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PION PRODUCTION SYSTEM FOR MUON COLLIDER ON THE BASE OF COLLECTION DEVICE OF MAGNETIC PARABOLIC MIRROR TYPE

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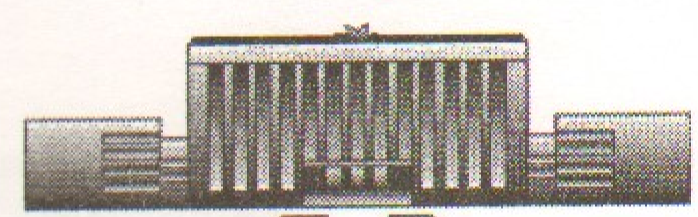

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Pion production system for muon collider on the base of collection device of magnetic parabolic mirror type

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Abstract

A pion production scheme for the muon collider with the collection angle of about 2π is suggested. The focusing system is the axial-symmetric parabolic mirror with pulsed magnetic field ~ 10 T. With the GEANT code the yield of pions generated by 24 GeV protons is calculated

1 Introduction

In the $\mu^+\mu^-$-collider project [1] to obtain pions one planes to use a long (1-2 nuclear lengths λ_i) target, placed inside a long solenoid with constant field of 20-28 T. The solenoid is estimated to require significant power up to 20 MW [2]. It collects pions emitted from a long target at large angles and in large momentum range and transports them to a long decay channel with 5-7 T magnetic field where the pions decay to muons. An optimal proton beam energy is 8-30 GeV. Targets made of different solid and liquid matters were considered and it was shown that one can obtain at the exit of solenoid ~1 pion per proton, decaying to muons suitable for ionization cooling.

An alternative scheme for collection of π-mesons in a large solid angle, based on usage of the collection device of the magnetic parabolic mirror type, is suggested in this work.

2 The main idea

The large aperture system for collection of π-mesons in the momentum range $100 < P_\pi < 300$ MeV/c, being emitted from the target at angle to axis $\sim 10^\circ - 90^\circ$ and more, is based on the idea of "magnetic parabolic mirror", which was considered in [3]. In such system the pions collection is performed by the axial-symmetric field of direct current which transforms a "cloud" of mesons, having large transverse momenta, to the beam where the direction of longitudinal momenta is ordered along the axis of symmetry of the system, which can be later transported to the decay channel with a large angular acceptance. As the decay channel one can consider a long solenoid with the longitudinal field of ~ 5 T, or a system with the longitudinal current and axially-symmetric field.

A general view of the collection system with the point source of pions on the axis is shown in Fig. 1. The outlet surface of the system is chosen in such a way, that 200 MeV/c pions being emitted from the focus, at the exit of the system have angle 0° to the axis. In difference to "the parabolic mirror" described in [3], the two-section system has a considerably larger optic strength of field.

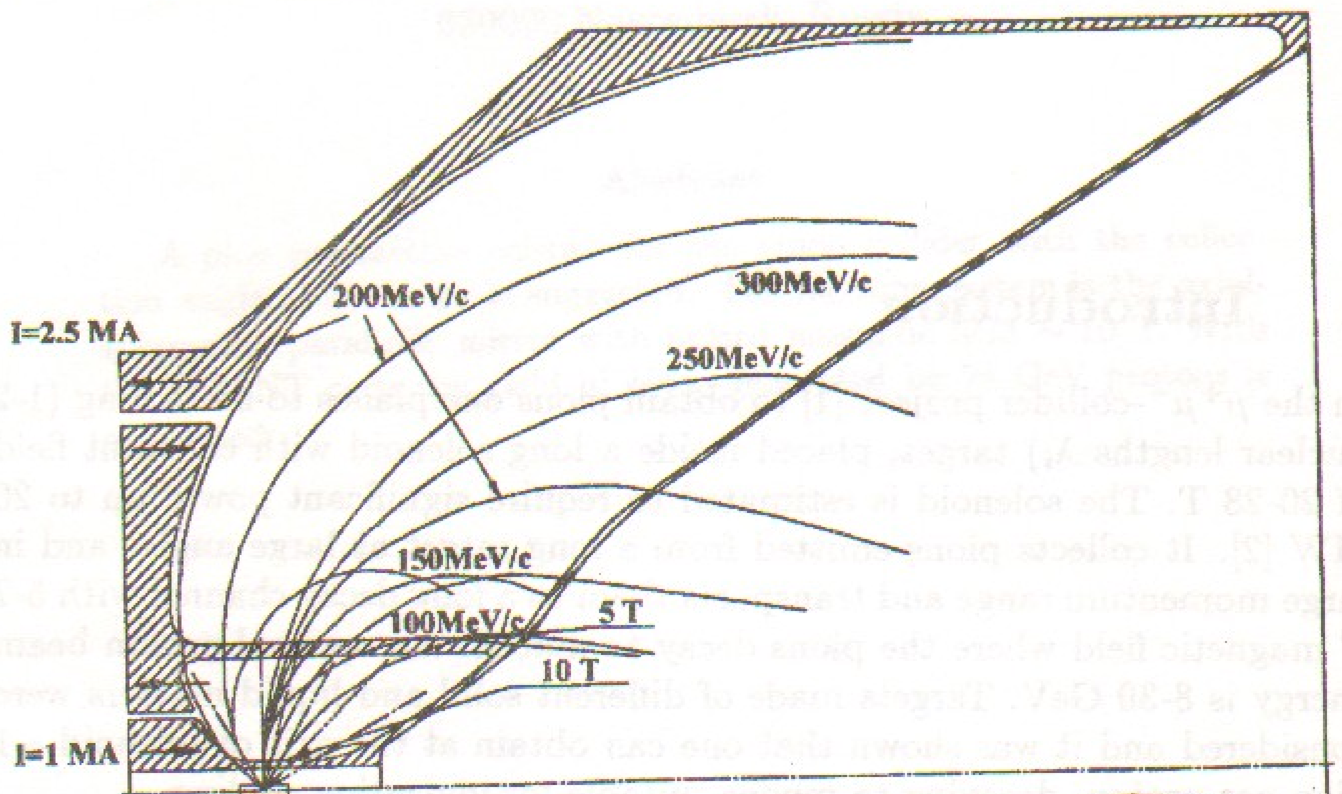

Figure 1: General view of parabolic mirror for collection of charged particles with momenta of several hundred MeV/c.

A large efficiency of such system can be shown by the collection of pions with fixed momentum, moving in $r-z$ plane. The particle trajectory in the field of direct current J (in z axis direction),

$$B(r)[T] = \frac{20J}{r},$$

where J is in MA and a distance to axis r is in cm, can be determined conveniently in the parametric form, using the angle to the z axis as the parameter [4]:

$$\alpha = arctg\frac{dr}{dz}.$$

The particle trajectory has the form:

$$r(\alpha) = r_0 e^{k(cos\alpha - cos\alpha_0)},$$

$$z(\alpha) = z_0 + r_0 k \int_{\alpha}^{\alpha_0} cos\alpha' e^{k(cos\alpha' - cos\alpha_0)} d\alpha', \qquad (1)$$

where $\alpha = \alpha_0$ at $r = r_0$ and $z = z_0$. Parameter k determines the optical strength of field; it depends on the value of longitudinal current and is given by the expression:

$$k = \frac{P_\pi}{eB_0 r_0} = \frac{P_\pi}{60J},$$

where the particle momentum P_π is in MeV/c, J is in MA.

In the geometry being considered the radius of the first current surface is r_1=2 cm and at J_1=1 MA the field at the surface will be B_1=10 T. The parameter k at P_π=200 MeV/c is k_1=3.3. The radius of the second current surface is r_2=10 cm and at J_2=2.5 MA the field at the surface will be B_2=5 T and k_2=1.33.

The main parameter of the system, which defines its geometric size, is the maximum transverse coordinate of the particle r_{max}, when the trajectory becomes parallel to the system axis, that is $\alpha = 0$.

Determine r_{max} for the 200 MeV/c particle entering the field of the first conductor at r_1=2 cm with angle of $\alpha_0 = \alpha_1 = 90°$, that is

$$r(\alpha) = r_1 e^{k_1 cos\alpha}.$$

At the surface of the second conductor at r_2=10 cm the particle will have the angle, determined from the equation

$$r_2 = r_1 e^{k_1 cos\alpha_2},$$

which gives

$$cos\alpha_2 = \frac{ln(r_2/r_1)}{k_1} = 0.49,$$

that is $\alpha_2 \approx 60°$. In the second section with $k_2 = 1.33$ the maximum coordinate r_{max} at $\alpha = 0$ will be

$$r_{max} = r_2 e^{k_2(1-cos\alpha_2)} = 19.2 \text{ cm}.$$

If the system has only one section with k=k_1=3.3 and r_1=2 cm, the maximum particle coordinate at $\alpha = 0$ would be

$$r_{max} = r_1 e^{k_1} = 54.2 \text{ cm}.$$

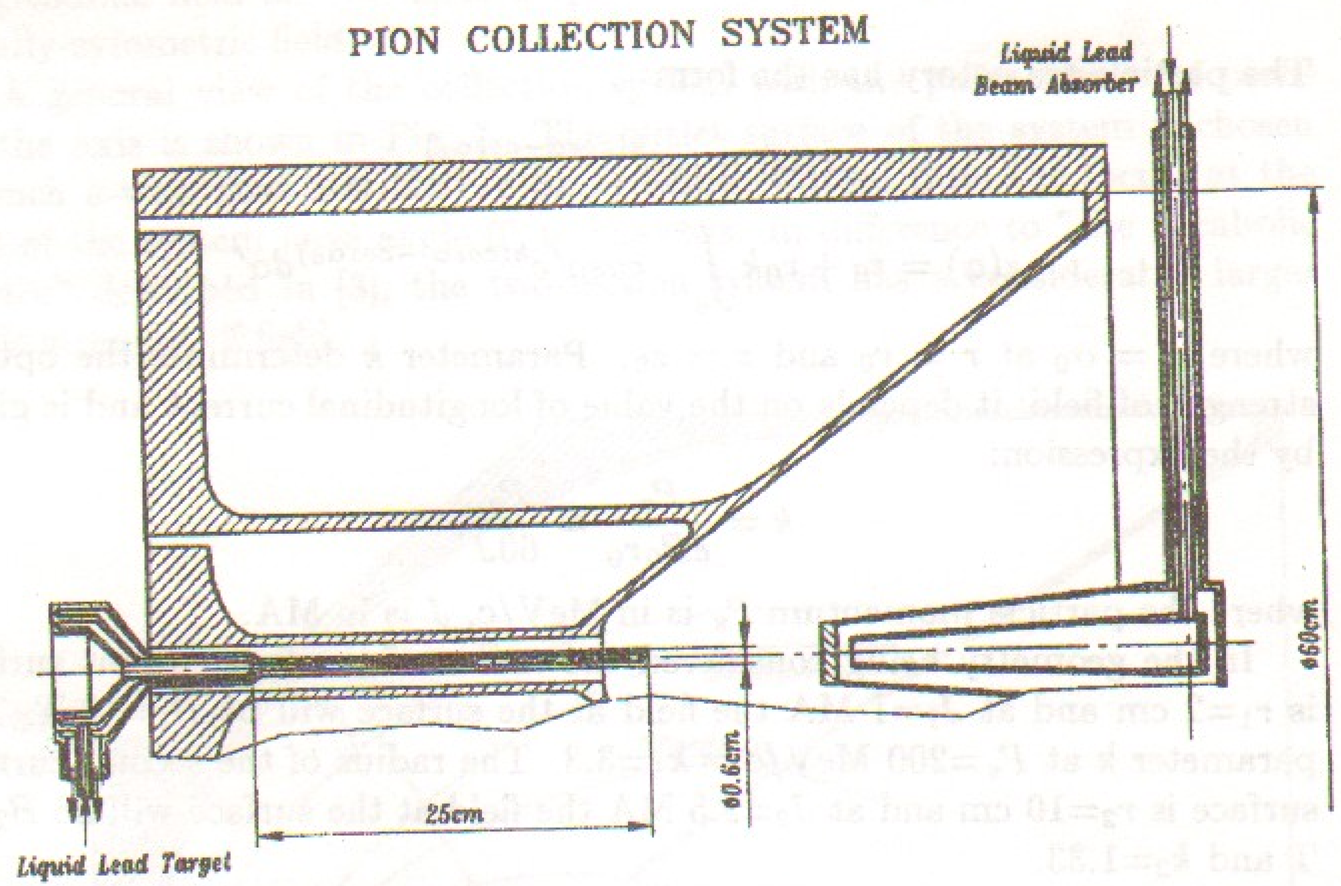

Figure 2: Possible construction of pion source based on magnetic parabolic mirror for charged particles collection with several hundred MeV/c momenta.

Fig. 2 shows the design of the system with the possible variant of position of the liquid lead target and absorber of the primary beam. This geometry was a base for the simulation program used for the numerical calculations of collection efficiency for pions produced at a long liquid lead target.

The system is supplied from two surge current generators by half-sine pulses of current with $\tau \simeq 100\ \mu s$ duration. For the Fig. 2 geometry and currents of 1 MA and 2.5 MA in the first and second sections, respectively, and at cycles frequency of 15 Hz the system consumes $\sim$ 200 kW power.

3 Position of target

An optimal target position at small target length ($L < \lambda_i$, interaction length) is one, when average coordinate of the 1-st inelastic interaction of proton with the target is in the focus of collection system. At larger lengths the average effective length of source should increase, since interactions of particles from the first inelastic interaction will contribute to the total production of pions.

But, since the particles produced in the nuclear interaction have mainly large angles to axis and the target radius is small, much smaller than λ_i, this contribution should be small.

Table 1: Coordinate of center of target in coordinate system with origin in the focus of system as a function of target's length.

Length of target, cm	Coordinate z_0, cm
2.5	-0.05
5.0	-0.10
10.0	-0.50
20.0	-1.90
30.0	-4.20

For the distribution of protons on range l before interaction

$$\frac{dW}{dl} = \frac{1}{\lambda_i} e^{-l/\lambda_i}$$

for the target of length equal to L, one obtains the following coordinate of the target center z_0 in the coordinate system, with the origin in the focus of collecting system:

$$z_0 = \lambda_i \frac{1 - e^{-L/\lambda_i}(L/\lambda_i + 1)}{1 - e^{-L/\lambda_i}} - L/2. \qquad (2)$$

Data of calculations according to (2) with λ_i=17 cm are given in Table 1.

Multiple scattering in the walls of lens can be considered as increase of the effective length of particles source. This should do the dependence of the optimal yield of pions on the target position more weak.

4 Short target

The system being considered performs optimal collection of 200 MeV/c particles produced with angles in wide interval from the point source, placed in the focus of the system. Really, the pion source is not point-like one. The 200 MeV/c particles emitted before the system focus will obtain in field the angle larger than optimal, after the focus – smaller than optimal. At the same time a non 200 MeV/c particle obtains 0° angle at the exit when, in general, it was emitted with definite angle not from the focus of the system.

To study the focusing properties of the parabolic mirror the Monte Carlo calculation for the short lead target of L=2.5 cm length was carried out.

The GEANT 3.14 code [5] was used for this calculation. The system geometry with sizes of blocks used in the simulation is shown in Fig. 3. The

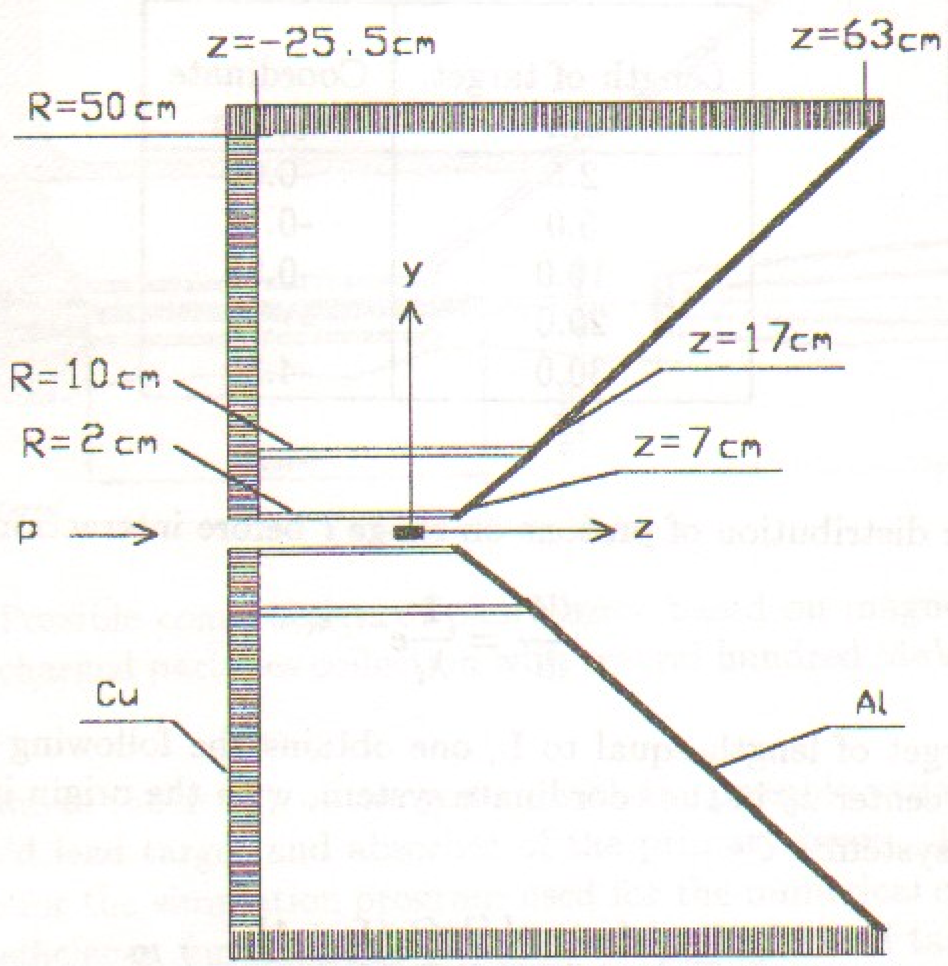

Figure 3: GEANT geometry used in calculation of production and collection of pions.

axially-symmetric magnetic field $\sim 1/r$ is only in the internal areas of the device. At the surface r_1=2 cm magnetic field is 10 T, at the surface r_2=10 cm it is 5 T. All the current surfaces of the system which are being crossed by particles are made of aluminum alloy. The thickness of the outer cylinder with r_2=10 cm is 1 cm, the thickness of the inner cylinder with r_1=2 cm is 0.5 cm, the thickness of the exit cone surfaces is 0.5 cm also.

The cylindrical lead target of 0.2 cm radius was used in the calculations. (In reality, the target radius can be larger, up to $\sim$ 1 cm). In this section the length of target is 2.5 cm or $0.15\lambda_i$ (the length of inelastic nuclear interaction

of protons in lead is λ_i=17 cm).

In the simulation the proton beam was considered having the zero radius, number of initial protons was 10000. Since such system collects the particles of one charge sign and defocuses the particles of other charge sign, here and later the calculations with the magnetic field, which focuses π^+-mesons, were performed.

For the 2.5 cm target the total π^+-meson yield with all moments in 4π solid angle is 0.66 per 1 initial proton with 24 GeV energy. (Here and later the GEANT cut on the pion momentum equal to 53 MeV/c is implied.)

In the decay channel of $\sim$ 100 m length the pions with moments up to 2500 MeV/c give contribution to muon production [1]. But because of the necessity of the ionization cooling, only the muons with momentum of several hundred MeV/c are necessary. The pion to muon and neutrino decay kinematics restricts the momentum of useful pions by the approximate interval 100–500 MeV/c (their yield in 4π sr is 0.39 π^+ per proton).

4.1 Momentum spectrum of π-mesons at target

The momentum spectrum of π^+-mesons leaving the target of 2.5 cm length and 0.2 cm radius for 24 GeV proton is shown in Fig. 4

Pions from the target can be collected by the parabolic mirror and then enter the decay channel, or if they have a small angle at the target, enter it without focusing by magnetic field. As one can see from Fig. 4, the focusing system suggested is adequate to the problem: the spectrum maximum lies at several hundred MeV/c pion momentum.

4.2 Distributions of π-mesons at exit of system

After the target and collecting system the pion beam will be transported to the decay channel of large radius or to several channels of smaller radius. We calculated different distributions of particles after the focusing system at the distance of 100 cm from the focus.

In Fig. 5 the distributions of charged mesons with momentum of 100–500 MeV/c on radius in plane z=100 cm are shown.

The particles cross this plane after the collection by parabolic mirror, or going directly from the target ("direct particles"). In the Figure π^--mesons, shown together with π^+-mesons, come in the pane z=100 cm directly from the target, without focusing. The distribution for π^- mesons gives the estimate of contribution from "direct" π^+-mesons in π^+-mesons distribution, while the difference between two distributions characterizes the efficiency of the

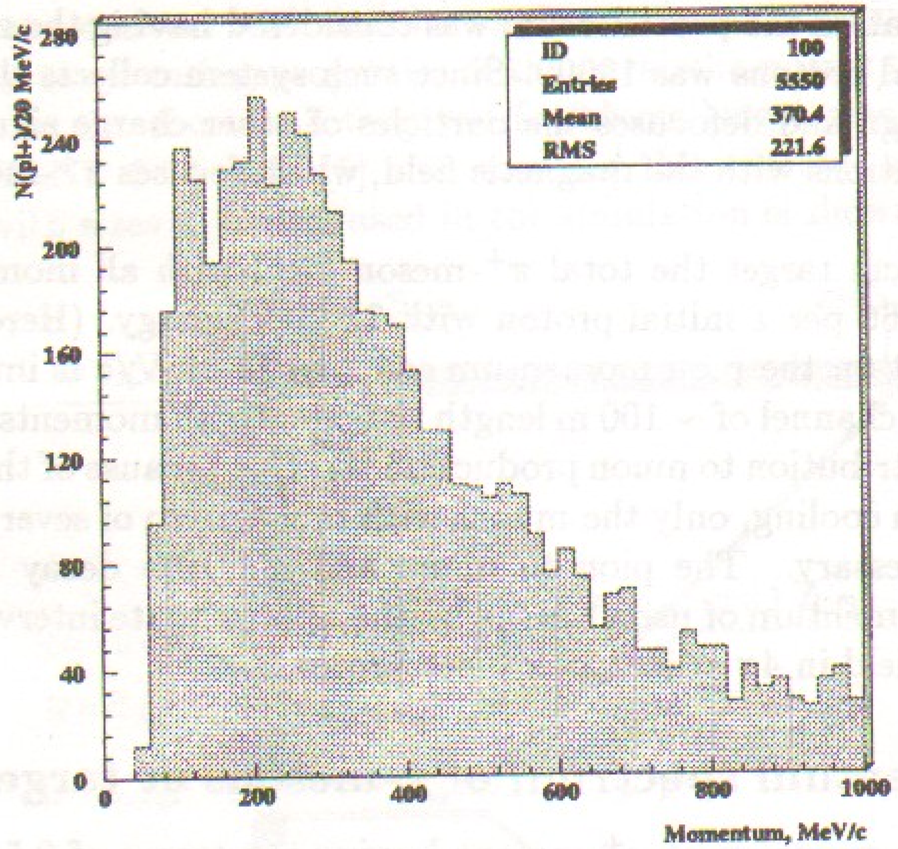

Figure 4: Momentum spectrum of π^+-mesons leaving the lead target of 2.5 cm length, 0.2 cm radius. Proton energy is 24 GeV.

collection device. As one can see from Fig. 5, the maximums of distributions are situated at radius 20–30 cm. About 77% of π^+-mesons and 81% of π^--mesons of their total numbers, crossing the plane z=100 cm, lie in the circle of 30 cm radius.

For further estimates we considered the distributions of particles in the circle of 30 cm radius in 100 cm distance from the focus of system ("at the exit of system").

In Table 2 the normalized to 1 integral angular distributions of pions with moments 100–500 MeV/c are given. The data shown are: in the 2-nd column – the data for π^+-mesons, leaving the target; in the 3-rd column – the data for π^+-mesons in plane z=100 cm, calculation with target without the focusing device; in the last two columns the data were obtained with the focusing device for particles crossing the plane z=100 cm in circle of 30 cm radius. The total yields of 100-500 MeV/c pions are given in the Table also.

To estimate the efficiency of the parabolic mirror compare the number of pions with angles to axis up to 10° crossing the circle of 30 cm radius in 100

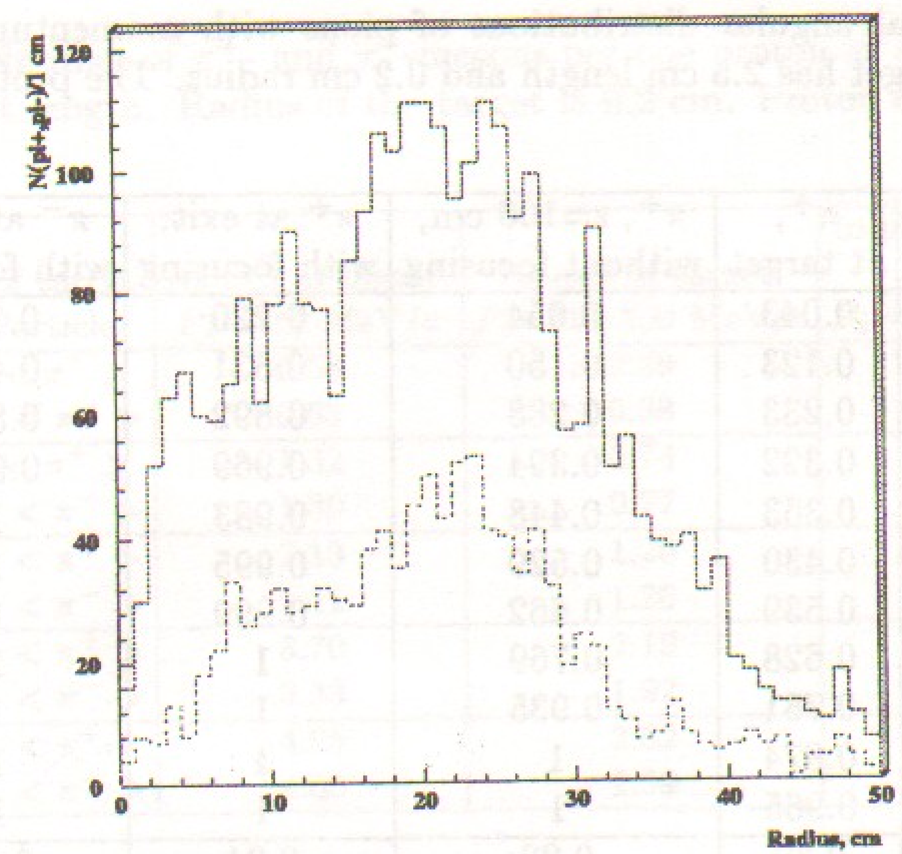

Figure 5: The 100-500 MeV/c π^+-mesons (upper curve) and π^--mesons distribution on radius in plane z=100 cm. Target of 2.5 cm length.

cm from the system focus with that without focusing. (If one uses quasi-paraxial optics at the exit of the system, the angle of $\approx 10°$ is a maximum one).

For the target without focusing the pions with angles to axis 10° in 1 m from the source cross a circle of smaller radius, about 20 cm. The yield of π^+-mesons for the momentum interval considered is y_0=0.044 per 1 proton; it can be taken as a "unit of measurement" for the target studied. This quantity is lower than y_0 obtained using distributions at target because of decays of a part of slow pions (the decay length is λ_π[cm]=5.6P_π[MeV/c]). Neglecting the angle of several degrees occupied by the absorber of initial beam the following yields per 1 proton can be obtained at the exit of the system in angle of 10° from the Table 2.

π^+, system with focusing 0.531×0.24=0.127
π^+, target without focusing 0.150×0.29=0.044
Ratio: 2.9
π^-, system with focusing 0.450×0.09=0.041 (32% of π^+).

Table 2: Integral angular distributions of pions with momentum 100-500 MeV/c. The target has 2.5 cm length and 0.2 cm radius. The proton energy is E_p=24 GeV.

Angle to axis	π^+, at target	π^+, z=100 cm, without focusing	π^+ at exit, with focusing	π^- at exit, with focusing
5^0	0.043	0.054	0.220	0.141
10^0	0.123	0.150	0.531	0.450
15^0	0.233	0.288	0.892	0.886
20^0	0.322	0.394	0.969	0.999
25^0	0.363	0.448	0.983	1
30^0	0.430	0.529	0.995	1
40^0	0.539	0.662	0.999	1
50^0	0.628	0.769	1	1
70^0	0.781	0.935	1	1
90^0	0.874	1	1	1
120^0	0.965	1	1	1
$<\pi^\pm>/p$	0.39	0.29	0.24	0.09

π^-, target without focusing 0.140×0.29=0.041.

Thus, the system efficiency can be estimated by the coefficient equal to 2.9. This focusing device adds to the yield of π^+-mesons y_0 about 30% of all π^+-mesons leaving the target with angles 10° and more.

5 Yield of pions as a function of target length

When the target becomes longer, the total pion yield increases, but the efficiency of the focusing device diminishes, since the source becomes longer.

The calculation results of yields of charged pions per one 24 GeV proton for targets of 2.5 – 30 cm length are collected in Table 3. The center of the target was at z_0 =-2 cm and z_0 =-4 cm for 20 cm target and 30 cm target, respectively (see Table 1). Besides the total yield of pions, the yield of pions with 100–500 MeV/c momentum, giving "useful" for cooling muons after decay is presented. The numbers of π^+- and π^--mesons per 1 incident proton with this restriction on the momentum at the focusing device exit in the circle of 30 cm radius in z=100 cm plane is shown in the 4-th column of the Table.

Table 3: Number of π^+- and π^--mesons per one proton as a function of the lead target length. Radius of the target is 0.2 cm. Proton energy equals 24 GeV.

Length of target (cm).	Particle	At exit of target, $P_\pi > 53$ MeV/c	At exit of target, P_π=100-500 MeV/c	In plane z=100 cm, $r < 30$ cm and P_π=100-500 MeV/c
2.5	$<\pi^+>$	0.66	0.39	0.24±0.01
	$<\pi^->$	0.63	0.38	0.09±0.01
5.0	$<\pi^+>$	1.32	0.74	0.45±0.01
	$<\pi^->$	1.30	0.77	0.16±0.01
10.0	$<\pi^+>$	2.13	1.20	0.67±0.01
	$<\pi^->$	2.11	1.26	0.24±0.01
20.0	$<\pi^+>$	3.70	2.19	1.14±0.08
	$<\pi^->$	3.33	1.92	0.20±0.03
30.0	$<\pi^+>$	4.95	2.82	1.39±0.08
	$<\pi^->$	4.33	2.59	0.19±0.03

From these calculations follows that 56÷59% of all particles leaving the target have momentum 100–500 MeV/c, independent of the target length. Number of pions "at the exit of the system" slowly changes from 20 cm to 30 cm target. Thus, the length 20–30 cm one can consider as about the optimal one. For the 30 cm target calculations with different positions of the target nearby the optimal position (that is when the target is shifted by 4 cm to the proton beam from the system focus) were performed. The results are shown in Table 4.

As one can see from the Table 4, there is not a maximum at the optimum position. The yields of pions almost agree within the statistical errors. Thus, in the last line, the values averaged over different target positions are given.

The momentum spectra of π^+- and π^--mesons in plane z=100 cm inside the circle of 30 cm radius are shown in Fig. 6. Number of π^+-mesons here is about 7 times larger, than number of π^--mesons, which mainly hit this plane passing the magnetic field and elements of the focusing device.

In Fig. 7 the radial distribution of π^+- and π^--mesons with momentum 100–500 MeV/c in plane z=100 cm is shown. The π^+-mesons distribution falls quickly at radius more than 20-30 cm.

In Table 5 the integral angular distributions of pions for 30 cm target are given.

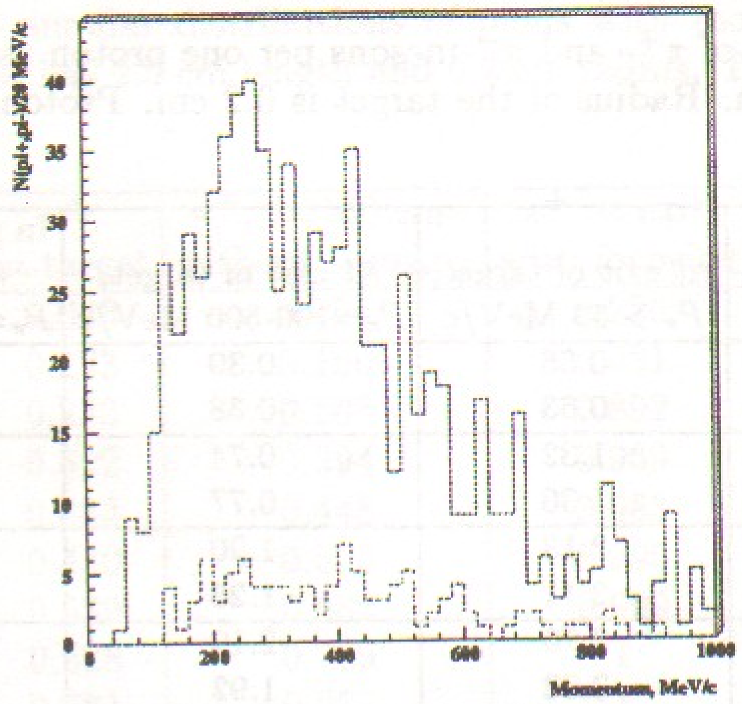

Figure 6: Momentum spectra of π^+-mesons (upper curve) and π^--mesons in plane z=100 cm in circle of 30 cm radius. Length of target is 30 cm. E_p=24 GeV.

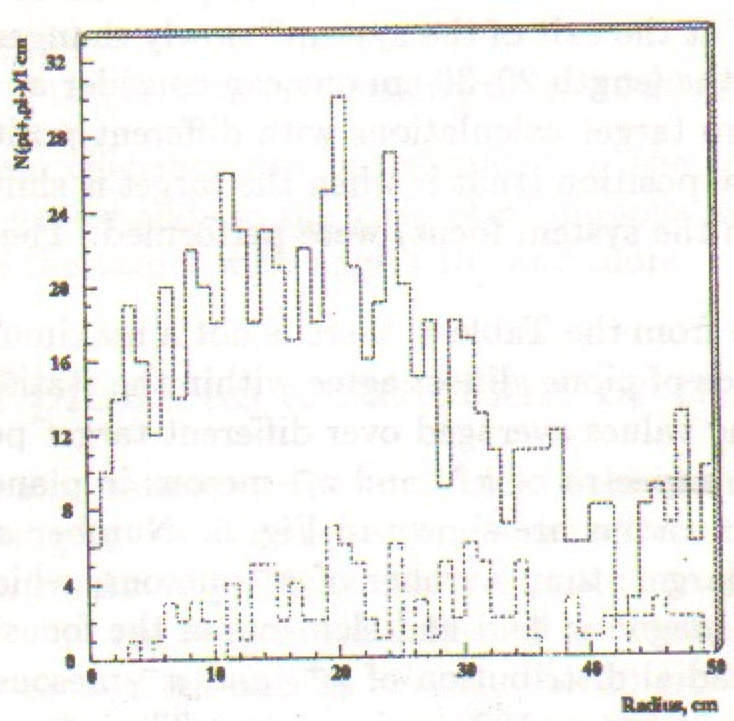

Figure 7: Radial distribution of π^+-mesons (upper curve) and π^--mesons with momentum 100-500 MeV/c in plane z=100 cm in circle of 30 cm radius. Length of target is 30 cm. E_p=24 GeV.

Table 4: Yield of π^+- and π^--mesons (N_+ и N_-) per 1 proton "at the exit of the system" (z=100 cm, r<30 cm) as a function of the 30 cm target position. The pion momentum is 100-500 MeV/c. E_p=24 GeV.

The center of target, z_0, cm	$<\pi^+>$	$<\pi^->$
0.0	1.48±0.12	0.34±0.03
-1.0	1.34±0.12	0.20±0.03
-2.0	1.17±0.12	0.21±0.03
-3.0	1.14±0.08	0.20±0.03
-4.0	1.39±0.08	0.19±0.03
-5.0	1.21±0.12	0.17±0.03
Average	1.3±0.1	0.16±0.03

As it was done for the 2.5 cm target, to estimate the efficiency of parabolic mirror compare the number of pions with angles restricted by 10° inside the circle of 30 cm radius in 1 m from the system focus with the number that could be in it without the focusing. For this target we have y_0=0.19. From the Table 5 we obtain the following yields of pions with angle up to 10° per proton at the exit of the system:

π^+, system with focusing: 0.508×1.39=0.71
π^+, target without focusing: 0.080×2.43=0.19
Ratio: 3.7
π^-, system with focusing: 0.427×0.19=0.081 (11% от π^+).
π^-, target without focusing: 0.083×2.0=0.17

From the data above we have that the efficiency of the collection for the 30 cm target system is 3.7. The focusing device adds about 20% of particles with 100-500 MeV/c momentum leaving the target with angle 10° and more. The background contribution from π^- for the system with focusing is lower considerably than gives the calculation without focusing due to the absorption of π^--mesons in elements of the focusing device.

To estimate the influence of walls of the mirror to the size of the pion beam with 100-500 MeV/c momentum at the exit of the system, we performed a calculation without aluminum walls, but with the field inside the device. This calculation showed that the yield of π^+ at the exit of the system agrees with

Table 5: Integral angular distribution of 100–500 MeV/c pions. Energy of proton is E_p=24 GeV. Target is of 30 cm length and 0.2 cm radius.

Angle to axis	π^+ at target	π^+, z=100 cm, without focusing	π^+ at exit, with focusing	π^- at exit, with focusing
5°	0.010	0.013	0.208	0.120
10°	0.066	0.080	0.508	0.427
15°	0.188	0.229	0.779	0.880
20°	0.297	0.365	0.934	1
25°	0.359	0.443	0.969	1
30°	0.437	0.537	0.995	1
40°	0.558	0.678	0.996	1
50°	0.645	0.782	0.998	1
70°	0.785	0.941	1	1
90°	0.872	1	1	1
120°	0.968	1	1	1
$<\pi>$ per proton	3.22	2.43	1.39	0.19

the calculation with the walls within statistical errors. Thus, the aluminum walls of 0.5-1 cm do not influence practically on the results of calculation.

In different Monte Carlo simulation programs different nuclear cascades models are used. To estimate the systematic error of the GEANT simulation we calculated the π^+ yield in forward direction from the copper target of 24 cm length and 1 cm radius for 24 GeV proton beam - as in Ref. [2]. Result of our calculation with the GEANT 3.14: 3.2±0.2 π^+-mesons per proton. The ARS code gave the yield equal to 2 (see Fig. 2 in [2]).

6 Conclusions

The principal possibility of usage of the parabolic magnetic mirror for collection of pions from long converter in wide angular range and with large momentum spread was shown in this work. The system suggested increases 3-4 times the yield of pions of one charge at not large angles to the axis as compared to the yield without the focusing system.

Future work to increase the efficiency of the device should include: a) optimization of forms and sizes of surfaces of the collecting system taking into account real space and momentum-coordinate distribution of pions leaving the target, currents in current surfaces; b) study of possibility to use for the target the materials with the smaller nuclear length, for example, tungsten, since for the focusing system considered a more short source is desirable; c) at sufficient computer power – a global optimization, taking into account generation of muons within the necessary momentum range in the decay channel.

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