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ON QUANTIZATION OF BLACK HOLES

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On Quantization of Black Holes

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Abstract

A simple argument is presented in favour of the equidistant spectrum in semiclassical limit for the horizon area of a black hole. The following quantization rules for the mass M_N and horizon area A_{Nj} are proposed:

$$M_N = m_p [N(N+1)]^{1/4};$$

$$A_{Nj} = 8\pi l_p^2 \left[\sqrt{N(N+1)} + \sqrt{N(N+1) - j(j+1)} \right].$$

Here both N and j are nonnegative integers or half-integers.

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The quantization of black holes was proposed long ago in the pioneering work [1]. The idea was based on the intriguing observation [2] that the horizon area A of a nonextremal black hole behaves in a sense as an adiabatic invariant. This last fact makes natural the assumption that the horizon area should be quantized. Once this hypothesis is accepted, the general structure of the quantization condition for large (generalized) quantum numbers n gets obvious, up to an overall numerical constant α (our argument here somewhat differs from that of the original paper [1]). The quantization condition should be

$$A_n = \alpha l_p^2 n. \tag{1}$$

Indeed, the presence of the Planck length squared

$$l_p^2 = \frac{G\hbar}{c^3} \tag{2}$$

in formula (1) is only natural. Then, for A to be finite in a classical limit, the power of n in expression (1) should be the same as that of $\hbar$ in l_p^2. The validity of this our argument can be checked by looking at any expectation value nonvanishing in the classical limit in usual quantum mechanics.

From different points of view the black hole quantization was discussed later in Refs. [3, 4]. However, although a lot of work has been done since on the subject (see recent review [5]), still one cannot say that the problem is solved. In particular, there are various prescriptions for the numerical constant α in formula (1), for instance, $\alpha = 4\ln 2$ [3, 5, 6]; $\alpha = 8\pi$ [7]. For $\alpha = 4\ln 2$, the mass spectrum looks as follows:

$$M_n = \sqrt{\frac{\ln 2}{4\pi}}\, m_p \sqrt{n}, \tag{3}$$

and the distance between neighbouring levels is

$$M_n - M_{n-1} = \sqrt{\frac{\ln 2}{\pi}}\,\frac{m_p^2}{4M_n}. \tag{4}$$

The emission spectrum of a black hole becomes discrete with the transition frequencies being multiples of (4). Its envelope corresponds to the Hawking temperature

$$T = \frac{m_p^2 c^2}{8\pi M}, \tag{5}$$

with the usual maximum (for bosons) at $T_{max} = 2.82\,T$.

We will address here the problem starting from the expression for the horizon area of the Kerr black hole, which can be written as

$$A = 8\pi l_p^2 \left[\frac{M^2}{m_p^2} + \sqrt{\frac{M^4}{m_p^4} - j(j+1)}\right]. \tag{6}$$

Here

$$m_p = \left(\frac{\hbar c}{G}\right)^{1/2} \tag{7}$$

is the Planck mass. As to the total angular momentum $\mathbf{J}$ of the black hole, according to the firmly established quantum-mechanical rule for any isolated system, it should be quantized:

$$\mathbf{J}^2 = \hbar^2 j(j+1). \tag{8}$$

As usual, j here is a nonnegative integer or half-integer.

In the case of an extreme Kerr hole, equation (6) leads to the following quantization rule for its mass M_e [8]:

$$M_e = m_p\,[j(j+1)]^{1/4}. \tag{9}$$

Although for the extreme black hole the horizon area does not behave as an adiabatic invariant, it is natural to look for the quantization rule generalizing (9) to nonextremal situations. Rather obvious generalization is:

$$M_N = m_p\,[N(N+1)]^{1/4}; \tag{10}$$

$$A_{Nj} = 8\pi l_p^2 \left[\sqrt{N(N+1)} + \sqrt{N(N+1) - j(j+1)}\,\right]. \tag{11}$$

Here N is a nonnegative integer or half-integer which is the maximum value of j. Of course, the dependence of the area A_{Nj} on two quantum numbers N and j, but not on one, in no way contradicts its adiabatic properties, which are so crucial for the whole problem.

If one goes over from N to integer quantum numbers $n = 2N$, the quantization rule (10) becomes for large n

$$M_n = \frac{1}{\sqrt{2}}\, m_p \sqrt{n}, \tag{12}$$

which differs from the quantization rule (3) by numerical factor $\sqrt{2\pi/\ln 2} = 3.01$.

For a Schwarzschild black hole a close quantization rule

$$M_n = \frac{1}{2}\, m_p \sqrt{n} \tag{13}$$

with integer n was obtained previously [7], starting with periodic boundary conditions in Euclidean time.

One cannot but notice a certain resemblance between our formula (11) and the quantization condition for a surface area

$$A_a = 8\pi l_p^2 \sum_i \sqrt{2j_{1i}(j_{1i}+1) + 2j_{2i}(j_{2i}+1) - j_{12i}(j_{12i}+1)}, \tag{14}$$

obtained in the loop approach to quantum gravity [9]. In formula (14) j_{1i}, j_{2i} are nonnegative integers or half-integers; j_{12i} run from $|j_{1i} - j_{2i}|$ to $j_{1i} + j_{2i}$. For usual closed surfaces both $\sum_i j_{1i}$ and $\sum_i j_{2i}$ are integers.

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