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Abstract

Discovery of superconducting materials that operate at high temperatures revive interest in the use of rf field for plasma confinement [1]. This paper discusses feasibility of a scheme where resonant rf cavities are attached to the mirror ends of an open system for plasma confinement.

1. Introduction

In the late 1950s, several papers were published regarding the use of rf electromagnetic field pressure to confine thermonuclear plasma by field buildup in a resonant cavity. These papers, as summarized by Glasstone [2], concluded that such an approach to fusion power was unpromising because Ohmic energy losses in cavity walls with normal conductors would be huge compared to all other energies involved, including possible thermonuclear yield, and because the electric field were impracticably large (exceeding 10^6 V/cm).

In the 1960s, high-Q superconducting cavities were developed [3] that could reduce the Ohmic energy losses in the cavity walls. In a typical calculation of fusion energy balance, cavity Qs in excess of 10^9 were required for the fusion power to exceed Ohmic losses. In fact, Qs exceeding 10^{10} have been achieved in empty cavities [4,5]. Also during the 1960s, the electric fields of the order of 10^6 V/cm were characterized as "about within the reach of current rf technology" [6]. A general survey of to-date viewpoint on the plasma confinement by rf field has been elucidated by S.O. Dean [1]. Here we discuss the use of superconducting rf-cavities to plug the plasma end losses from a mirror device. We restrict our consideration to only one but the key problem of the approach, namely that of damping of the rf oscillations at the plasma boundary.

The principal scheme of the device under consideration is shown in Fig. 1. To be more specific, we assume that the resonant cavities have cylindrical shape. Making this choice, we take into account that the axial symmetry of plasma body is crucial for reduction of transverse plasma transport.

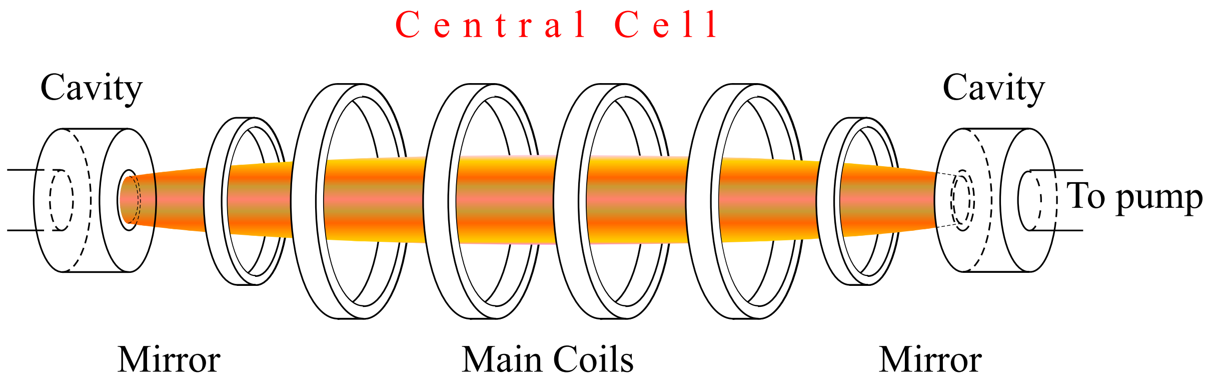

Figure 1: Sketch of an open confinement system with the resonant rf-cavities attached to the mirrors plugs in order to reduce the plasma end losses. MHD stability may be provided by rf-stabilizers (additional antennas installed at central cell of the device; not shown).

2. Pressure balance

If rf electromagnetic radiation inpinding on the plasma boundary is largely reflected, it exerts the "pressure"

$$\left\langle \frac{E_\tau^2 + B_\tau^2}{8\pi} - \frac{E_n^2 + B_n^2}{8\pi} \right\rangle \tag{1}$$

on the reflecting surface, where subscripts n and τ denote components of the electromagnetic field normal and tangent to the surface respectively. As long as the plasma is considered to be an ideal conductor, $E_\tau = 0$ and $B_n = 0$ at its boundary. Then the equation (1) reduces to

$$\left\langle \frac{B^2 - E^2}{8\pi} \right\rangle . \tag{2}$$

The magnetic component of rf field really exerts a pressure while the electric field yields "tension of the field lines" and acts in the opposite direction. Thus we conclude that for better confinement the electric field must be zero at that part of the resonant cavity shell that faces the plasma.

A confined plasma exerts an outward pressure nT. This must be balanced by the radiation pressure $\langle B^2/8\pi \rangle$. The overall equilibrium requires the ratio of these two pressures

$$\tilde{\beta} = \frac{8\pi nT}{\langle B^2 \rangle} \tag{3}$$

to be less than or at least equal to 1. Since the transverse equilibrium also requires a similar condition $\beta = 8\pi nT/H^2 < 1$ to be satisfied, the amplitude of rf field

$E_{\max} \sim B_{\max}$ should be of the order of the ambient magnetic field H. We will refer the inequality $H \ll B$ as the case of magnetic confinement, and the opposite inequality $H \gg B$ as the case of rf-confined plasma.

The ideal case of complete normal reflection, discussed above, will be difficult to achieve in a resonant cavity that plugs longitudinal losses in an open confinement system. Indeed, the plasma boundary will not be smooth and flat because of radial dependence of the plasma pressure, it will not be sharp as the plasma may do some degree penetrate into the cavity. As a result, refraction and field rearrangement will occur. Furthermore, evanescent tails of the cavity's electromagnetic field will penetrate into the plasma shell through the cavity's opening and interact with the shell itself instead of the plasma. Thus, $\tilde{\beta} \leq 1$ is only a rough upper estimate, while the realistic value of $\tilde{\beta}$ will be lower, depending on the system design and plasma parameters. In particular, one possibility is the "close wave guide regime", when the field frequency is so low that a corresponding wave cannot propagate along the waveguide formed by the plasma shell. We will briefly discuss this possibility in the concluding section.

3. Choice of frequency

In the case of unmagnetized plasma, $H = 0$, an electromagnetic wave impinging on the plasma boundary is reflected if rf radiation frequency ω is less than the electron plasma frequency ω_{pe} so that the operational range of frequencies is restricted only from above, $\omega < \omega_{pe}$. In a plasma, immersed into a magnetic field, the range of frequencies is limited also from below, moreover, it shrinks to zero in low density plasma.

Consider for example waves propagating along the magnetic field lines of $\boldsymbol{H}$. Eigen waves are circular. Electric vector of the right-hand wave (R) rotates in electron direction while that of the left-hand wave (L) co-revolves with ions. The square of the refractive index

$$N_{\mathrm{L,R}}^2 = 1 - \frac{\omega_{pe}^2/\omega}{\omega \pm \Omega_e} - \frac{\omega_{pi}^2/\omega}{\omega \mp \Omega_i} \tag{4}$$

is positive for propagating waves while it is negative for evanescent oscillations. All notations here are of common use with the only point to notice that the cyclotron frequencies $\Omega_{e,i} = |e|H/m_{e,i}c$ are assumed to be positive both for ions and electrons. Simple algebra reveals that $N_{\mathrm{L}}^2 < 0$ if the frequency ω comes into the range

$$\Omega_i < \omega < \sqrt{\omega_{pe}^2 + \Omega_e^2/4 + \Omega_e\Omega_i/2} - \Omega_e/2 + \Omega_i/2 \equiv \omega_{\mathrm{L}} \tag{5}$$

while $N_{\rm R}^2 < 0$ if

$$\Omega_e < \omega < \sqrt{\omega_{pe}^2 + \Omega_e^2/4} + \Omega_e/2 \equiv \omega_{\rm R}. \tag{6}$$

The frequency bands (5) and (6) partially overlap provided that

$$\omega_{pe} > \sqrt{2}\Omega_e. \tag{7}$$

If the above condition is satisfied, any rf radiation with the frequency in the range

$$\Omega_e < \omega < \omega_{\rm L} \tag{8}$$

is reflected from the plasma boundary. The dependence of $N_{\rm L,R}^2$ on ω for the case $\omega_{pe} > \sqrt{2}\Omega_e$ is shown in Fig. 2b. Fig. 2a illustrates the opposite case $\omega_{pe} < \sqrt{2}\Omega_e$ where an electromagnetic wave does not penetrate into the plasma if and only if it is circularly polarized and its frequency falls in either of the two bands (5), (6). We will see in the next Section that oscillations in a resonant cavity cannot have circular polarization in the whole volume of the cavity which limits plasma parameters to the inequality (7). In practical units, the latter leads to

$$n_e > 2{\cdot}10^{11} H^2\,[{\rm cm}^{-3}/{\rm kG}^2]. \tag{9}$$

Hence, the ambient magnetic field should be as small as $20\ {\rm kG}$ at the mirrors to be plugged for the plasma with "typical" thermonuclear density $n_e = 10^{14}\ {\rm cm}^{-3}$. Notice that the condition (7) can be cast into the form

$$\beta > 4\,\frac{T}{m_e c^2}. \tag{10}$$

Hence, for T in the thermonuclear range of temperatures $100 \div 200\ {\rm keV}$ it can be satisfied if $\beta \approx 1$.

4. RF field in cylindrical cavity

Consider a cylindrical cavity with radius R and width h as shown on the Fig. 3. We assume that the coordinate z is directed along the axis of the cavity and that the planar walls are placed at $z = 0$ and $z = h$. Maxwell equations in the cylindrical system of co-ordinates take the form:

$$-\frac{1}{c}\frac{\partial B_r}{\partial t} = \frac{1}{r}\frac{\partial E_z}{\partial \phi} - \frac{\partial E_\phi}{\partial z}, \qquad \frac{1}{c}\frac{\partial E_r}{\partial t} = \frac{1}{r}\frac{\partial B_z}{\partial \phi} - \frac{\partial B_\phi}{\partial z}, \tag{11a}$$

$$-\frac{1}{c}\frac{\partial B_\phi}{\partial t} = \frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r}, \qquad \frac{1}{c}\frac{\partial E_\phi}{\partial t} = \frac{\partial B_r}{\partial z} - \frac{\partial B_z}{\partial r}, \tag{11b}$$

$$-\frac{1}{c}\frac{\partial B_z}{\partial t} = \frac{1}{r}\frac{\partial r E_\phi}{\partial r} - \frac{1}{r}\frac{\partial E_r}{\partial \phi}, \qquad \frac{1}{c}\frac{\partial E_z}{\partial t} = \frac{1}{r}\frac{\partial r B_\phi}{\partial r} - \frac{1}{r}\frac{\partial B_r}{\partial \phi}. \tag{11c}$$

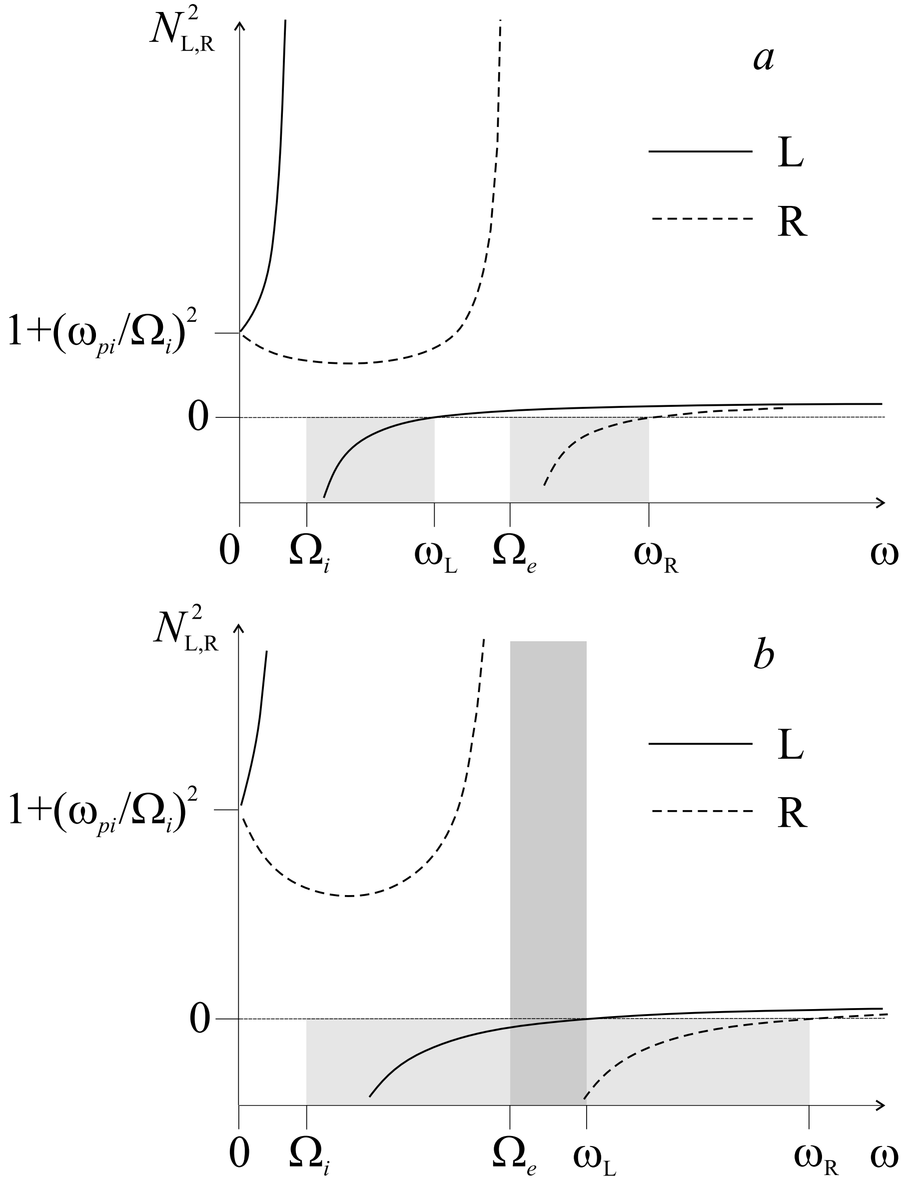

Figure 2: Dependence of $N^2 \equiv k^2c^2/\omega^2$ on ω for the left-hand (L) and right-hand (R) circular waves propagating parallel to the ambient magnetic field:
(a)—$\omega_{pe} < \sqrt{2}\Omega_e$, rf field penetrates into the plasma at any frequency ω;
(b)—$\omega_{pe} > \sqrt{2}\Omega_e$, rf field is reflected from plasma boundary if the frequency ω falls into the range $\Omega_e < \omega < \omega_L$ where $\omega_L = \sqrt{\omega_{pe}^2 + \Omega_e^2/4 + \Omega_e\Omega_i/2} - \Omega_e/2 + \Omega_i/2$. Regions where the waves are evanescent are shaded.

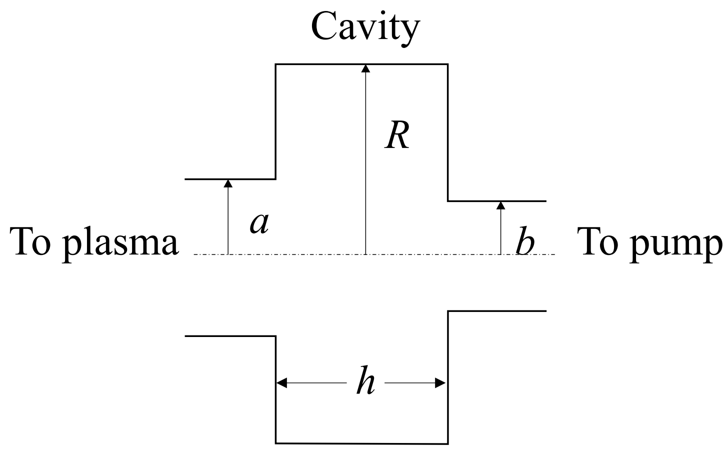

Figure 3: Resonant cavity. R is the radius of the cavity, h is its width, a and b are the radius of waveguides for plasma and pumpout respectively.

We first consider an empty cavity with all the walls made of a superconducting material. Then $E_r = E_\phi = B_z = 0$ at $z = 0$ and $z = h$, while $E_\phi = E_z = B_r = 0$ at $r = R$. All oscillations that can be excited in such a cavity are separated into two types. Oscillations of the electric type has no magnetic rf field along the axis z of the cavity, $B_z = 0$. Oscillations of the magnetic type are characterized by $E_z = 0$. As it is clear from discussion of the previous Section, the electric oscillations are of less interest as they produce non-zero electric field at the cavity's planar walls which are exposed by plasma. Putting $E_z = 0$ into (11), it is easy to reveal that B_z obeys the equation

$$\Delta B_z + \frac{\omega^2}{c^2} B_z = 0. \tag{12}$$

All other components of the electromagnetic field can be expressed in terms of B_z. General solution of the equation (12), which satisfies all of the above cited boundary conditions, depends on few constant: the amplitude B_0 of B_z, the azimuthal angle ψ_m and the phase shift ξ_m:

$$\begin{aligned}
B_r &= B_0 \frac{k}{\kappa} J_m^{'}(\kappa r) \sin(m\phi + \psi_m) \cos kz \cos(\omega t + \xi_m), \\
B_\phi &= B_0 \frac{mk}{\kappa^2 r} J_m(\kappa r) \cos(m\phi + \psi_m) \cos kz \cos(\omega t + \xi_m), \\
B_z &= B_0 J_m(\kappa r) \sin(m\phi + \psi_m) \sin kz \cos(\omega t + \xi_m), \\
E_r &= B_0 \frac{m\omega}{\kappa^2 cr} J_m(\kappa r) \cos(m\phi + \psi_m) \sin kz \sin(\omega t + \xi_m), \\
E_\phi &= -B_0 \frac{\omega}{\kappa c} J_m^{'}(\kappa r) \sin(m\phi + \psi_m) \sin kz \sin(\omega t + \xi_m) \\
E_z &= 0.
\end{aligned} \tag{13}$$

Here $k = \pi l/h$ with $l = 1, 2, \ldots$ being integer,

$$\omega^2 = c^2(\kappa^2 + k^2), \tag{14}$$

κ is to be found from the equation

$$J'_m(\kappa R) = 0, \tag{15}$$

J_m is the Bessel function, and J'_m is its derivative.

The solution (13) describes linearly polarized mode. Combination of two modes (13) with different B_0, ψ_m and/or ξ_m produces, in general, a mode with elliptic polarization. A special choice of ψ_m and ξ_m for given B_0 in the combination can yield an oscillation circularly polarized near the cavity axis:

$$\begin{aligned}
B_r &= B_0 \frac{k}{\kappa} J_m^{'}(\kappa r)\cos(kz)\sin(m\phi \mp \omega t),\\
B_\phi &= B_0 \frac{mk}{\kappa^2 r} J_m(\kappa r)\cos(kz)\cos(m\phi \mp \omega t),\\
B_z &= B_0 J_m(\kappa r)\sin(kz)\sin(m\phi \mp \omega t),\\
E_r &= \mp B_0 \frac{m\omega}{\kappa^2 c r} J_m(\kappa r)\sin(kz)\sin(m\phi \mp \omega t),\\
E_\phi &= \mp B_0 \frac{\omega}{\kappa c} J_m^{'}(\kappa r)\sin(kz)\cos(m\phi \mp \omega t)\\
E_z &= 0.
\end{aligned} \tag{16}$$

The upper signs in (16) corresponds to the R mode which gyrates in electron direction; the lower sings correspond to the L mode revolving together with ions. At the cylinder wall the modes (16) are linearly polarized, near the axis they are of circular polarization, and they have elliptic polarization between the axis and the cylinder wall. To avoid possible misunderstanding, we emphasize that saying about polarization of rf field we imply the shape of the curve which is drawn by the end of the vector $\boldsymbol{E}$ at a *fixed* point (r, ϕ, z). Snapshot of rf electric field lines for the mode with the azimuthal number $m = 1$ is shown on Fig. 4. Fig. 5 shows the electric field lines map for the $m = 2$ mode. Best choice for the purpose of plasma plugging is provided by the mode with azimuthal number $m = 1$ as the pressure exerted by the $m = 1$ mode on the planar walls $z = 0$ and $z = h$ reaches its maximal value at the cavity's axis. Also it is clear that the lowest radial mode provides better homogeneity of rf pressure at the planar wall. The lowest root of the equation (15) for $m = 1$ is $\kappa R = 1.841$. Pressure of higher radial modes has zero nodes that may play role of holes through which plasma would escape from the device. Figs. 6 and 7 show isobars of rf field pressure at the planar walls for the $m = 1$ and $m = 2$ modes.

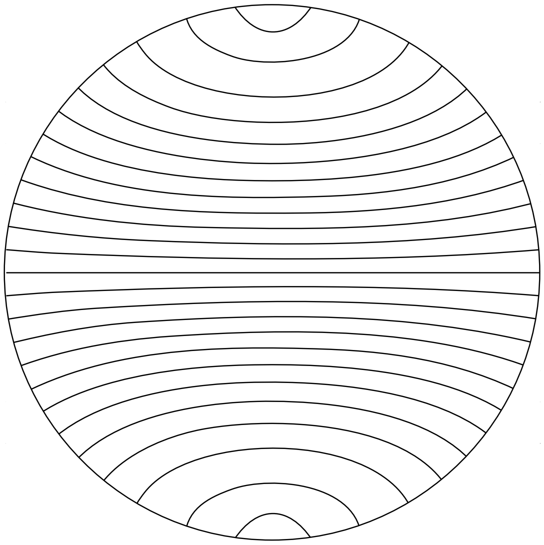

Figure 4: Map of the electric field lines for the $m = 1$ mode in the resonant cavity. The map rotates in due course of time if the mode is circular at the cavity's axis.

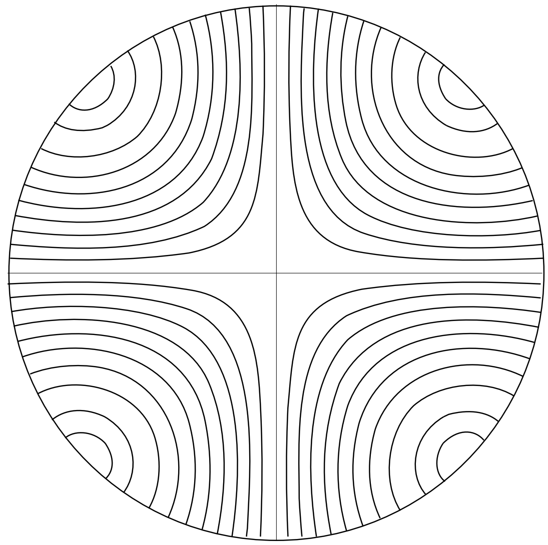

Figure 5: The same map as in Fig. 4 but for the $m = 2$ mode.

As we have seen in the previous Section, rf field does not penetrate into the plasma provided the inequality (9) holds and the frequency ω falls into the range (8). Hence, to confine plasma with the density of $n_e = 10^{14}\,\mathrm{cm}^{-3}$ without any leakage of the rf power into the plasma, the rf frequency should be as large as the electron cyclotron frequency $\Omega_e = 3.5{\cdot}10^{11}$ evaluated at $H = 20\,\mathrm{kG}$. The wavelength of the oscillations with $\omega = 3.5{\cdot}10^{11}$ is as small as $0.5\,\mathrm{cm}$. This means that the oscillations with very big number l of the wavelengths per the cavity's width should be excited in the resonant cavity of reasonable sizes. Since using high-l modes may cause severe technology obstacles, below in this Section we consider a range of lower frequencies where only the L mode is totally reflected while the R mode penetrates into the plasma core thus reducing the cavity's Q.

Now we take into account that one of the planar walls has a window for the plasma where reflecting surface is formed by the plasma boundary instead of a superconducting material. Rf power can penetrate into the plasma core thus introducing a damping mechanism for rf oscillations excited by an external source in the resonant cavity. For the sake of simplicity, we assume that the waves, outgoing from the cavity into the plasma, propagate almost parallel to the ambient magnetic field, i.e., $k \gg \kappa$. Notice that $B_z \ll B_{r,\phi}$ in this case. With this assumption being

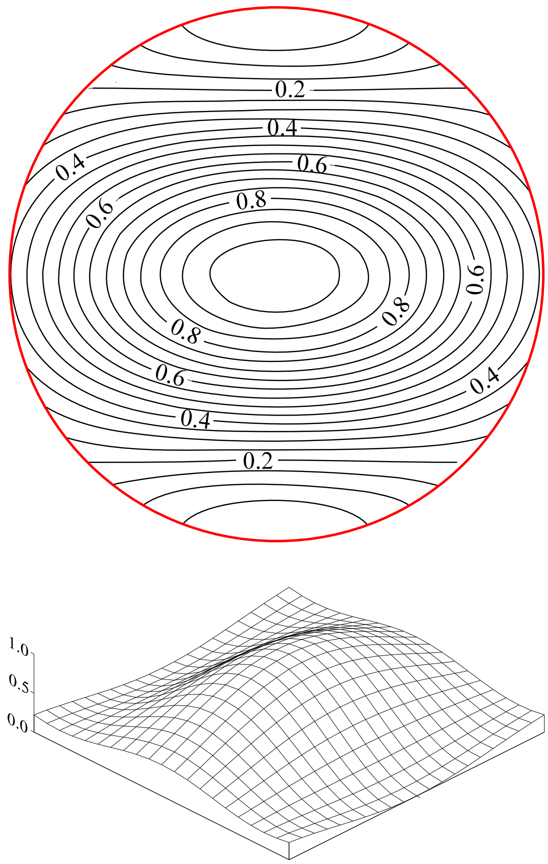

Figure 6: Instant distribution of rf field pressure at the planar wall of the resonant cavity for the $m = 1$ mode. Revolution of the mode leads to averaging of the pressure over the azimuthal angle.

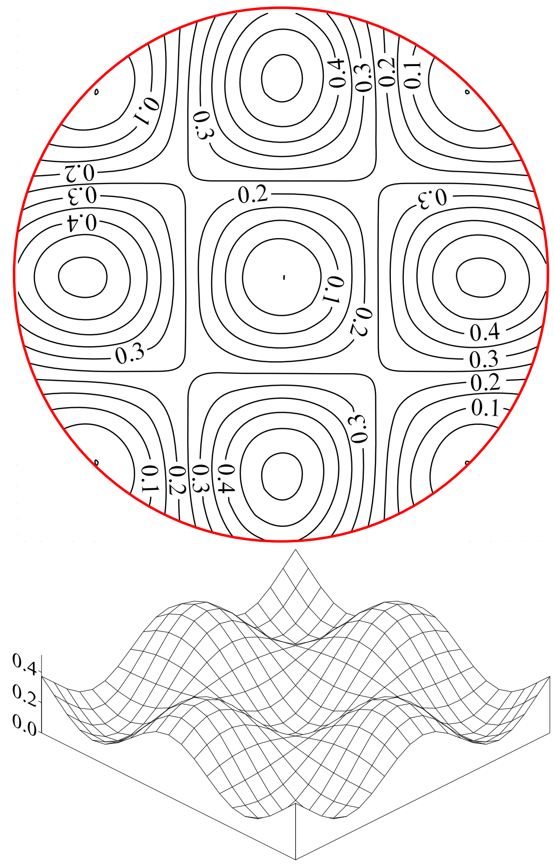

Figure 7: The same as in Fig. 6 but for the $m = 2$ mode.

adopted, boundary conditions at the plasma take the form

$$\boldsymbol{E}_{\mathrm{L,R}} = \frac{1}{N_{\mathrm{L,R}}}[\boldsymbol{B}_{\mathrm{L,R}}, \boldsymbol{n}] \tag{17}$$

where $\boldsymbol{n}$ is the unit vector normal to the surface of the boundary and directed outwards the plasma.

For the damping rate γ of rf oscillations to be small, two conditions are required. Firstly, the window for the plasma in the superconducting wall should comprise small part of the surface of cavity's walls. Secondly, polarization of excited wave should be close to circular near the window. With tuning rf frequency to the band where the excited circular wave does not penetrate into the plasma, one can reduce losses of rf power. Fig. 2 indicates that appropriate mode gyrates in the ion direction (L-wave) and its frequency satisfies to inequalities $\Omega_i < \omega < \omega_{\mathrm{L}}$.

Assuming damping rate of the mode to be small and using a perturbation technique (see, e.g., [7]), we get

$$\gamma = \frac{c}{h} \frac{\int\limits_{N_{\mathrm{L}}^2>0} dr\, r J_0^2(\kappa r)/N_{\mathrm{L}} + \int\limits_{N_{\mathrm{R}}^2>0} dr\, r J_2^2(\kappa r)/N_{\mathrm{R}}}{\int_0^R dr\, r[J_0^2(\kappa r) + J_2^2(\kappa r)]} \tag{18}$$

for the $m = 1$ mode given by Eq. (16) with the lower sign. Integration in the numerator of (18) goes on that part of the window where corresponding refractive index is real. Using definition $Q = 2\gamma/\omega$ and substituting ω with $\omega \approx \pi c l/h$ we find the cavity's Q:

$$Q = \frac{2}{\pi l} \frac{\int\limits_{N_{\mathrm{L}}^2>0} dr\, r J_0^2(\kappa r)/N_{\mathrm{L}} + \int\limits_{N_{\mathrm{R}}^2>0} dr\, r J_2^2(\kappa r)/N_{\mathrm{R}}}{\int_0^R dr\, r[J_0^2(\kappa r) + J_2^2(\kappa r)]}. \tag{19}$$

According to our assumption, N_{L} in the above formulae is imaginary almost everywhere in the window,[1] therefore first term in the numerator must be omitted. Other term, containing N_{R}, is small since $J_2(\kappa r)$ tends to zero near the cavity's axis $r = 0$.

5. Collisional dissipation of energy

If rf field does not penetrate into the plasma core it nevertheless dissipates its energy in the plasma sheath. It is customary [8,2] to compute the rf dissipation in the plasma

[1] Plasma density depends on the radius so the refractive indices $N_{\mathrm{L,R}}$ do.

by using a model which is only valid for weak fields which do not appreciably alter the presumed Maxwellian distribution of the sheath. Motz and Watson [6, p. 237] note the absence of an applicable high-field theory and conclude that the weak-field computation "probably enormously exaggerates the heating effect. In this Section we extend the theory of Coulomb collisions to the case when oscillation velocity of plasma electrons under the action of the rf field is as large as their thermal velocity. We will see that the above cited prediction of Motz and Watson is mainly confirmed. Our calculations indicate that dissipated power almost does not depend on plasma temperature and that external magnetic field slightly decreases Coulomb dissipation.

5.1. Unmagnetized plasma

We first consider plasma without external magnetic field, $H = 0$. We also notice that in order to calculate local deposition of rf power one can treat both the plasma density n and rf field amplitude E_0 to be homogeneous since a particle excursion v/ω for the period $2\pi/\omega$ of rf oscillations is small as compared with the penetration length $c/\omega|N_{\mathrm{L,R}}|$ of the rf field into the plasma as long as the particle motion is non-relativistic, $v \ll c$. Ignoring space dependence of the rf field, we put $\boldsymbol{E} = \boldsymbol{E}_0 \cos\omega t$. As the ions are not effectively involved in high-frequency motion and their thermal velocity is much less than that of the electrons we assume the ions to be unmovable. In contrast to the case of stationary current in plasma, dissipation of the oscillatory motion of electrons in the rf field occurs mainly due to the electrons scattering on the ions. One can show that contribution of the electron-electron collisions into the dissipation is small provided that frequency ω of the field is much greater that the frequency of collisions ν_{ei}. We assume the inequality

$$\omega \gg \nu_{\mathrm{ei}} \tag{20}$$

to be the case under consideration.

Motion of an electron is described by the equation

$$m_e \frac{d\boldsymbol{v}}{dt} = -e\boldsymbol{E} - \frac{4\pi\Lambda e^2 {e_i}^2 n_i}{m_e v^3}\boldsymbol{v} \tag{21}$$

where Λ is the Coulomb logarithm, and other notations are standard. Though, as we said above, the electron-electron collisions almost do not contribute to the rf power dissipation, they play very significant role. In particular, they preserve the shape of electron distribution function to be of shifted Maxwellian type:

$$f_e(t,\boldsymbol{v}) = \frac{n_e}{\pi^{3/2} v_{Te}^3} \exp\left[-\frac{(\boldsymbol{v} - \tilde{\boldsymbol{v}}(t))^2}{v_{Te}^2}\right] \tag{22}$$

where $\tilde{\boldsymbol{v}}(t)$ is the oscillatory velocity of the plasma electrons. Multiplying the equation (21) by the distribution function (22) and making integration over the velocity $\boldsymbol{v}$, we get the equation for $\tilde{\boldsymbol{v}}$

$$n_e m_e \frac{d\tilde{\boldsymbol{v}}}{dt} = -en_e \boldsymbol{E} - \frac{8\pi\Lambda e^2 {e_i}^2 n_e n_i}{m_e v_{Te}^2} G\left(\frac{\tilde{v}}{v_{Te}}\right)\frac{\tilde{\boldsymbol{v}}}{\tilde{v}}. \tag{23}$$

Here

$$G(\xi) = \frac{2}{\sqrt{\pi}\xi^2}\int_0^\xi du\, u^2\, \mathrm{e}^{-u^2} = \frac{\mathrm{erf}(\xi) - \xi\,\mathrm{erf}'(\xi)}{2\xi^2} \tag{24}$$

is Chandrasekhar's function. Multiplying (23) by $\tilde{\boldsymbol{v}}$ yields the equation of energy balance

$$n_e \frac{d}{dt}\frac{m_e \tilde{v}^2}{2} = \boldsymbol{j}\boldsymbol{E} - \frac{8\pi\Lambda e^2 {e_i}^2 n_e n_i}{m_e v_{Te}^2} G(\tilde{v}/v_{Te})\tilde{v}. \tag{25}$$

In a steady state, the time average of the left-hand side is equal to zero. Hence, the power $q = \langle \boldsymbol{j}\boldsymbol{E}\rangle$ dissipated in a unit volume of plasma is equal to

$$q = \frac{8\pi\Lambda e^2 {e_i}^2 n_e n_i}{m_e v_{Te}^2}\langle \tilde{v} G(\tilde{v}/v_{Te})\rangle \tag{26}$$

where $\langle \ldots \rangle$ stands for the time average

$$\frac{\omega}{2\pi}\int_0^{2\pi/\omega}(\ldots)dt.$$

Since collisions frequency is small compared to the field frequency ω, we can substitute $\tilde{v}(t)$ with

$$\tilde{v} = -\frac{eE_0}{m_e\omega}\sin\omega t,$$

which is derived from the equation (23) without last term in the right-hand side. Thus, we get

$$q = \frac{8\pi\Lambda e^2 {e_i}^2 n_e n_i}{m_e v_{Te}} g\left(\frac{eE_0}{m_e\omega v_{Te}}\right), \tag{27}$$

where

$$\begin{aligned} g(\xi) &= \frac{1}{2\pi}\int_0^{2\pi} d\tau\, \xi\, \sin\tau\, G(\xi\sin\tau) \\ &= \frac{4}{\pi^{3/2}\xi}\int_0^{\pi/2}\frac{d\tau}{\sin\tau}\int_0^{\xi\sin\tau} du\, u^2 \mathrm{e}^{-u^2}. \end{aligned} \tag{28}$$

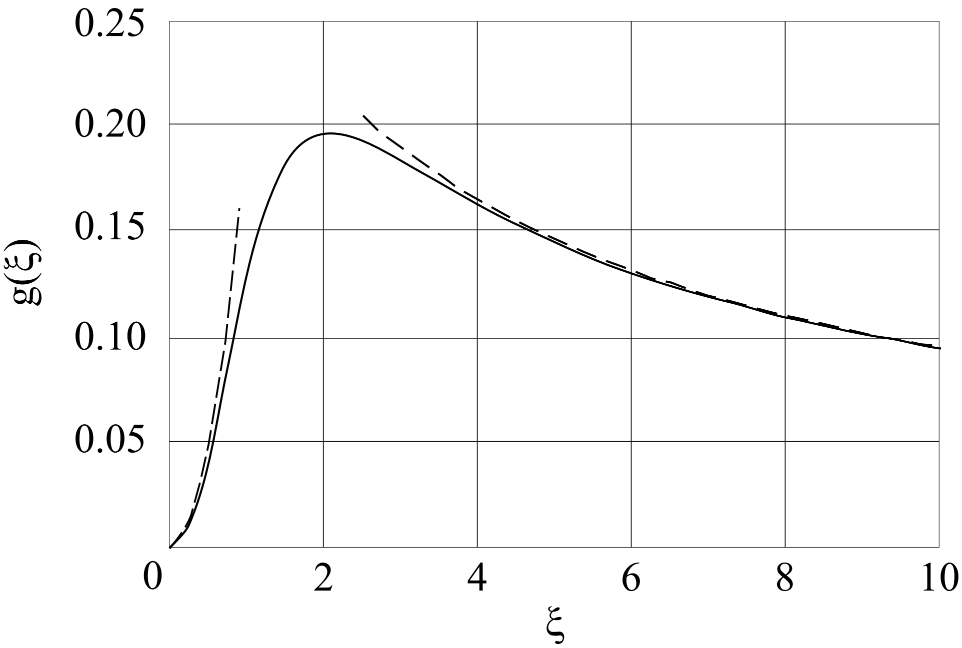

Figure 8: Plot of function $g(\xi)$. Dotted lines show its asymptotics (29) and (32) for small and large ξs respectively.

The function $g(\xi)$ is plotted on Fig. 8. If $\xi \ll 1$ (i.e., $\tilde{v} \ll v_{Te}$), the function $g(\xi)$ reduces to

$$g(\xi) \approx \frac{\xi^2}{3\sqrt{\pi}}. \tag{29}$$

It yields well known result (cf. [9, §4.6])

$$q = \nu_{\rm ei}\, \frac{\omega_{pe}^2}{\omega^2}\, \frac{E_0^2}{8\pi} \tag{30}$$

with

$$\nu_{\rm ei} = \frac{4}{3}\sqrt{\frac{2\pi}{m_e}}\,\frac{\Lambda e^2 {e_i}^2 n_i}{{T_e}^{3/2}}. \tag{31}$$

being the frequency of electron-ions collisions.

In the opposite case of large ξ, we come to

$$g(\xi) \approx \frac{\ln(2\xi)}{\pi\xi}, \tag{32}$$

and (27) can be cast into the form

$$q = \frac{8\Lambda e^2 {e_i}^2 n_e n_i \omega}{eE_0}\, \ln\left(\frac{2eE_0}{m_e \omega v_{Te}}\right). \tag{33}$$

The power absorption is inversely proportional to the rf amplitude E_0. It logarithmically depends on electron temperature.

5.2. Magnetized plasma

Consider an rf oscillation with arbitrary polarization $\boldsymbol{E}$ excited in a plasma immersed into the steady-state-magnetic field $\boldsymbol{H}$, directed along the axis z. We shall not assume that magnetic field $\boldsymbol{B}$ of rf oscillations is smaller than the steady-state magnetic field $\boldsymbol{H}$ allowing for arbitrary ratio B/H. Nevertheless we neglect rf part $\frac{e}{c}[\boldsymbol{v}, \boldsymbol{B}]$ of the Lorentz's force acting on a plasma electron as it is small in comparison with electric force $e\boldsymbol{E}$ as far as $v \ll c$. Then the motion of an electron is governed by the equation

$$\dot{\boldsymbol{v}} = -\frac{e}{m_e}\boldsymbol{E} - [\boldsymbol{v}, \boldsymbol{\Omega}_e] - \frac{4\pi\Lambda e^2 e_i^2 n_i}{m_e v^3}\,\boldsymbol{v} \tag{34}$$

where $\boldsymbol{\Omega_e} = |e|\boldsymbol{H}/m_e c$.

Arbitrary rf field has elliptical polarization, therefore in the most general case we can set

$$\begin{aligned} E_x(t) &= \hat{E}_x \cos\omega t, \\ E_y(t) &= \hat{E}_y \sin\omega t, \\ E_z(t) &= \hat{E}_z \cos(\omega t + \psi). \end{aligned} \tag{35}$$

Relations between the amplitudes $\hat{E}_x$, $\hat{E}_y$, $\hat{E}_z$, and the phase shift ψ follow from solution of an appropriate dispersion problem. For example, circular waves propagating along the external magnetic correspond to $\hat{E}_x = \pm\hat{E}_y$, $\hat{E}_z = 0$.

As far as $\nu_{\rm ei} \ll \omega$ one can use, with minor amendments, the approach developed in the previous subsection. In particular, distribution function of electrons keeps the form of "shifted Maxwellian" distribution (22) where oscillatory velocity $\tilde{\boldsymbol{v}}$ is to be found from (34) without last (i.e. collisional) term:

$$\begin{aligned} \tilde{v}_x &= -\frac{e}{m_e}\frac{\omega\hat{E}_x + \Omega_e\hat{E}_y}{\omega^2 - \Omega_e^2}\sin\omega t &&= A\sin\omega t, \\ \tilde{v}_y &= \frac{e}{m_e}\frac{\Omega_e\hat{E}_x + \omega\hat{E}_y}{\omega^2 - \Omega_e^2}\cos\omega t &&= B\cos\omega t, \\ \tilde{v}_z &= -\frac{e\hat{E}_z}{m_e\omega}\sin(\omega t + \psi) &&= C\sin(\omega t + \psi). \end{aligned} \tag{36}$$

The vector $\tilde{\boldsymbol{v}}(t)$ draws an ellipsis with the half-axes

$$v_{\max} = \sqrt{\frac{1}{2}(A^2+B^2+C^2) + \sqrt{(A^2-B^2)^2 + 2C^2(A^2-B^2)\cos 2\psi + C^4}},$$
$$v_{\min} = \sqrt{\frac{1}{2}(A^2+B^2+C^2) - \sqrt{(A^2-B^2)^2 + 2C^2(A^2-B^2)\cos 2\psi + C^4}}. \tag{37}$$

Its absolute magnitude is

$$\tilde{v}(t) = \sqrt{v_{\max}^2 \cos^2 \omega t + v_{\min}^2 \sin^2 \omega t}. \tag{38}$$

Notice that the magnitude of $\tilde{v}$ is finite even for $\omega = \Omega_e$ if one takes into account the relations between components of polarization vector $\boldsymbol{E}$ which follow from solution of appropriate dispersion problem.

After straightforward calculations we obtain

$$q = \frac{8\pi\Lambda e^2 {e_i}^2 n_e n_i}{m_e v_{Te}} g\left(\frac{v_{\max}}{v_{Te}}, \frac{v_{\min}}{v_{Te}}\right) \tag{39}$$

where

$$g(\xi,\eta) = \frac{1}{2\pi}\int_0^{2\pi} d\tau \sqrt{\xi^2\cos^2\tau + \eta^2\sin^2\tau}\, G\left(\sqrt{\xi^2\cos^2\tau + \eta^2\sin^2\tau}\right). \tag{40}$$

Surface plot of $g(\xi,\eta)$ is shown on Fig. 9.

If $\xi \ll 1$ and $\eta \ll 1$, one can use the approximation similar to (29):

$$g(\xi,\eta) \approx \frac{1}{3\sqrt{\pi}}(\xi^2 + \eta^2). \tag{41}$$

In this case our calculation recovers the result of linear theory (cf. [9, §5.6]):

$$q = \frac{\omega_{pe}^2 \nu_{\text{ei}}}{8\pi}\left\{\left(\frac{\omega \hat{E}_x + \Omega_e \hat{E}_y}{\omega^2 - \Omega_e^2}\right)^2 + \left(\frac{\Omega_e \hat{E}_x + \omega \hat{E}_y}{\omega^2 - \Omega_e^2}\right)^2 + \frac{\hat{E}_z^2}{\omega^2}\right\}. \tag{42}$$

If $\xi \gg 1$ but $\eta \ll 1$ or, vise versa, $\xi \ll 1$ but $\eta \gg 1$ the asymptotics (32) gives correct result (with additional substitution ξ with η for the latter case).

For the special case $\xi \approx \eta \gg 1$ that stands for almost circular rf wave, one can approximate $g(\xi,\eta)$ with

$$g(\xi,\xi) \approx 1/2\xi. \tag{43}$$

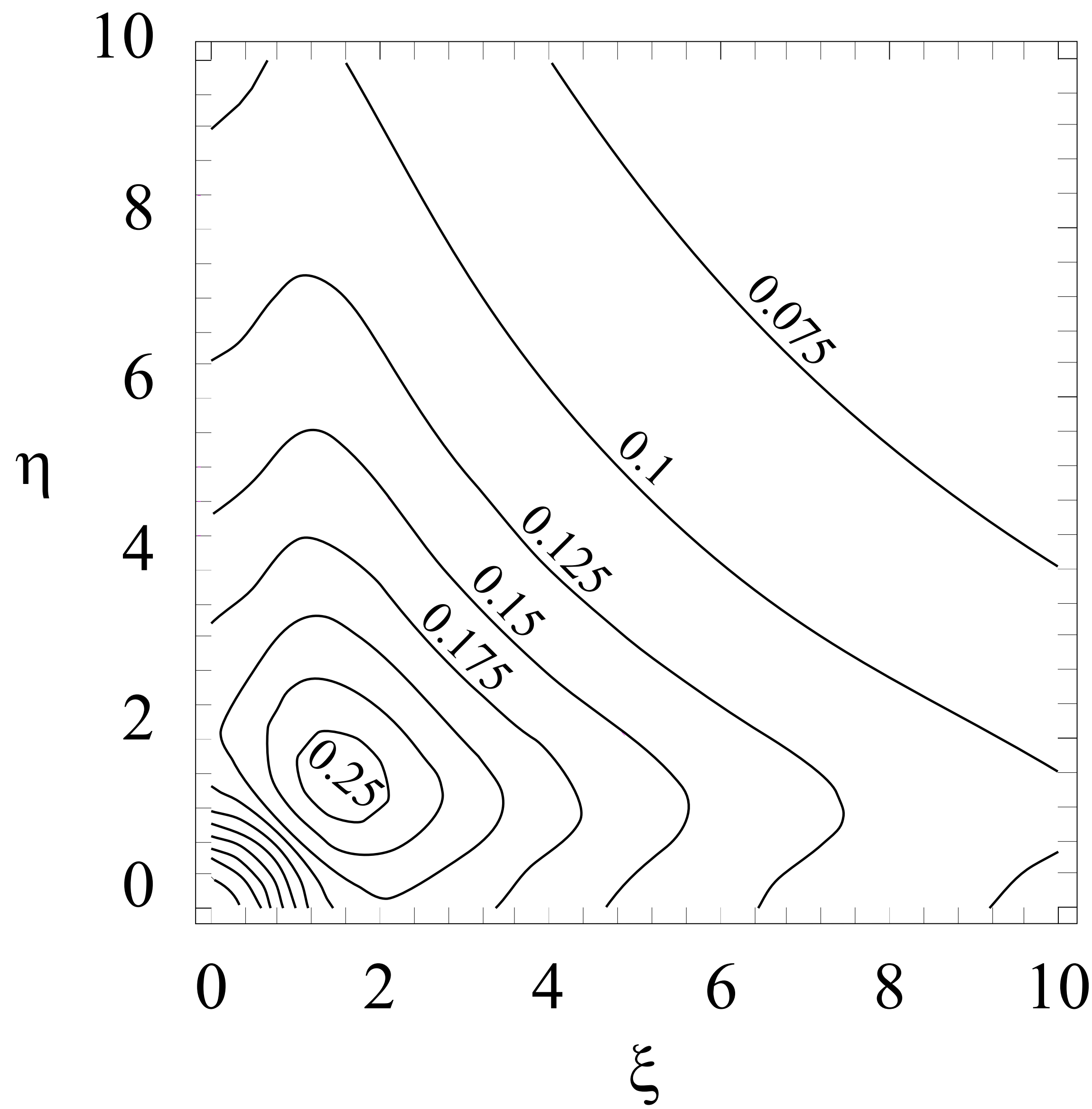

Figure 9: Surface plot of function $g(\xi,\eta)$.

Coulomb dissipation of circular wave decreases inversely proportional to its amplitude as it does in the case of linearly polarized wave propagating in unmagnetized plasma (considered in previous Section) but now relatively large logarithmic factor $\ln 2\xi$ is absent.

Having calculated the power q absorbed in an unit volume of the plasma, we can estimate total power absorption P in the plasma boundary sheath as the product of q by the penetration length λ of the rf field into the plasma and by the plasma cross section πa^2, $P \sim \pi a^2 \lambda q$. The penetration length is equal to $c|N_{\mathrm{R,L}}|/\omega$ provided that $N_{\mathrm{R,L}}$ is imaginary, i.e., rf field does not penetrate into the plasma core.

To evaluate q we note that required amplitude of rf field E must be of order of the steady-state magnetic field H as discussed in Sec. 2. For the frequency range ω close to the electron cyclotron frequency Ω_e the ordering $E \sim H$ leads to the conclusion that electron's motion becomes relativistic since $\tilde{v} \sim eE/m_e\omega \sim c$. Though our theory is not applicable to relativistic motion, we can use it just to evaluate dissipation at the plasma sheath in order of magnitude. For $\xi \sim \eta \sim c/v_{Te}$, $N_{\mathrm{R,L}} \sim 1$ we obtain from (39) and (43) that

$$P \sim \frac{8\pi\Lambda e^4 n^2}{m_e v_{Te}} \frac{v_{Te}}{2c} \frac{c}{\omega} \pi a^2 .$$

Cavity's Q is equal to $\omega W/P$ with $W = VB^2/8\pi$ being the energy, stored in the volume V of the cavity. Combining all together, we obtain

$$Q \sim \frac{2}{\Lambda\beta\tilde{\beta}} \left(\frac{T}{m_e c^2}\right)^2 \frac{V}{\pi a^2 r_e}$$

where $r_e = e^2/m_e c^2$ is the classical radius of electron, and ω has been substituted by Ω_e. For $\tilde{\beta} \sim \beta \sim 1$, $T = 10\,\mathrm{keV}$, $\Lambda = 15$, $V/\pi a^2 \sim 10\,\mathrm{cm}$ calculated resonator cavity's $Q = 2{\cdot}10^9$ satisfies the requirement formulated in [1] for Q to be larger than 10^9. This estimation indicate principal feasibility of rf plugging of plasma in open systems provided that technical solution to sustain rf mode specified in Sec. 4 will be found.

6. Conclusions

We considered two basic mechanism of damping of the rf field in a resonant cavity attached to the mirror ends of an open system for plasma confinement. In particular, we have calculated the damping rate of the rf field due to Coulomb collisions in the sheath at the plasma boundary for arbitrary large amplitude of rf field. We concluded that the power absorption in the plasma sheath due to collisional dissipation decreases to a suitable level as the rf field increases.

We also found that possible leakage of the rf power through the plasma waveguide imposes severe restrictions on the choice of the rf mode to be excited in the resonant cavities. We note that the rf frequency should be too high in order to provide conditions where rf field does not penetrate into the plasma core. Attempts to lower the frequency (and consequently, to lower the mode number to be excited) may exploit two ideas. One of them is to decrease the window in the cavity's wall for plasma. We have shown that the rf power escaping from the cavity decreases as the size of the window decreases. Other possibility is to decrease the radius of the plasma waveguide. A rf oscillation does not penetrate into the waveguide if its wavelength is larger than the waveguide radius a, i.e. if $cN/\omega > a$. However, simple estimation shows that this requirements leads to the inequality $c/\omega_{pi} > a$ which limits a to few centimeters for plasma with "thermonuclear" density of order of $10^{14}\ \mathrm{cm}^{-3}$. However more detailed analysis of the rf field penetration into the plasma waveguide is required. It may reveal that the modes with frequencies of order of $(0.01 \div 0.1)\Omega_e$ may provide suitable level of the rf power losses. Another anxious problem is the calculation of the rf field absorption due to wave transformation in imhomogeneous plasma.

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