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Budker INP 98-27

http://www.inp.nsk.su/publications

Novosibirsk
1998

Improvement of Resolving Power of Spatially-Encoded Fourier Spectrometer by the Method of Numerical Correction of Interferogram

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Abstract

The method is suggested of numerical correction of the errors of an interferogram that is recorded by a position-sensitive detector in a spatially-encoded Fourier spectrometer; these errors are conditioned by the fact that optical elements are not ideal, and adjustment is not precise. This correction, that in principle is not possible for a conventional "temporally-encoded" Fourier spectrometer with a moving mirror, allows at the stage of an interferogram processing by a computer to improve the resolving power of a spectrometer almost to the theoretical limit, even if the quality of optical elements is not high. The method of processing has been experimentally approved with the Fourier spectrometer based on the Sagnac interferometer with the photodiodes array for the radiation sources with various combinations of line and continuous spectra at the scope of 0.3–1.6 micrometers. While an interferogram processing the correction was carried out of its spectral components period that was distorted because of light dispersion in a plane-parallel light divider. The device reliably reconstructed all kinds of spectra. An interferogram correction improved the resolution power of a spectrometer by six times. The resolving power R=200 has been achieved, when the theoretical limit defined by Nyquist frequency is about 250.

1 Problem statement

In the previous paper [1] we have shown that a spatially-encoded Fourier spectrometer in the whole complex of its parameters (a geometric factor, a resolving power, and a multiplex-factor) surpasses both a dynamic Fourier spectrometer and dispersive spectrographs including multiplex ones. In this paper[1]. we demonstrate another advantage of a spatially-encoded Fourier spectrometer – the possibility of obtaining spectra with high resolution, even if optical elements of inferior quality were used.

The theoretical resolving power of a spatially-encoded Fourier spectrometer is equal (in accordance with the theory [2]) to a half of total number of interferogram resolving elements. While recording with a multi-element position-sensitive detector (MPSD) this limit is $n/2$, where n – a number of elements. In practice this magnitude may occur less owing to many reasons, such as imperfection of longitudinal, transverse and azimuth adjustments, errors of manufacturing of MPSD and optical elements, aberrations. Adjustment errors are determined by the quality of the device mechanics. Chromatic aberration could be minimized by means of using of a good achromatic objective or the reflecting optics. Monochromatic aberrations and interferogram distortions generated by optical elements could be corrected considerably at the stage of processing of an interferogram and reconstruction of a source spectrum.

In this work is suggested and realized an algorithm for improvement the quality of a spectrum retrieved by inverse Fourier transform of an interferogram. One could correct the errors by means of the special calibration of the system while recording a well-known monochromatic line, for which the "true" interferogram is known a priori. Comparing the recorded and calculated "ideal" interferograms, one can find some correction function, depending on a detector coordinate, which allows to correct (for the given spectrograph adjustment) interferograms obtained with the sources of unknown spectral composition and to improve the resolution in the reconstructed spectra. This feature of a Fourier spectrometer is similar to a property of two-exposure holographic interferometry to deduct errors connected with inferior transparent elements placed into the interferometer [3]. The errors are also deducted there at the stage of the spectrum reconstruction, though in holography it is fulfilled by optical means, and in SFS - at the stage of computer processing.

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[1] Present work have been performed in BINP and Novosibirsk State University.

2 Algorithm of interferogram correction

Schematic of the Fourier spectrometer being used in the experiment is shown in the Fig. 1. Light from the source 1 enters a Sagnac interferometer formed by a beam-splitter 2 and two mirrors 3, 4. At the output of the interferometer there is a lens 5, through which the light, reflecting from the mirrors 6, 7, falls onto the photodiode array 8 with the number of elements $N = 1024$. The source and the

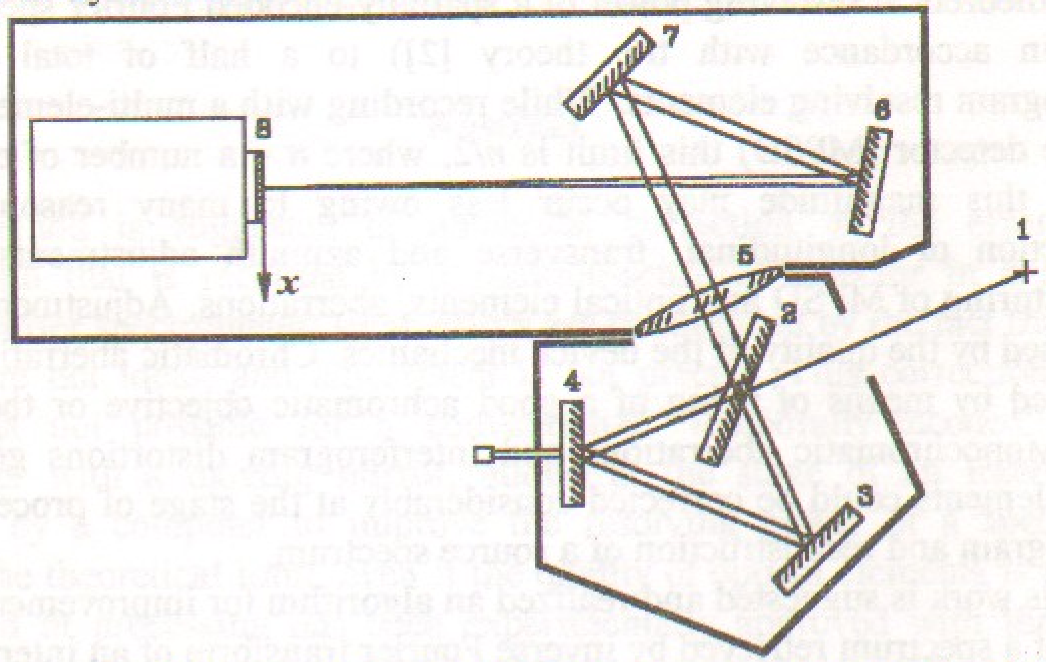

Fig.1. Schematic of the spatially-encoded Fourier spectrometer :1 – the source; 2 – a beam-splitter; 3,4 –interferometer mirrors; 5 –achromatic lens; 6,7 –mirrors; 8 –photodiode array with a control unit and connection with a computer.

photodiode array are placed in the front and back focal planes of the lens, correspondingly. If the mirror 4 is placed from its symmetric position[2], the interferometer forms in the front focal plane of the lens 5 two imaginary sources which produce behind the lens two plane waves. An angle between their directions depends on the magnitude of the mirror 4 displacement. These waves interfere forming in the back focal plane of the lens (in accordance with a well-known lens property) an exact Fourier-image of an initial spectrum of incident radiation.

$$I(x) = \int I_0(\sigma)B(\sigma)\left(1+\cos\frac{2\pi a\sigma x}{F}\right)d\sigma\,, \qquad (1)$$

[2] "Symmetric position" of the mirror corresponds to the coincidence of both virtual sources, *i.e.* zero angle between interfering beams.

where a – a distance between imaginary sources, F – a focal distance of a lens, x – a coordinate along the array, and $B(\sigma)$ – a spectral sensitivity of the system. Hence, it is evident, that after inverse Fourier transform of $I(x)$ we find a magnitude $I_0(\sigma)B(\sigma)$, from which if the spectral sensitivity of a system is known, one can find the source spectrum.

The errors mentioned here may, however, considerably distort the results. In Fig. 2a is shown the spectrum of the line 632.8 nm of a HeNe laser reconstructed by means of the discrete Fourier transform of an interferogram recorded after setting into the interferometer low-grade mirrors and a beam-splitter. Here s is a harmonic number in a reconstructed spectrum, that is proportional to a wave number σ. At a given distance between the imaginary sources (a = 6 mm) an theoretical spectral resolution is 6 nm or two pixels. It is evident, however, that the real resolution is seven times worse. It means that interferograms contain distortions. Assuming that the distortions do not depend on the radiation wavelength and connect only with optics and array imperfections one can calibrate the system comparing the real spectrum inteferogram of a He-Ne laser with the calculated one. At the reflection coefficient of beam-splitter R the equation (1) transforms into

$$I(x) = (1-2R+2R^2)\int I_0(\sigma)B(\sigma)\left[1+\left(\frac{1}{1-2R+2R^2}-1\right)\cos\frac{2\pi a\sigma x}{F}\right]d\sigma \qquad (2)$$

If $R \neq 0.5$ an interferogram contrast decreases.

Turning to a real interferogram, let the relative distribution of the intensity of an interferogram located on the surface of an N-element photodiode (or CCD) array is $f(\xi) \sim I(\xi)/I_0$, where ξ denotes a coordinate along the array. Because of possible influence of vignetting, parasitic illumination and detector noise the light intensity can change along the array, we take it into account introducing into (2) the factor $A(\xi)$. Now assume that interfering wave fronts are not plane owing to inaccuracies of optical elements manufacturing. This leads to distortion of an interference pattern in comparison with an "ideal" one. In this case one may consider the scale ξ as the "ideal" scale x deformed in some way and introduce some differential factor of a space distortion

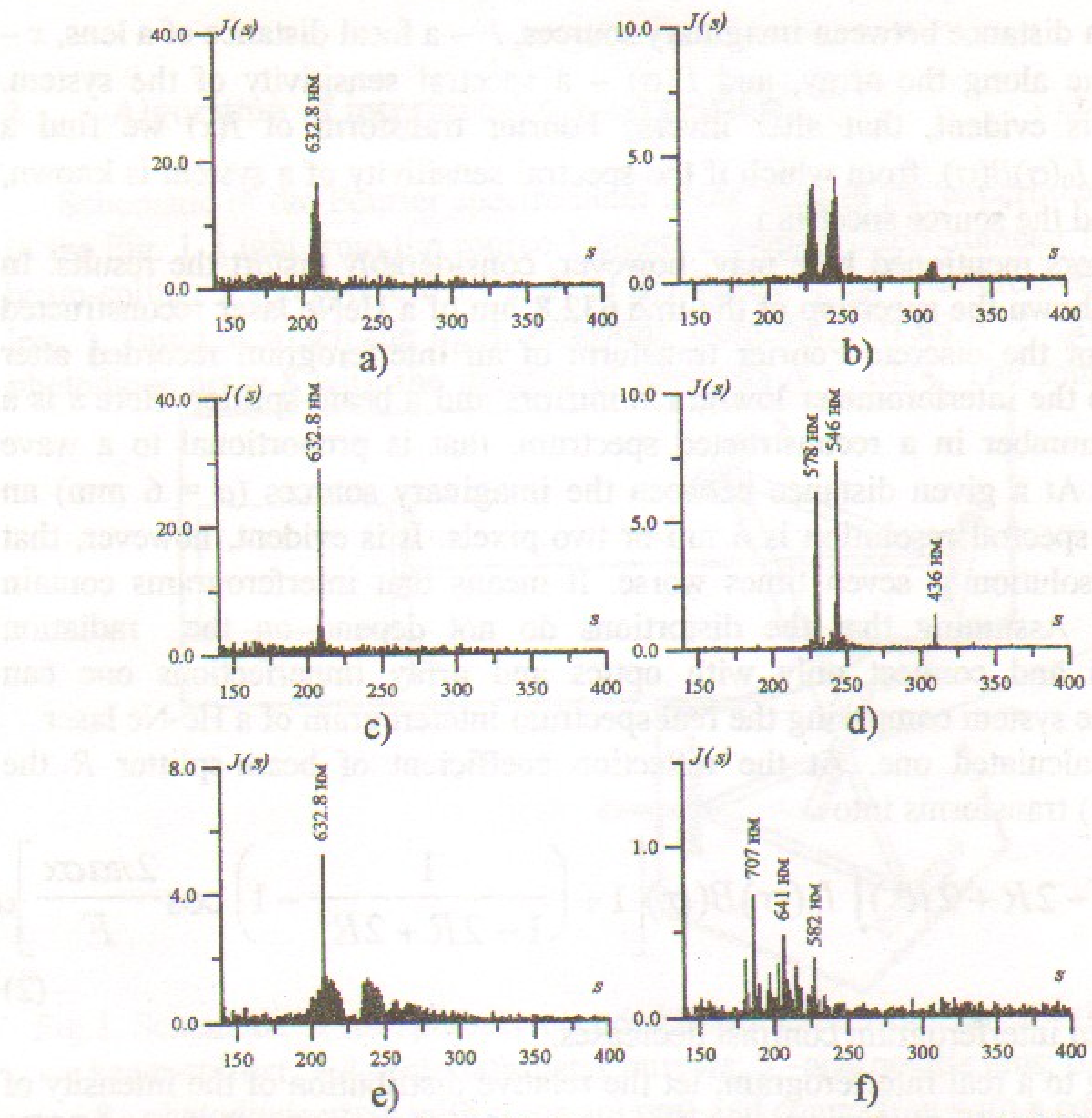

Fig.2. a,b) the spectrum of a NeHe laser and a mercury lamp reconstructed by a standard method; c,d) the same spectra after the procedure of interferograms correction; e,f) corrected spectra of a HeNe laser combined with a continuous spectrum and a neon lamp.

$$\gamma(\xi) = \frac{dx}{d\xi}. \qquad (3)$$

Note, that if, besides optical errors, there are also errors in the array period they are automatically included into the factor γ. Since the expected relative deformation of the scale is small, we do not consider here the change of an interferogram amplitude connected with deformation. Thus, the real interferogram on the scale ξ, if take into account (2) and (3), is

$$f(x) \sim A(\xi)\int f_0(\sigma)B(\sigma)\left[1+\beta\cos\left(\frac{2\pi a\sigma}{F}\int_0^{\xi}\gamma(\xi)d\xi\right)\right]d\sigma, \qquad (4)$$

where β is the factor before the cosine in the expression (2).

From the above-stated algorithm it is evident that the "ideal" interferogram can be reconstructed if $\gamma(\xi)$ has been measured. We have N values of an interferogram intensity $f_i \equiv f(\xi_i)$ obtained experimentally for equal intervals $\Delta\xi$. The scale ξ is normalized for the array length L

$$\sum_{i=1}^{N-1}\Delta\xi = L. \qquad (5)$$

The intensity values f_i are located on the coordinate scale x irregularly at the points

$$x(\xi_i) = \sum_{i=0}\gamma(\xi_i)\Delta\xi \qquad (6)$$

for the full scale length

$$L_x = \sum_{i=0}^{N-1}\gamma(\xi_i)\Delta\xi. \qquad (7)$$

Dividing the scale x into N sections of the length $\Delta x = L_x/(N\text{-}1)$ and interpolating with some method the function $f(x)$ using measured values f_i, we find the values $f_m \equiv f(x_m)$ at the points x_m (m = 1...N–1). In that way we obtain the corrected interferogram from which we restore the initial spectrum using the discrete Fourier transform. The examples of spectra of various sources obtained before and after the interferogram correction are shown in Fig. 2.

The function $\gamma(\xi)$ could be defined experimentally by recording an interferogram of a source for which the "ideal" interferogram is known a priori. If as a source is chosen a HeNe laser generating practically monochromatic radiation $f(\sigma) = \delta(\sigma_0)$, the interferogram is represented by a harmonic function. In this case the equation (4) becomes

$$f(x) \sim A(\xi)\left[1+\beta\cos\left(\frac{2\pi a\sigma_0}{F}\int_0^{\xi}\gamma(\xi)d\xi\right)\right] \qquad (8)$$

Subtracting the constant component from $f(x)$, we get an oscillating function intersecting the zero at the points $\xi_0, \xi_1, \ldots \xi_n, \xi_{n+1}, \ldots$. For neighboring points the condition following from (8) fulfills

$$\frac{a\sigma_0}{F}\int_{\xi_n}^{\xi_{n+1}}\gamma(\xi)d\xi = 1. \tag{9}$$

Assuming that $\gamma(\xi)$ changes slowly on the interval $\delta\xi_n \equiv \xi_{n+1} - \xi_n$, one could express $\gamma(\xi)$ from (9)

$$\gamma(\xi_n) = \frac{F}{a\sigma_0 \cdot \delta\xi_n}. \tag{10}$$

Thus, a differential coefficient of distortion one could get, having measured the periods of the oscillating interferogram $\delta\xi_n$ and substituting them into the formula (10). It should be noted that the equations (8)–(10) assume a continuous function $f(x)$ but actually this function is initially defined at discrete points x_i only. To specify the interferogram more exactly at intermediate points between x_i we used a special method based on theorems of discrete mathematics.

In Fig. 3 as an example is given the function $\gamma = \gamma(\xi)$, that was obtained for our spectrometer. Using a given $\gamma(\xi)$ for correction of an interferogram obtained

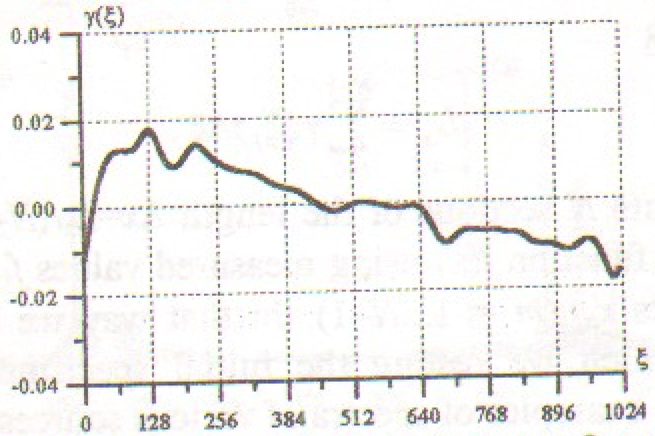

Fig.3. A differential factor of spatial distortion as a function of an element number of the photodiode array.

for a HeNe laser and executing the Fourier transform once more, we obtain the spectrum shown in Fig. 2c. It is obvious that the resolution has improved considerably and approached to the theoretical one. Having fulfilled the same procedure for the other spectra, we have obtained the spectra with a high resolution (Fig. 2d-f). However, the more distant was a wavelength from the calibrating one (632.8 nm), the more the obtained value of a wavelength differed from the true one. The reason of this phenomenon is a dispersion in a beam-splitter that manifests itself in dependence of the coefficient of refraction from the wavelength $n = n(\lambda)$.

3 Correction of dispersion of beam-splitter

In Fig. 4 are shown the beam traces in a spectrometer for two wavelengths. The wave with the shorter wavelength spreads along the way represented by the a dotted line. From the picture it is seen the dispersion influence onto the interferogram is developed owing to the difference of the optical length on the intervals $MNPQF$ (for the long wave) and $MN'P'Q'F$ (for the short wave). Simple geometric calculations give the dependence of the interferometer base a from the index of refraction of the beam-splitter:

$$a = a_0 + h\cdot\left(\frac{1}{\sqrt{n^2-0.5}} - \frac{1}{\sqrt{n_0^2-0.5}}\right), \tag{11}$$

where a_0 and n_0 are the base and the refraction index for the calibrating wavelength λ_0, h – the plate thickness.

Thus, in the expression (2) under the cosine the factor a is the function of the refraction coefficient that, at the presence of a dispersion, depends on the wavelength. Taking that into consideration, we obtain a formula connecting the wavelength with the number of a harmonic in the discrete Fourier image taken from the expression (2). To the lines which the harmonics number is s corresponds the wave number

$$\sigma = \frac{F}{aL}s, \tag{12}$$

where L – the length of a photodiode array. Now the wave number is

$$\lambda = \frac{1}{\sigma} = \left[a_0 + h\cdot\left(\frac{1}{\sqrt{n^2-0.5}} - \frac{1}{\sqrt{n_0^2-0.5}}\right)\right]\cdot\frac{L}{sF}. \tag{13}$$

The interferometer base a_0 at the callibrating wave number λ_0 obeys the formula

$$a_0 = \frac{F}{L}\lambda_0 s_0, \tag{14}$$

here s_0 – the harmonic number for the line obtained while measuring the spectrum of the source with the wavelength λ_0. Taking that into account, we get the equation that allows to determine the relation between the harmonic number of the discrete spectrum s and the wavelength λ

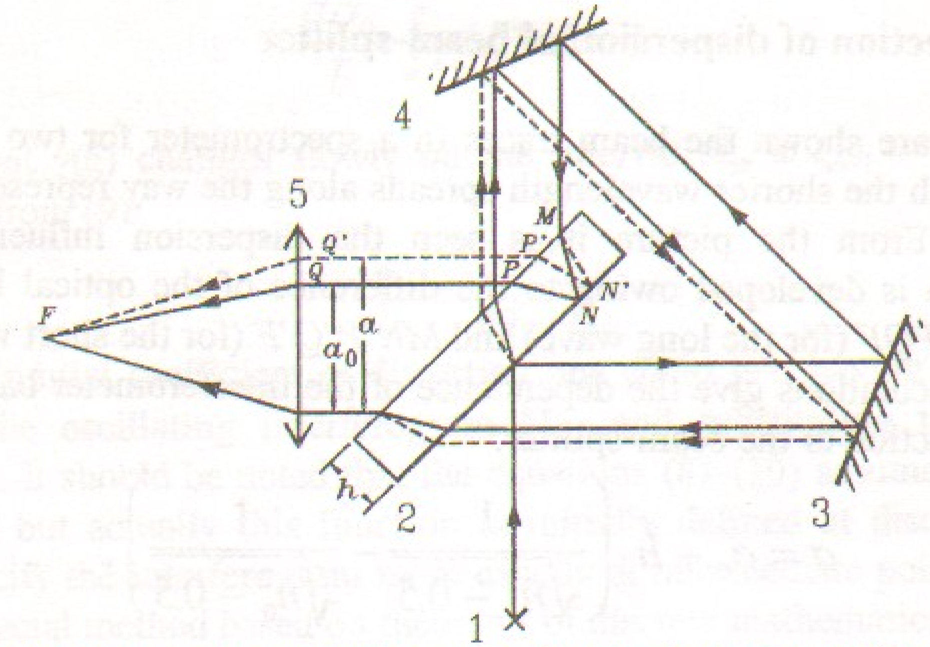

Fig.4. Traces of the rays in a Fourier spectrometer for two wavelengths: a dashed line corresponds to a ray with a less wavelength: 1 – source; 2 – beam-splitter; 3,4 –interferometer mirrors; 5 –lens.

$$\lambda = \frac{s_0}{s}\lambda_0 + \frac{hL}{sF}\cdot\left(\frac{1}{\sqrt{n^2(\lambda)-0.5}} - \frac{1}{\sqrt{n^2(\lambda_0)-0.5}}\right). \quad (15)$$

The obtained equation is solved as a first approximation, if assume for the refraction index $n(\lambda) \approx n(s_0\lambda_0 / s)$. In fig. 2 the spectral lines are marked by the values of the wavelengths, obtained from the formula (15) at the known relation $n(\lambda)$. It is evident that these values for the lines of a mercurial lamp are in good agreement with the well-known ones: 577-579.1, 546.1, 435.8 nm.

4 Conclusion

The presented results indicate that if the methods of numerical correction of interferograms suggested in this paper are used one could obtain the spectra of high quality by means of a spatially-encoded Fourier spectrometer even if the quality of optical elements is low. That permits to develop a widely adopted cheap spectral device which may be used for industrial and educational purposes. In the mode of imaging spectroscopy, when a CCD-matrix is used as a photodetector, the interferograms recorded at each matrix array (and corresponding to different spatial coordinates) may be corrected separately if use the method described above. In conclusion we note, although this subject is out of the frame of this paper, that numerical correction of optical errors offers an opportunity of miniaturization of a spatially-encoded Fourier spectrometer down to microdimensions and its usage in the systems of integrated optics.

Acknowledgments

This work has been carried out under partial support of U.S. Civilian Research and Development Foundation (Award No. RP1-239), the program "Russia Universities" and the program "Integration of Science and Education" (grant No. 274) of the Ministry of Education of Russian Federation.

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Budker INP 98-27

Ответственный за выпуск А.М. Кудрявцев
Работа поступила 21.04. 1998 г.

Сдано в набор 22.04.1998 г.
Подписано в печать 22.04.1998 г.
Формат бумаги 60×90 1/16 Объем 0.8 печ.л., 0.7 уч.-изд.л.
Тираж 110 экз. Бесплатно. Заказ N° 27

Обработано на IBM PC и отпечатано на
ротапринте ИЯФ им. Г.И. Будкера СО РАН,
Новосибирск, 630090, пр. академика Лаврентьева, 11.