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PRODUCTION OF POLARIZED POSITRONS IN INTERACTION OF HIGH-ENERGY ELECTRONS WITH LASER WAVE

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Production of polarized positrons in interaction of high-energy electrons with laser wave

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Abstract

Creation of polarized positrons is considered in two-step process of interaction of unpolarized high-energy electrons with circularly polarized soft (laser) photon. The first step is the Compton scattering in which high-energy circularly polarized photon appears. The second step is pair creation in subsequent interaction of this photon with another circularly polarized laser photon. Direct electroproduction of electron-positron pair in interaction of high-energy electron with laser photon (trident production) is considered also. It is shown that high degree of the longitudinal polarization of created positrons can be obtained. An analysis is carried out in the Born approximation.

1 Introduction

Projects of electron-positron linear colliders with the energies of the order of TeV are now being under discussion in several laboratories. For a program of physics research with such collider it will be quite important to have opportunity to work with longitudinally polarized particles.

The are a few proposals to obtain polarized positrons:

1. Longitudinally polarized positrons are created in thin target by circularly polarized photons radiated from high-energy electrons in an appropriate undulator [1], [2].

2. Longitudinally polarized positrons are created at collision of high energy photon with circularly polarized laser photon. A radiation of high-energy electrons in oriented crystals is proposed as a source of high-energy photons [3].

In the present paper creation of longitudinally polarized positrons in interaction of high-energy electrons with photons of circularly polarized laser wave is proposed. We consider two-step process: the first stage is the Compton scattering of circularly polarized soft photon on high-energy unpolarized electron with creation of high-energy partially circularly polarized photon; the second stage is pair creation in interaction of this photon with circularly polarized soft photon from laser wave. Direct electroproduction of electron-positron pair (where a positron is polarized) in interaction of a high-energy electron with a circularly polarized laser photon (trident production) is considered also using the equivalent photon method.

For unpolarized particles such two-step (or cascade) process in an external field [4] and in laser wave [5] was recently considered.

2 Cross sections of basic processes

For convenience, the cross sections of basic processes with polarization under discussion will be presented first. Let for the Compton scattering k and p_c are the initial 4-momenta of a photon and an electron respectively and k' and p_c' are their final momenta, so that $p_c + k = p_c' + k'$.

Let us introduce invariant variables $x_c = s_c/m^2 - 1$, $y_c = 1 - u_c/m^2$; $s_c = (p_c + k)^2$, $u_c = (p_c - k')^2$. The covariant form of description of photon polarization is given in detail in [6]. Polarization effects in Compton scattering have been analyzed in many papers, see e.g. references in [6]. The complete set of polarization effects, which are written down in covariant form, has been calculated recently in [7], where the method of [6] was used for description of photon polarization.

The cross section of Compton scattering for unpolarized electrons can be written in the form [7] (this cross section can be found also in [6]):

$$\frac{d\sigma_c}{dy_c\,d\phi} = \frac{\alpha^2}{2m^2 x_c^4 y_c^2} \sum_{jj'} R_{0j}^{0j'}(x_c, y_c)\xi_{cj}\xi'_{cj'}, , \tag{1}$$

where ξ_{cj} and $\xi'_{cj'}$ are the Stokes parameters of the initial and the final photons. Note that the Stokes parameters describing linear polarization are defined in [6] and [7] with opposite signs. In (1) summation over final photon polarization is not carried out, so for the unpolarized final photon: $d\sigma = \frac{1}{2} d\sigma_{unpol}$. The right-hand side depends on ϕ because the polarizations are defined relative to the scattering plane, After integration over angle ϕ dependence on linear polarizations vanishes.

The final photon polarization is

$$\xi'_{cj'} = \frac{1}{R} \sum_j R_{0j}^{0j'}(x_c, y_c)\xi_{cj}, \tag{2}$$

where $R = R_{00}^{00}(x_c, y_c)$.

Components $R_{0j}^{0j'}(x_c, y_c)$ depending on photon's polarizations are presented below:

$$\begin{aligned}
R_{00}^{00}(x_c, y_c) &= x_c^3 y_c - 4x_c^2 y_c + 4x_c^2 + x_c y_c^3 + 4x_c y_c^2 - 8x_c y_c + 4y_c^2, \\
R_{00}^{03}(x_c, y_c) &= -4(x_c - y_c)(x_c y_c - x_c + y_c), \\
R_{01}^{01}(x_c, y_c) &= 2x_c y_c \left(x_c y_c - 2x_c + 2y_c\right), \\
R_{02}^{02}(x_c, y_c) &= \left(x_c^2 + y_c^2\right)\left(x_c y_c - 2x_c + 2y_c\right), \\
R_{03}^{03}(x_c, y_c) &= 2\left(x_c^2 y_c^2 - 2x_c^2 y_c + 2x_c^2 + 2x_c y_c^2 - 4x_c y_c + 2y_c^2\right)
\end{aligned} \tag{3}$$

Now the process $\gamma\gamma \to e^+e^-$ will be discussed [8]. The initial photons momenta are k, k' and momenta of created electron and positron are p and p', so that $k + k' = p + p'$. The Mandelstam invariants are $t = (k - p)^2 = m^2(1 - x)$, $u = (k - p')^2 = m^2(1 - y)$, $s = m^2(x + y)$.

It is convenient to use the basis

$$n_0 = \frac{Q}{v}, \quad n_1 = \frac{K}{v}, \quad n_2 = \frac{v}{2w} P_\perp, \quad n_3^\mu = -\frac{1}{2vw} \varepsilon^\mu{}_{\alpha\beta\gamma} Q^\alpha K^\beta P^\gamma, \tag{4}$$

where $Q = k + k' = p + p'$, $K = k - k'$, $P = p' - p$, $P_\perp = P - \frac{PK}{K^2}K$; $v = \sqrt{x + y}$, $w = \sqrt{xy - x - y}$. Then the particles' momenta are

$$\begin{aligned}
p &= \frac{(x + y)n_0 - (x - y)n_1 - 2wn_2}{2v}, \quad p' = \frac{(x + y)n_0 + (x - y)n_1 + 2wn_2}{2v}, \\
k &= \frac{v}{2}(n_0 + n_1), \quad k' = \frac{v}{2}(n_0 - n_1).
\end{aligned} \tag{5}$$

The vectors $\mathbf{n}_1$, $\mathbf{n}_2$, $\mathbf{n}_3$ form a right–handed system.

The vectors n_2 and n_3 can be used as polarization vectors of both photons. For the photon with momentum k, the vectors $\mathbf{n}_2$, $\mathbf{n}_3$, $\mathbf{k}$ form a right–handed system (in the c. m. frame).

The electron and positron density matrices are $\rho = \frac{1}{2}(\hat{p} - m)(1 - \gamma_5 \hat{a})$ and $\rho' = \frac{1}{2}(\hat{p}' + m)(1 - \gamma_5 \hat{a}')$. Let's introduce two bases

$$\begin{aligned} e_0 &= \frac{p}{m}, \quad e_1 = \frac{(x+y-2)p - 2p'}{muv}, \quad e_2 = \frac{2wn_1 - (x-y)n_2}{uv}, \quad e_3 = n_3; \\ e_0' &= \frac{p'}{m}, \quad e_1' = \frac{(x+y-2)p' - 2p}{muv}, \quad e_2' = \frac{2wn_1 + (x-y)n_2}{uv}, \quad e_3' = n_3; \end{aligned} \tag{6}$$

where $u = \sqrt{x+y-4}$. Then $a = \sum_{i=1}^{3} \zeta_i e_i$, where in c. m. frame ζ_1 is the longitudinal polarization, ζ_2 is the transverse polarization in the reaction plane, and ζ_3 is the transverse polarization perpendicular to this plane. Introducing formally $\zeta_0 = 1$, we have $\rho = \frac{1}{2}\sum_{i=0}^{3} \zeta_i \rho_i$, where $\rho_0 = \hat{p} - m$, $\rho_i = -\rho_0 \gamma_5 \hat{e}_i$. Similarly, $\rho' = \frac{1}{2}\sum_{i'=0}^{3} \zeta_{i'}' \rho_{i'}'$, where $\rho_0' = \hat{p}' + m$, $\rho_i' = -\rho_0' \gamma_5 \hat{e}_i'$.

The cross section of the process $\gamma\gamma \to e^+e^-$ may be written in the form

$$\frac{d\sigma_p}{dt\,d\varphi} = \frac{\alpha^2}{4s^2x^2y^2} \sum_{ii'jj'} F_{jj'}^{ii'}(x,y)\xi_j \xi_{j'}' \zeta_i \zeta_{i'}', \tag{7}$$

The right–hand side of (7) depends on φ because the polarizations are defined relative to the reaction plane. The final particles' polarizations ζ_i, $\zeta_{i'}'$ describe probabilities of their registration by the detector; when they are absent, $d\sigma = \frac{1}{4} d\sigma_{\text{unpol}}$ [6]. The cross section summed over the final particles' polarizations is

$$\frac{d\sigma_p}{dt\,d\varphi} = \frac{\alpha^2}{s^2x^2y^2} F, \quad F = \sum_{j'j} F_{jj'}^{00}(x,y)\xi_j \xi_{j'}'. \tag{8}$$

Polarizations of the final particles themselves are

$$\zeta_i^{(f)} = \frac{1}{F}\sum_{jj'} F_{jj'}^{i0}(x,y)\xi_j \xi_{j'}', \quad \zeta_{i'}^{(f)\prime} = \frac{1}{F}\sum_{jj'} F_{jj'}^{0i'}(x,y)\xi_j \xi_{j'}'. \tag{9}$$

The four–vectors of the final particles' polarization are evidently $a = \sum_{i=1}^{3} \zeta_i e_i$, $a' = \sum_{i'=1}^{3} \zeta_{i'}' e_{i'}'$.

The components $F_{jj'}^{ii'}(x,y)$ needed for our problem are presented below (where the notation $\overline{F}_{jj'}^{ii'}(x,y) = F_{jj'}^{ii'}(y,x)$ is used).

$$\begin{aligned} F_{00}^{00}(x,y) &= x^3y + 4x^2y - 4x^2 + xy^3 + 4xy^2 - 8xy - 4y^2, \\ F_{03}^{00}(x,y) &= F_{30}^{00}(x,y) = 4v^2w^2, \\ F_{02}^{01}(x,y) &= \overline{F}_{20}^{01}(x,y) = -\left(x^2y - 2x^2 - xy^2 - 2xy + 4x + 4y\right) v^3/u, \\ F_{02}^{02}(x,y) &= -F_{20}^{02}(x,y) = 4v^2w^3/u, \\ F_{22}^{00}(x,y) &= -\left(x^2 + y^2\right)\left(xy - 2x - 2y\right), \\ F_{23}^{01}(x,y) &= 4v^3w^2/u, \\ F_{23}^{02}(x,y) &= -2\left(y - 2\right) v^4w/u, \end{aligned} \tag{10}$$

3 Cascade creation of the longitudinally polarized positrons

Using probabilities given in previous section (probability of a process is dW, $d\sigma = \frac{dW}{J}$, J is the flow of the initial particles, in our case $J = 1 + v_z \simeq 2$, v_z is the velocity of the initial electron) as kernels of corresponding kinetic equation one can calculate characteristics of a cascade caused by a initial high-energy electron.

Here the method of successive approximations will be used. This method, generally speaking, is applicable if the total probability of cascade is relatively small. In this case for probability of cascade electroproduction one has

$$\frac{dw_{cas}}{d\omega'} = \frac{dW_c}{d\omega'}\left[L - \frac{1}{W_p(\omega')}\left(1 - \exp(-W_p(\omega')L)\right)\right], \tag{11}$$

where ω' is the energy of the final photon in the Compton effect, $\frac{dW_c}{d\omega'}$ is the probability of the Compton scattering, $W_p(\omega')$ is the total probability of pair creation, L is the interaction length, notation is used:$dW = \frac{dw}{dL}$. When $W_p(\omega')L \ll 1$ one can expand exponent in (11):

$$\frac{dw_{cas}}{d\omega'} = \frac{1}{2}\frac{dW_c}{d\omega'}LW_p(\omega', \xi_2 = \xi'_{c2})L, \quad \frac{dn_c}{d\omega'} = \frac{dW_c}{d\omega'}L, \tag{12}$$

where n_c is the total number of final photons in Compton effect.

Let us introduce kinematic variables:

- ω is the energy if the initial photon in the Compton scattering;
- ω' is the energy if the final photon in the Compton scattering;
- ε is the energy if the initial electron in the Compton scattering;
- $\varepsilon_p = \epsilon$ is the energy if the created electron in the pair creation process;
- $\varepsilon_{p'} = \epsilon'$ is the energy if the created positron in the pair creation process;
- $x_c = \lambda = \frac{2kp}{m^2} = \frac{4\varepsilon\omega}{m^2}$ is the energy invariant for Compton scattering;
- $z = \frac{\epsilon'}{\varepsilon}$, $z' = \frac{\omega'}{\varepsilon}$ are the dimensionless variables.

The differential cross sections written down in previous section present spectral distributions over energy of the final particles. Using these distributions, one can obtain the spectral distribution of created positrons in the cascade process. In terms of the introduced variables it has the form:

$$\frac{dw_{cas}}{dz} = \frac{L^2}{2}\int_{z'_1}^{z'_2} dz'\frac{dW_c}{dz'}\frac{dW_p(z', \xi_2 = \xi'_{c2})}{dz}, \tag{13}$$

where

$$z_1' = \frac{z^2\lambda}{z\lambda - 1}, \quad z_2' = \frac{\lambda}{1+\lambda}. \tag{14}$$

Value of z varies within limits: $z_1 \le z \le z_2$, where

$$z_1 = \frac{\lambda - f(\lambda)}{2(\lambda+1)}, \quad z_2 = \frac{\lambda + f(\lambda)}{2(\lambda+1)}, \quad f(\lambda) = \sqrt{\lambda^2 - 4\lambda - 4}. \tag{15}$$

At the threshold of the cascade process $f(\lambda) = 0$, so that $t_{th} = 2(1+\sqrt{2})$ (compare [5]). The limits in (14), (15) follow from simple kinematic consideration.

Substituting into (13) the explicit expressions for probabilities of Compton scattering (1),(3) and pair creation (7),(10) one obtains for the probability of the cascade production of a positron in interaction of a high-energy electron with a soft (laser) photon

$$\frac{dw_{cas}}{dz} = \frac{(2\pi\alpha^2 L)^2}{2m^2\lambda^2}\left(I_0 + \zeta_1'\xi_{c2}I_\zeta\right), \tag{16}$$

where ζ_1' describes probability of registration of the longitudinal polarization of the positron by detector, ξ_{c2} is the circular polarization of the initial soft photon,

$$\begin{aligned} I_0 &= \int_{z_1'}^{z_2'} \frac{dz'}{z'^2}\left[R_{00}^{00}(x_c,y_c)F_{00}^{00}(x,y) + R_{02}^{02}(x_c,y_c)F_{22}^{00}(x,y)\xi_{c2}^2\right], \\ I_\zeta &= \int_{z_1'}^{z_2'} \frac{dz'}{z'^2}\left[R_{02}^{02}(x_c,y_c)F_{02}^{01}(x,y) + R_{00}^{00}(x_c,y_c)F_{20}^{01}(x,y)\right], \end{aligned} \tag{17}$$

here one has to substitute

$$x_c = \lambda, \quad y_c = \lambda(1 - z'), \quad x = \lambda z, \quad y = \lambda(z' - z). \tag{18}$$

We took into account in (16) that $dt = -\lambda dz$.

The longitudinal polarization of the created positron itself is

$$\zeta_1^{(f)\prime} = \frac{I_\zeta}{I_0}\xi_{c2} \tag{19}$$

The results obtained are illustrated in Fig.1-3 for $\lambda = 5$, $\lambda = 8$ and $\lambda = 50$ respectively. Figures (a) are the spectral distributions of the probability of cascade process for unpolarized particles with correlation term of the circular polarizations (for $\xi_{c2}^2 = 1$) of photons (I_0 (17)) in units $\dfrac{(2\pi\alpha^2 L)^2}{2m^2\lambda^2}$. Figures (b) present longitudinal polarization of the created positrons plotted vs z for $\xi_{c2} = 1$. In Fig.1 the situation near threshold of cascade process ($\lambda_{th} = 4.83$) is shown. The maximum of the spectral distribution is near the middle of the available interval of z (15), slightly shifted to the left. There is sizable longitudinal polarization of positrons for high-energy tail of positrons only. In Fig.2 the situation far from threshold of cascade process is shown. The maximum of the spectral distribution is shifted noticeably to the left. There is sizable longitudinal polarization of positrons in the whole interval of z. Especially high degree of polarization is attained both in soft and hard part of the spectrum. In Fig.3 the situation in high-energy region of cascade process is shown. There is pronounced peak in the spectral distribution of created positrons near soft boundary of the spectrum. Here also there is sizable longitudinal polarization of positrons in the whole interval of z. Especially high degree of polarization (up to $\zeta_1^{(f)\prime} = 1$) is attained both in soft and hard part of the spectrum. Collecting positrons created within the peak one obtains polarized beam of positrons.

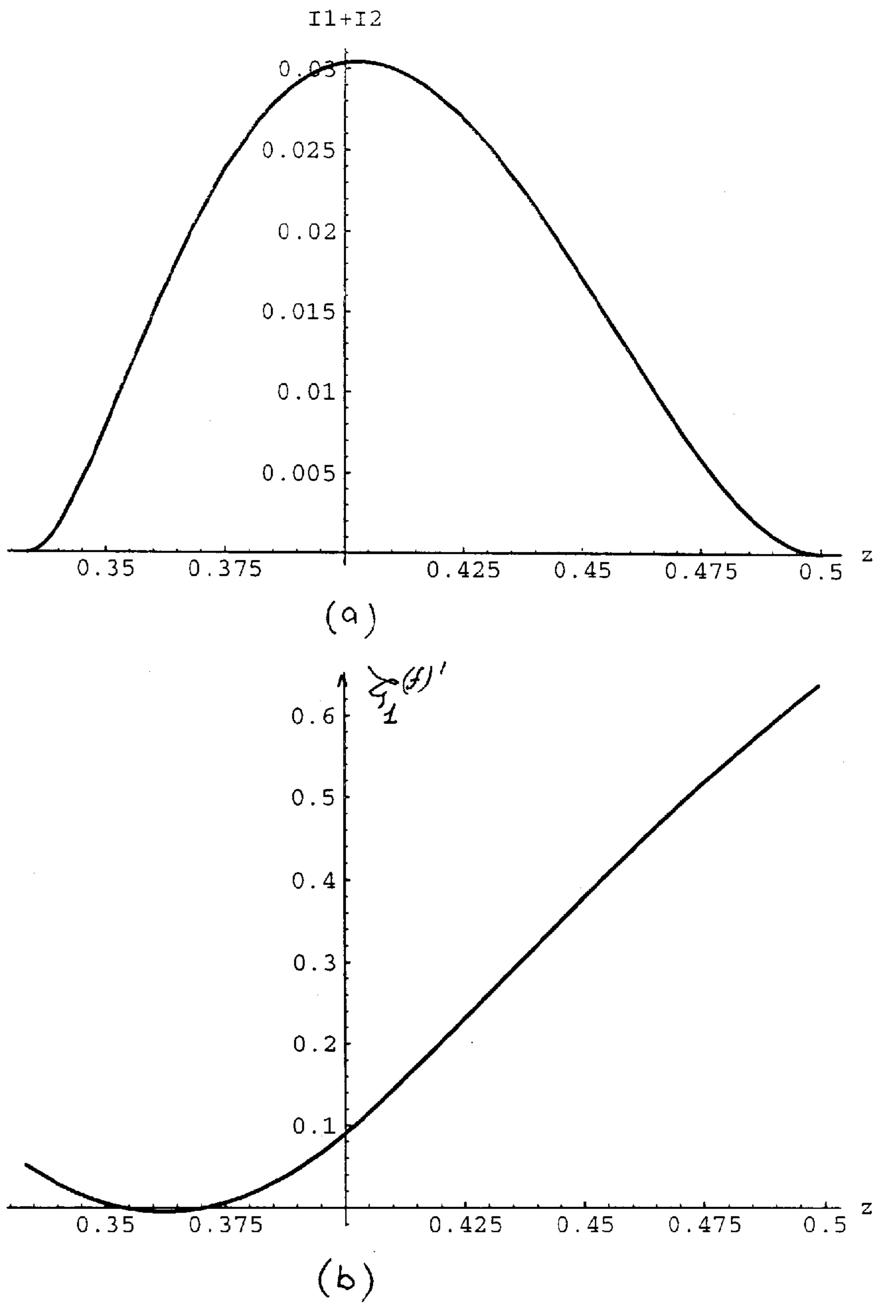

Figure 1: Characteristics of the cascade process for $\lambda = 5$. The spectral distribution of the probability of cascade process for unpolarized particles with correlation term of the circular polarizations (for $\xi_{c2}^2 = 1$) of photons (I_0 in eqs.(16), (17)) in units $\dfrac{(2\pi\alpha^2 L)^2}{2m^2\lambda^2}$ (a). Longitudinal polarization of the created positrons plotted vs z for $\xi_{c2} = 1$ (b).

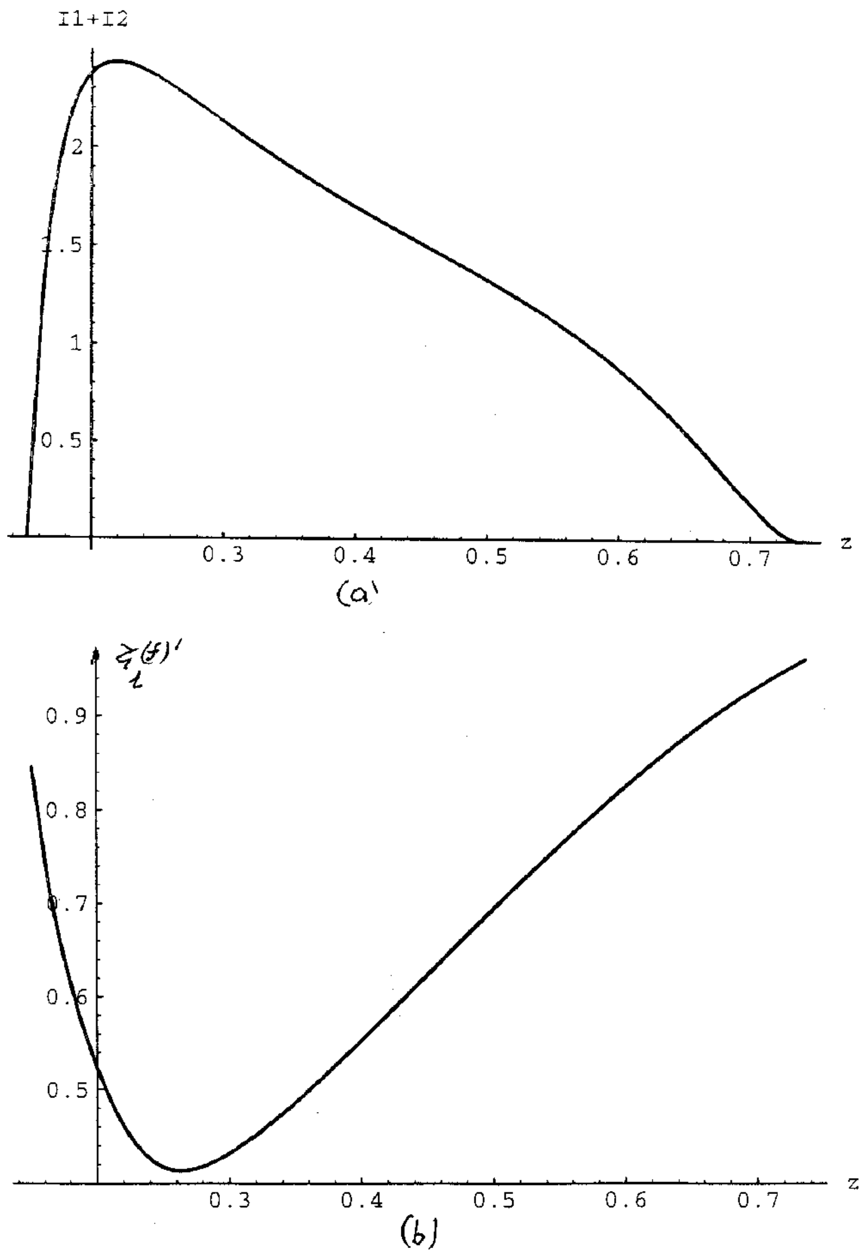

Figure 2: Same as Fig.1 but for $\lambda = 8$.

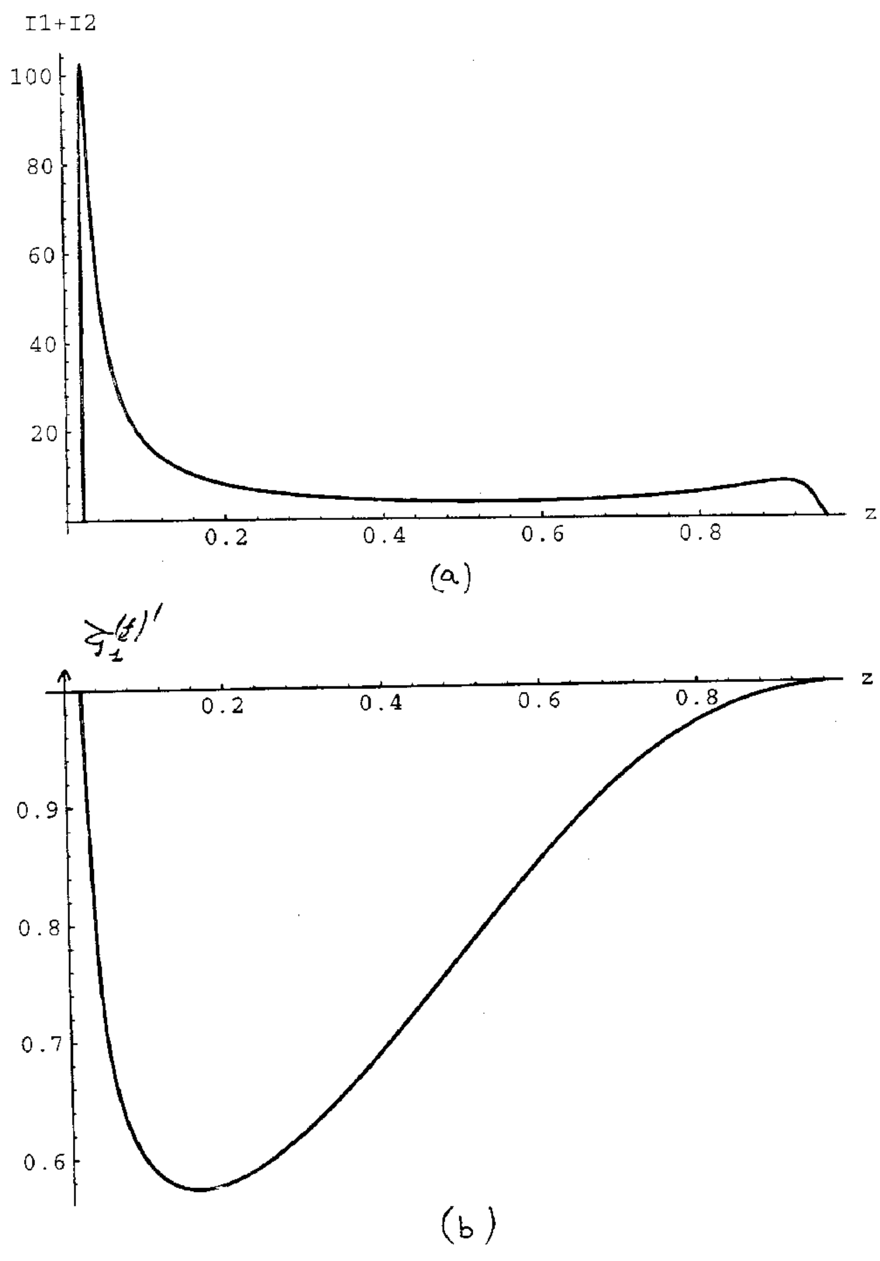

Figure 3: Same as Fig.1 but for $\lambda = 50$.

4 Direct electroproduction of polarized positron

Another mechanism is a direct electroproduction of electron-positron pair. The main contributions give the diagrams shown in Fig.4. Cross section of this process can be obtained using the method of equivalent photons:

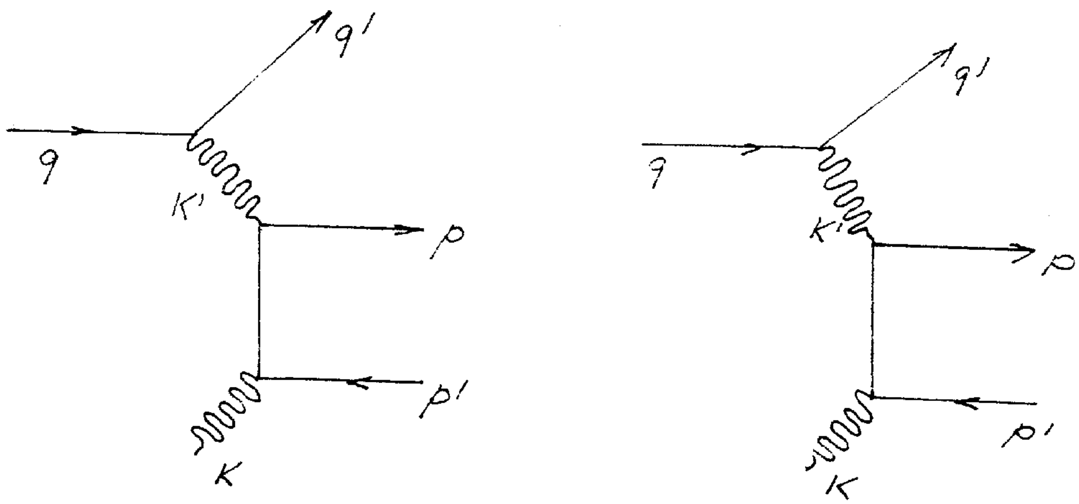

Figure 4: Diagrams of direct electroproduction of electron-positron pair.

$$\frac{d\sigma_{ep}}{dz} = \frac{\alpha^3 \ln \lambda}{m^2 \lambda} \left(J_0 + \zeta_1' \xi_{c2} J_\zeta \right), \tag{20}$$

where ζ_1' describes probability of registration of the longitudinal polarization of the positron by detector, ξ_{c2} is the circular polarization of the initial soft photon,

$$\begin{aligned} J_0 &= \lambda^2 \int_{z_1'}^{z'_{max}} \frac{dz'}{z'} \left(1 - z' + \frac{z'^2}{2} \right) \frac{F_{00}^{00}(x,y)}{s^2 x^2 y^2}, \\ J_\zeta &= \lambda^2 \int_{z_1'}^{z'_{max}} \frac{dz'}{z'} \left(1 - z' + \frac{z'^2}{2} \right) \frac{F_{01}^{02}(x,y)}{s^2 x^2 y^2}, \end{aligned} \tag{21}$$

here

$$\lambda = \frac{2kq}{m^2}, \quad s = \lambda z', \quad x = \lambda z, \quad y = \lambda(z' - z), \quad z'_{max} = 1 - \frac{1}{2\lambda}, \tag{22}$$

and z_1' is defined in (14). The longitudinal polarization of the created positron itself for this case is

$$\zeta_1^{(f)\prime} = \frac{J_\zeta}{J_0} \xi_{c2} \tag{23}$$

Formally, these results are valid when $\ln \lambda \gg 1$. Note, that in (20) summation over final particle polarizations is not carried out.

The results obtained in this section are illustrated in Fig.5-6 for $\lambda = 20$ and $\lambda = 50$ respectively. Figures (a) are the spectral distributions of the probability of direct electroproduction process for unpolarized particles and photons in units $\dfrac{\alpha^3 \ln \lambda}{m^2 \lambda}$ (the term J_0 in eq.(20)). Figures (b) present longitudinal polarization of the created positrons plotted vs z for $\xi_{c2} = 1$. In Fig.5 the situation enough far from threshold ($\lambda_{th} = 8$) of direct electroproduction process is shown. The maximum of the spectral distribution

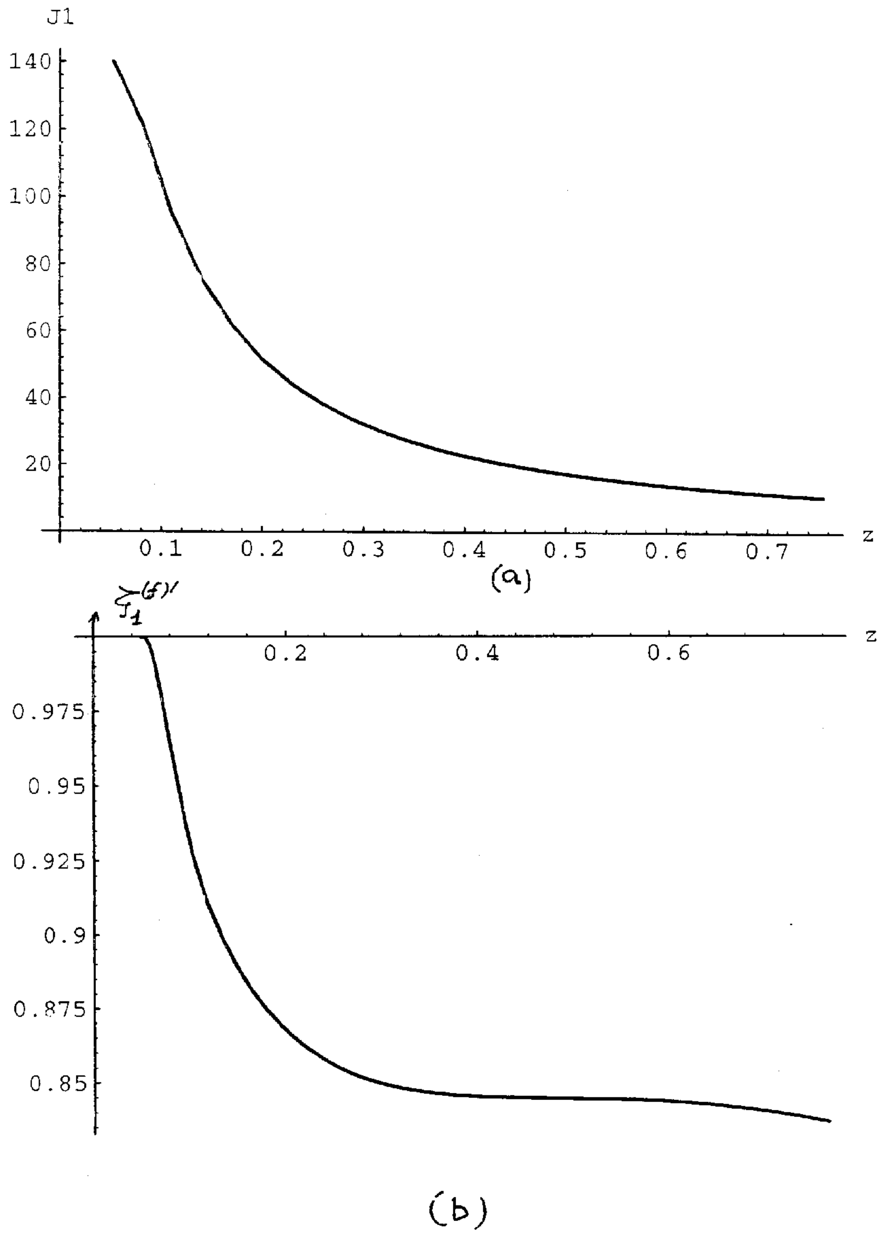

Figure 5: Characteristics of the direct electroproduction process for $\lambda = 20$. The spectral distributions of the probability of direct electroproduction process for unpolarized particles and photons in units $\dfrac{\alpha^3 \ln \lambda}{m^2 \lambda}$ (the term J_0 in eq.(20)) (a). Longitudinal polarization of the created positrons plotted vs z for $\xi_{c2} = 1$ (b).

is lying in the soft part of the spectrum. There is sizable longitudinal polarization of positrons in the region of the peak. Especially high degree of polarization (up to $\zeta_1^{(f)'} = 1$) is attained in soft of the spectrum. In Fig.6 the situation in high-energy region of direct electroproduction process is shown. There is pronounced peak in the spectral distribution of created positrons near soft boundary of the spectrum just as in the case of cascade process. Especially high degree of polarization (up to $\zeta_1^{(f)'} = 1$) is attained in the redion of the peak. Collecting positrons created within the peak one obtains polarized beam of positrons. So, independent of mechanism of positron production created positrons are longitudinally polarized especially in soft part of the spectrum.

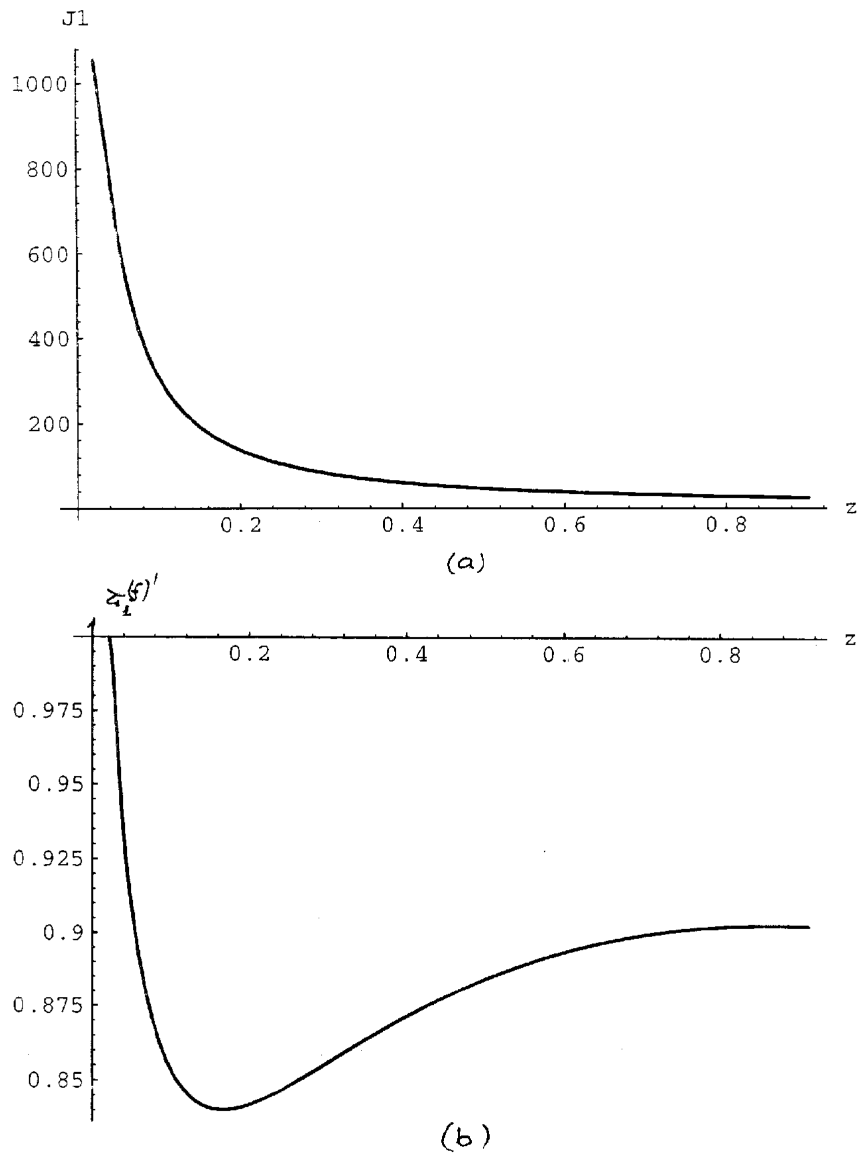

Figure 6: Same as Fig.5 but for $\lambda = 50$.

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