \tag{3}$$

where $n_\omega(ct-z)$ is a piecewise function which is nonzero in the regions of both focusing and decelerating field only:

$$\begin{array}{ll} n_\omega = 0 & \left\{\begin{array}{l}\text{if } q\langle E_z\rangle > 0,\\ \text{or } \Phi(r) > \Phi(r_1)\ \text{ for any } r_1 > r;\end{array}\right. \\ n_\omega = n_{b0} & \text{otherwise}\ . \end{array} \tag{4}$$

Here $\langle E_z\rangle$ is the z-component of the electric field averaged over the driver cross-section:

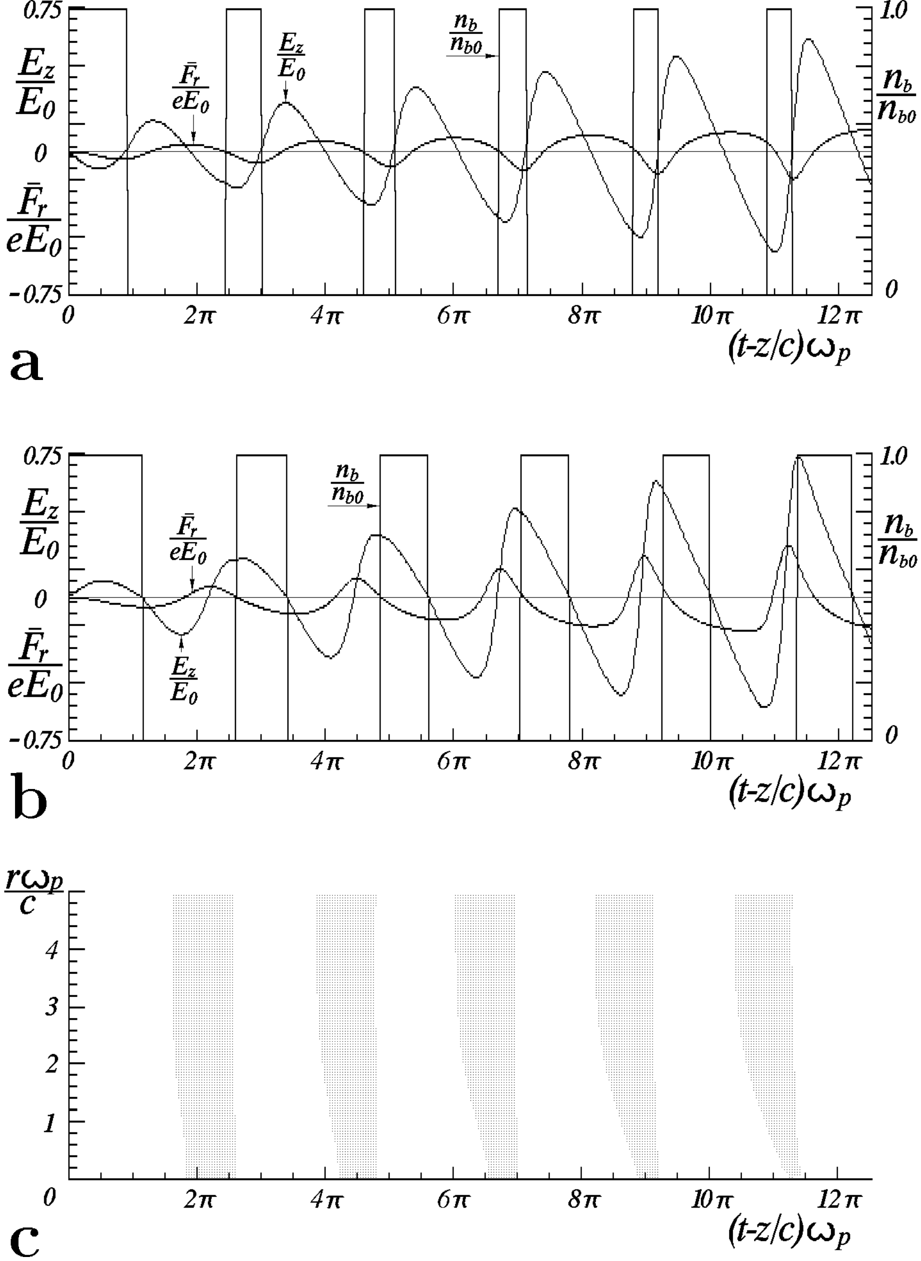
$$\langle E_z\rangle = \frac{1}{a^2}\int\limits_0^\infty E_z \exp\left(-\frac{r^2}{2a^2}\right)\ r\,dr, \tag{5}$$

and Φ is the force potential (see, e. g., [11], p. 172):

$$\Phi(r, ct-z) = \int\limits_r^{-\infty} F_r(r', ct-z)\,dr', \qquad F_r = -\frac{\partial\Phi}{\partial r}, \quad qE_z = -\frac{\partial\Phi}{\partial z}. \tag{6}$$

The bunch sequence thus constructed is the optimum one in the sense that all its “slices” give up energy to the plasma wave, and the driver itself, being focused everywhere, propagates over a long distance (determined by deceleration) in plasma. Two examples of calculated optimized drivers are shown in Fig. 1a,b.

Two remarks should be made here to justify the validity of the above method. First, regions of focusing and defocusing field do not intermix in driver cross-section: if the near-axis part of the driver is focused, the driver periphery is focused either (Fig. 1c). Narrow regions of alternating focusing force sign (also seen in Fig. 1c) do not appreciably affect the field structure



$\frac{E_z}{E_0}$
$\frac{\bar{F}_r}{eE_0}$
$\frac{n_b}{n_{b0}}$
0.75
0
-0.75
1.0
0
2π
4π
6π
8π
10π
12π
$(t\text{-}z/c)\omega_p$
a
b
$\frac{r\omega_p}{c}$
1
2
3
4
c

Figure 1: Optimized proton (a) and electron (b) drivers of radius $a = c/\omega_p$ and peak density $n_{b0} = 0.2\, n_0$. Also shown are the electric field E_z on axis and the focusing force $\bar{F}_r$ (averaged over the bunch radius) as functions of the longitudinal coordinate $(ct - z)$. (c) Regions of focusing (white) and defocusing (gray) field (defined according to (4)) for the electron driver shown above.

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and, therefore, can be treated with some inconsistency (as if the "partly defocused" slices of the driver were initially injected into the plasma, but all defocused particles were instantly lost).

Second, for the radial driver density distribution to be the same (for example, Gaussian) in every bunch cross section, the initial angular distribution of driver particles should be of a rather peculiar shape. In particular, parts of the driver experiencing stronger focusing should have greater angular spread. It is not the case in real situations, where the bunch radius is a function of z-coordinate. However, essential features of the realistic optimum bunch sequence can be understood from the above simplified model.

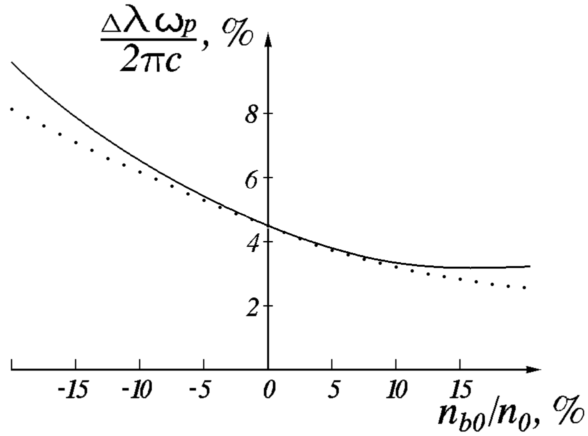

Figure 2: Shift of the driver modulation period ($\Delta\lambda = \lambda_{opt} - 2\pi c/\omega_p$) as a function of the peak driver density n_{b0} for $a = c/\omega_p$. Negative values of n_{b0} correspond to electron drivers, positive values — to proton ones. Shown by dots is the dependence determined by (9).

It is seen from Fig. 1 that the bunches in the optimized driver are equidistant to a high accuracy and, therefore, can be characterized by the repetition frequency ω_{opt} and the period $\lambda_{opt} = 2\pi c/\omega_{opt}$ (to be rigorous, we define λ_{opt} as one fifth of the distance between back fronts of the first and the sixth bunches). It is also seen that $\omega_{opt} \neq \omega_p$ (Figures 1 and 2). There are several effects which cause the observed deviation of the frequency.

The primary effect is the current neutralization of the driver. Charged particle beams (including modulated ones) are known to induce the plasma current directed to compensate the current of the beam. Hence plasma electrons in PWFA acquire the average velocity

$$|\bar{v}_z| \simeq c\,\frac{\bar{n}_b}{n_0} \cdot \frac{a^2}{a_p^2}, \tag{7}$$

where $\bar{n}_b$ is the average driver density, and a_p is the radius of plasma field localization $[a_p \simeq \max(a, c/\omega_p)]$. The electron motion results in inhomogeneous longitudinal drift of the plasma wave (which, possibly, accounts for the alteration (with radius) of the focusing force sign observed in [8,10]). Therefore, account of the current neutralization alone gives us the value of bunch repetition frequency

$$\omega_{opt} \simeq \omega_p \cdot (1 \pm |\bar{v}_z|/c) \tag{8}$$

($\omega_{opt} > \omega_p$ for the proton driver, and $\omega_{opt} < \omega_p$ for the electron one). The drift of the plasma wave caused by the current neutralization is clearly seen in computer simulations (Fig. 3): in the beam region ($r\omega_p/c \lesssim 1$) the spatial period of the oscillations is longer than that at the beam periphery (where it equals to $2\pi c/\omega_p$ since there is no neutralizing current there).

Certain positive shift of the driver period present even at small n_{b0}/n_0 (also seen in Fig. 2) is not a manifestation of any nonlinear effect. It appears because of a weak non-periodicity of generated plasma wave (and, consequently, of the bunch sequence) and shows the "error" in the definition of λ_{opt}.

Relativistic reduction of the plasma oscillation frequency can also lead to a driver frequency shift. It is easy to show that, for $|E_z| \sim E_0$, the velocity of plasma electrons is comparable to c, and, consequently, there appears a relativistic "increase" of the electron mass. The effect manifests itself in a weak increase of the bunch-to-bunch distance as $|E_z|$ increases and in some additional positive shift of $\Delta\lambda$ at high n_{b0}/n_0.

One more nonlinear effect is visible in Figures 1 and 3. Namely, because of the observed plasma wave nonharmonicity, the regions of favorable focusing ($F_r < 0$) for proton drivers turn out to be shorter than those for electron ones. This is because positively charged particles are focused by narrow

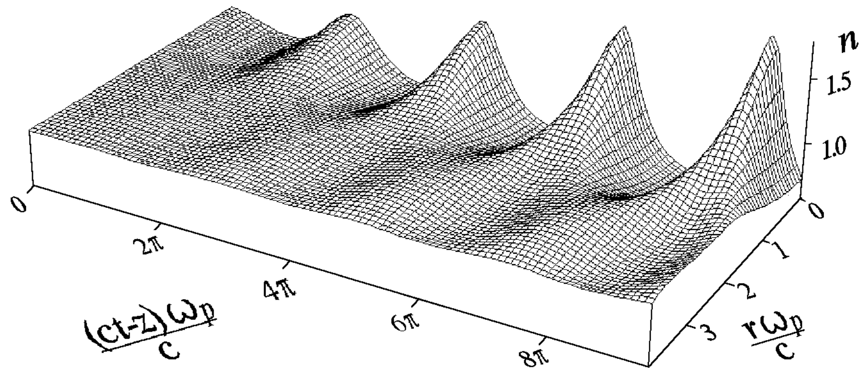

Figure 3: Plasma electron density oscillations generated by the optimized electron driver of radius $a = c/\omega_p$ and peak density $n_{b0} = 0.3\, n_0$.

peaks of the electron density, while electrons are focused by extended regions where $n < n_0$ (Fig. 3). Therefore, for given N, n_{b0}, and a, the electron driver generates a stronger wake-field than the proton one does.

The shift of driver modulation period determined by the approximating formula

$$\Delta\lambda = 2\pi c/\omega_p \left(A + B \frac{\bar{n}_b}{n_0} \right), \tag{9}$$

which takes into account both the current neutralization and the nonharmonicity driven "elongation" of the plasma wave, is also shown in Fig. 2. The coefficients A and B are chosen so that $\Delta\lambda$ and $d(\Delta\lambda)/dn_{b0}$ at $n_{b0} = 0$ are the same as for the numerically calculated dependence; the ratio $\bar{n}_b/n_0$ is taken as it is in optimized driver. It is seen that the numerically obtained deviation of the driver period, indeed, can be explained by aforementioned effects.

The accelerated bunch ("witness") must be placed inside the region where the force is both focusing and accelerating. The location of this region turns out to be very sensitive to the ratio of plasma frequency and driver frequency (Fig. 4). The driver (or plasma) frequency deviation of 5% from the designed value is seen to result in a twofold reduction of E_z. Hence, the requirements imposed on a system of plasma density maintenance and bunch position control are rather high.

In conclusion we note that, for an equidistant bunch sequence with the

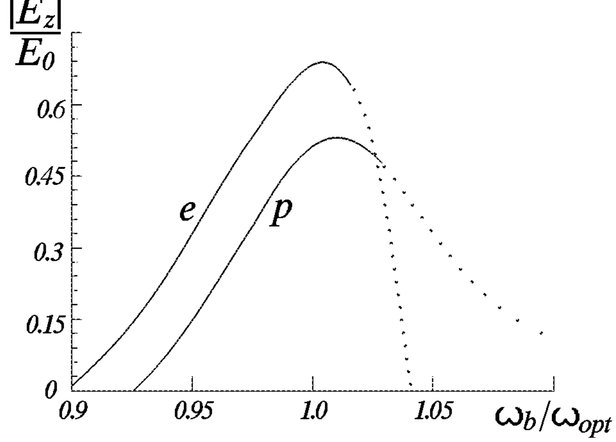

Figure 4: Electric field on axis at certain ("optimum" for electron witness) point behind the sixth bunch as a function of driver frequency for electron (e) and proton (p) drivers of radius $a = c/\omega_p$ and peak density $n_{b0} = 0.2\, n_0$. Dotted lines indicate the defocusing radial force.

frequency ω_{opt}, the accelerating wakefield, with a high accuracy, turns out to be almost the same as for the optimized driver calculated above. Therefore, the optimization of the driver does not require an individual non-equidistant placing of each bunch.

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