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BUDKER INSTITUTE OF NUCLEAR PHYSICS

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Budker INP 98-54

http://www.inp.nsk.su/publications

NOVOSIBIRSK
1998

The generalized non-forward BFKL equation and the "bootstrap" condition for the gluon reggeization in the NLLA[1]

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Abstract

The generalization of the BFKL equation for the case of non-forward scattering is considered. The kernel of the generalized equation in the next-to-leading approximation is expressed in terms of the gluon Regge trajectory and the effective vertices for particle production in Reggeon collisions. The "bootstrap" equations for the gluon Reggeization are presented.

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[1] Work supported in part by the Ministero italiano dell'Universitá e della Ricerca Scientifica e Tecnologica, in part by INTAS and the Russian Fund of Basic Researches.

1 Introduction

One of the remarkable properties of QCD is the Reggeization of elementary particles. Contrary to QED, where the electron does Reggeize in perturbation theory [1], but the photon remains elementary [2], in QCD the gluon [3,4] does Reggeize as well as the quark [5]. The gluon Reggeization plays the key role in the derivation of the BFKL equation [6] for the cross sections at high C.M.S. energy $\sqrt{s}$ in perturbative QCD. This equation is very important for the theory of high energy processes. It is used [7] together with the DGLAP equation [8] for the description of deep inelastic scattering processes at a small value of the Bjorken variable x. The equation was derived [6] in the leading logarithm approximation (LLA) more than twenty years ago, and recently the calculation of radiative corrections [9-15] to the kernel of the equation was completed and the equation in the next-to-leading logarithmic approximation (NLLA) was obtained [16].

The famous BFKL equation is a particular case of the equation for the t-channel partial waves of the elastic amplitudes [6] for the forward scattering, i.e. $t = 0$ and vacuum quantum numbers in the t-channel. Evidently, it is very important to obtain in the NLLA not only the equation for this particular case, but the equation for the non-forward scattering as well. Besides the fact that the last equation is much more general, it permits to check in the NLLA the gluon Reggeization, the base of the whole program of the calculation of the radiative corrections formulated in Ref. [9] and fulfilled in Refs. [9-15]. Remind that in the LLA the Reggeization was noticed in the first several orders of the perturbation theory. After that, assuming that it is correct in all orders, the equation for the t-channel partial waves of the elastic scattering amplitudes [6] was derived. It is clear that, for the gluon quantum numbers in the t-channel, the solution of this equation

must reproduce the gluon Reggeization, as it was explicitly demonstrated in Ref. [6]. This "bootstrap" supports the idea of the Reggeization in such a strong way that practically no doubts remains that it is correct. Nevertheless, strictly speaking, the "bootstrap" cannot be considered as a rigorous proof. Therefore, such a proof was specially constructed in Ref. [17]. In the NLLA till now we have only the simple check of the Reggeization in the first three orders of perturbation theory in α_S [12].

In this paper we present the representation for the scattering amplitudes in QCD at high energies $\sqrt{s}$ and fixed momentum transfer $\sqrt{-t}$ in the NLLA in terms of the impact factors of the scattered particles and the Green function for the Reggeized gluon scattering. The representation is obtained on the base of the gluon Reggeization. The impact factors and the kernel of the equation for the Green function are expressed in terms of the gluon Regge trajectory and the effective vertices for Reggeon-Reggeon and Reggeon-particle interaction. The requirement of the selfconsistency leads to the "bootstrap" equations for the gluon Reggeization.

In the next Section we discuss the meaning of the gluon Reggeization. In Section 3 we show the representation of the scattering amplitudes in terms of the impact factors and the Green function. In Section 4 the "bootstrap" equations are derived. The summary is given in Section 5.

2 The gluon Reggeization in QCD

The notion "Reggeization" of elementary particles in perturbation theory is usually related to the absence of non analytic terms in the complex angular momentum plane [1-3]. We use this notion in a much stronger sense. Talking about the gluon Reggeization in QCD we mean not only the existence of the Reggeon with gluon quantum numbers, negative signature and trajectory

$$j\,(t) = 1 + \omega(t) \tag{1}$$

passing through 1 at $t = 0$. We mean also that in each order of perturbation theory this Reggeon gives the leading contribution to the amplitudes of the processes at large relative energies of the participating particles and fixed (i.e. not increasing with s) momentum transfers.

Let us explain this in more details. Consider the elastic scattering process $A + B \to A' + B'$ at large s and fixed t:

$$s = (p_A + p_B)^2 \to \infty\,, \quad t = q^2\,, \quad q = p_A - p_B\,. \tag{2}$$

For the sake of brevity, the term "gluon Reggeization" used by us means that the elastic scattering amplitude with the gluon quantum numbers in the t-channel has the Regge form

$$(\mathcal{A}_8)^{A'B'}_{AB} = \Gamma^c_{A'A}\left[\left(\frac{-s}{-t}\right)^{j(t)} - \left(\frac{+s}{-t}\right)^{j(t)}\right]\Gamma^c_{B'B}\,. \tag{3}$$

Here c is a color index and $\Gamma^c_{P'P}$ are the particle-particle-Reggeon (PPR) vertices which do not depend on s. Notice that the form (3) represents correctly the analytical structure of the scattering amplitude, which is quite simple in the elastic case. In the derivation of the BFKL equation it is assumed that this form is valid in the NLLA as well as in the LLA.

Together with the form (3) of the elastic amplitude the derivation of the BFKL equation in the LLA and NLLA is based on the Reggeized form of production amplitudes in the multi-Regge kinematics (MRK). For the production of n particles with momenta k_i, $i = 1 \div n$, in the process $A + B \to \tilde{A} + \tilde{B} + n$ this kinematics implies that the invariant masses of any pair of produced particles are large and all the transferred momenta are fixed (not increasing with s). More definitely, let us put $p_{\tilde{A}} \equiv k_0$, $p_{\tilde{B}} \equiv k_{n+1}$ and introduce the Sudakov decomposition

$$k_i = \beta_i p_1 + \alpha_i p_2 + k_{i\perp}\,, \qquad s\alpha_i\beta_i = k_i^2 - k_{i\perp}^2 = k_i^2 + \vec{k}_i^{\,2}\,, \tag{4}$$

where $p_{1,2}$ are the light cone momenta such that

$$p_A = p_1 + \frac{m_A^2}{s}p_2\,, \qquad p_B = p_2 + \frac{m_B^2}{s}p_1\,, \qquad 2p_1p_2 = s \tag{5}$$

(we admit all particles to have non zero masses, reserving the possibility to consider each of them as a compound state or as a group of particles) and the vector sign is used for the transverse components. Then in the MRK we have

$$\frac{\vec{p}_{\tilde{A}}^{\,2} + m_{\tilde{A}}^2}{s} \approx \alpha_0 \ll \alpha_1 \ldots \ll \alpha_n \ll \alpha_{n+1} \approx 1\,,$$
$$\frac{\vec{p}_{\tilde{B}}^{\,2} + m_{\tilde{B}}^2}{s} \approx \beta_{n+1} \ll \beta_n \ldots \ll \beta_1 \ll \beta_0 \approx 1\,. \tag{6}$$

Due to Eqs. (4)-(6) the squared invariant masses

$$s_i = (k_{i-1} + k_i)^2 \approx s\beta_{i-1}\alpha_i = \frac{\beta_{i-1}}{\beta_i}(k_i^2 + \vec{k}_i^{\,2}) \tag{7}$$

are large compared with the squared transverse momenta of the produced particles, which have the order of the squared momentum transfers:

$$s_i \gg \vec{k}_i^2 \sim \mid t_i \mid = \mid q_i^2 \mid , \tag{8}$$

where

$$q_i = p_A - \sum_{j=0}^{i-1} k_j = -\left(p_B - \sum_{j=i}^{n+1} k_j \right) \approx \beta_i p_1 - \alpha_{i-1} p_2 - \sum_{j=0}^{i-1} k_{j\perp} ,$$

$$t_i = q_i^2 \approx q_{i\perp}^2 = -\vec{q}_i^{\,2} , \tag{9}$$

and the product of s_i is proportional to s:

$$\prod_{i=1}^{n+1} s_i = s \prod_{i=1}^{n} (k_i^2 + \vec{k}_i^2) . \tag{10}$$

It is necessary to remind that, contrary to the elastic amplitude, the production amplitudes have a complicated analytical structure (see, for instance, Refs. [10,18]). Fortunately, only the real parts of these amplitudes are used in the derivation of the BFKL equation in the NLLA as well as in the LLA. The term "gluon Reggeization" used by us means that the real parts of the production amplitudes in the MRK have a simple factorized form and can be presented as

$$A_{AB}^{\tilde{A}\tilde{B}+n} = 2s\Gamma_{\tilde{A}A}^{c_1} \left[\prod_{i=1}^{n} \frac{1}{t_i} \gamma_{c_i c_{i+1}}^{P_i}(q_i, q_{i+1}) \left(\frac{s_i}{\sqrt{\vec{k}_{i-1}^2 \vec{k}_i^2}} \right)^{\omega(t_i)} \right]$$

$$\times \frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{\sqrt{\vec{k}_n^2 \vec{k}_{n+1}^2}} \right)^{\omega(t_{n+1})} \Gamma_{\tilde{B}B}^{c_{n+1}} , \tag{11}$$

where $\gamma_{c_i c_{i+1}}^{P_i}(q_i, q_{i+1})$ are the so-called Reggeon-Reggeon-particle (RRP) vertices, i.e. the effective vertices for the production of the particles P_i with momenta $k_i = q_i - q_{i+1}$ in the collision of the Reggeons with momenta q_i and q_{i+1} and colour indices c_i and c_{i+1}. Pay attention that we have taken definite energy scales in the Regge factors in Eq. (11) as well as in Eq. (3). In principle, we could take an arbitrary scale s_R; in this case the PPR and RRP vertices would become dependent on s_R. Of course, physical results do not depend on the scale.

In the LLA only one particle can be produced in the RRP vertex, and since our Reggeons are Reggeized gluons, this particle can be only a gluon. The situation is quite different in the NLLA. In this case we have to consider the so-called quasi-multi-Regge kinematics (QMRK) [9], where any (but only one) pair of the produced particles has a fixed (not increasing with s) invariant mass. We can treat this kinematics using the effective vertices $\gamma_{c_i c_{i+1}}^{G_1 G_2}(q_i, q_{i+1})$ [9,13] and $\gamma_{c_i c_{i+1}}^{Q\bar{Q}}(q_i, q_{i+1})$ [13,15] for the production of two gluons and a quark-antiquark pair respectively in Reggeon-Reggeon collisions, as well as the effective vertices $\Gamma_{P^* P}^{c}$ for the production of the "excited" state (containing an extra particle) in the fragmentation region of the particle P in the process of scattering of this particle off the Reggeon. Introducing these vertices the production amplitudes of $n+1$ particles in the QMRK are given by Eq. (11) with one of the vertices $\gamma_{c_i c_{i+1}}^{P_i}$ or $\Gamma_{\tilde{P}P}^{c}$ substituted with the vertices $\gamma_{c_i c_{i+1}}^{P_1 P_2}$ or $\Gamma_{P^* P}^{c}$ respectively.

Since in the limit of large invariant masses of all pairs of the final state particles the QMRK amplitudes must turn into the MRK ones, in this limit the effective vertices $\gamma_{c_i c_{i+1}}^{G_1 G_2}$ satisfy the factorization properties

$$\gamma_{c_i c_{i+1}}^{G_1 G_2}(q_i, q_{i+1}) = \gamma_{c_i c}^{G_1}(q_i, q_i - l_1) \frac{1}{(q_i - l_1)^2} \gamma_{c c_{i+1}}^{G_2}(q_i - l_1, q_{i+1}) \tag{12}$$

at $(p_B l_2) \ll (p_B l_1)$, $(p_A l_1) \ll (p_A l_2)$, where $l_{1,2}$ are the momenta of the produced gluons. The vertices $\Gamma_{P^* P}^{c}$ in the case in which the "excited" state P^* contains the gluon, i.e. $P^* = G\tilde{P}$, have the property

$$\Gamma_{G\tilde{P}P}^{c}(q) \simeq \Gamma_{\tilde{P}P}^{\tilde{c}} \frac{1}{(q+l)^2} \gamma_{\tilde{c}c}^{G}(q+l, q) \tag{13}$$

at $(p_P l) \gg (p_P p_{\tilde{P}})$, l being the gluon momentum and $q = p_P - p_{\tilde{P}}$.

The BFKL equation is straightforwardly obtained [6] if the amplitudes (3) and (11) are used in the unitarity relation for the s-channel imaginary part of the elastic scattering amplitude. Remind that the representations (3) and (11) for the amplitudes with the gluon quantum numbers in the t_i-channels were rigorously proved [17] in the LLA.

3 The generalized BFKL equation in the NLLA

Decomposing the elastic scattering amplitudes $\mathcal{A}_{AB}^{A'B'}$ in the parts with definite irreducible representation $\mathcal{R}$ of the colour group in the t-channel:

$$\mathcal{A}_{AB}^{A'B'} = \sum_{\mathcal{R}} \left(\mathcal{A}_{\mathcal{R}}\right)_{AB}^{A'B'} \ , \tag{14}$$

and using the amplitudes (11) and their generalization for the QMRK, we get for the s-channel imaginary part of the amplitudes $\mathcal{A}_{\mathcal{R}}$ (details will be given elsewhere)

$$\mathcal{I}m_s\left(\left(\mathcal{A}_{\mathcal{R}}\right)_{AB}^{A'B'}\right) = \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}\left(\vec{q}_1-\vec{q}\right)^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}\left(\vec{q}_2-\vec{q}\right)^2} \sum_{\nu} \Phi_{A'A}^{(\mathcal{R},\nu)}\left(\vec{q}_1;\vec{q};s_0\right)$$

$$\times \frac{s}{(2\pi)^{D-2}} \int_{\delta-\infty}^{\delta+\infty} \frac{d\omega}{2\pi i} \left[\left(\frac{s}{s_0}\right)^{\omega} G_{\omega}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2,\vec{q}\right)\right] \Phi_{B'B}^{(\mathcal{R},\nu)}\left(-\vec{q}_2;-\vec{q};s_0\right) \ . \tag{15}$$

Here s_0 is the energy scale (which can be, in principle, arbitrary), the index ν enumerates the states in the irreducible representation $\mathcal{R}$, $\Phi_{P'P}^{(\mathcal{R},\nu)}\left(\vec{q}_1;\vec{q};s_0\right)$ are the impact factors and $G_{\omega}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2,\vec{q}\right)$ is the Mellin transform of the Green function for the Reggeon-Reggeon scattering. The impact factors and the Green function appear as the generalization of those defined in Refs. [16,19] for the case of non-forward scattering and non-vacuum quantum numbers in the t-channel. The Green function obeys the equation

$$\omega G_{\omega}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2,\vec{q}\right) =$$

$$\vec{q}_1^{\,2}\left(\vec{q}_1-\vec{q}\right)^2 \delta^{(D-2)}\left(\vec{q}_1-\vec{q}_2\right) + \int \frac{d^{D-2}q'}{\vec{q}^{\,\prime 2}\left(\vec{q}^{\,\prime}-\vec{q}\right)^2} \mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}^{\,\prime};\vec{q}\right) G_{\omega}^{(\mathcal{R})}\left(\vec{q}^{\,\prime},\vec{q}_2;\vec{q}\right) . \tag{16}$$

Here the kernel

$$\mathcal{K}^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)$$

$$= \left[\omega\left(-\vec{q}_1^{\,2}\right) + \omega\left(-\left(\vec{q}_1-\vec{q}\right)^2\right)\right] \vec{q}_1^{\,2}\left(\vec{q}_1-\vec{q}\right)^2 \delta^{(D-2)}\left(\vec{q}_1-\vec{q}_2\right) + \mathcal{K}_r^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right) \tag{17}$$

is given as the sum of the "virtual" part, defined by the gluon trajectory, and the "real" part $\mathcal{K}_r^{(\mathcal{R})}$, related to the real particle production in Reggeon-Reggeon collisions. The "real" part can be written in the NLLA as

$$\mathcal{K}_r^{(\mathcal{R})}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right) = \int \frac{ds_{RR}}{(2\pi)^D} \mathcal{I}m\mathcal{A}_{RR}^{(\mathcal{R})}\left(q_1,q_2;\vec{q}\right) \theta\left(s_{\Lambda}-s_{RR}\right)$$

$$-\frac{1}{2}\int \frac{d^{D-2}q'}{\vec{q}^{\,\prime 2}\left(\vec{q}^{\,\prime}-\vec{q}\right)^2} \mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}_1,\vec{q}^{\,\prime};\vec{q}\right)$$

$$\times \mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}^{\,\prime},\vec{q}_2;\vec{q}\right) \ln\left(\frac{s_{\Lambda}^2}{\left(\vec{q}^{\,\prime}-\vec{q}_1\right)^2\left(\vec{q}^{\,\prime}-\vec{q}_2\right)^2}\right) \ . \tag{18}$$

In this equation $\mathcal{A}^{(\mathcal{R})}\left(q_1,q_2;\vec{q}\right)$ is the scattering amplitude of the Reggeons with initial momenta q_1 and $-q_2$ and momentum transfer q, for the representation $\mathcal{R}$ of the colour group in the t-channel, $s_{RR}=\left(q_1-q_2\right)^2$ is the squared invariant mass of the Reggeons; $\mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)$ is the part of the kernel at the Born (i.e. LLA) order related to the real particle production, which is given by the first term in the R.H.S. of Eq. (18) taken in the Born approximation. The expression for the s_{RR}-channel imaginary part $\mathcal{I}m\mathcal{A}^{(\mathcal{R})}\left(q_1,q_2;\vec{q}\right)$ in terms of the effective vertices for the production of particles in Reggeon-Reggeon collisions is given below. The intermediate parameter s_{Λ} in Eq. (18) must be taken tending to infinity, so that the dependence on s_{Λ} disappears in Eq. (18), because of the factorization property (12) of the two gluon production vertex.

The impact factors can be expressed through the imaginary part of the particle-Reggeon scattering amplitudes. In the NLLA the representation takes the form

$$\Phi_{P'P}^{(\mathcal{R},\nu)}\left(\vec{q}_R;\vec{q};s_0\right) = \int \frac{ds_{PR}}{2\pi s} \mathcal{I}m\mathcal{A}_{P'P}^{(\mathcal{R},\nu)}\left(p_P,q_R;\vec{q};s_0\right) \theta\left(s_{\Lambda}-s_{PR}\right)$$

$$-\frac{1}{2}\int \frac{d^{D-2}q'}{\vec{q}^{\,\prime 2}\left(\vec{q}^{\,\prime}-\vec{q}\right)^2} \Phi_{P'P}^{(\mathcal{R},\nu)B}\left(\vec{q}^{\,\prime},\vec{q}\right) \mathcal{K}_r^{(\mathcal{R})B}\left(\vec{q}^{\,\prime},\vec{q}_R\right) \ln\left(\frac{s_{\Lambda}^2}{\left(\vec{q}^{\,\prime}-\vec{q}_R\right)^2 s_0}\right) \ . \tag{19}$$

In Eq. (19) $s_{PR}=\left(p_P-q_R\right)^2$ is the squared particle-Reggeon invariant mass while $\mathcal{I}m\mathcal{A}_{P'P}^{(\mathcal{R},\nu)}\left(p_P,q_R;\vec{q};s_0\right)$ is the s_{PR}-channel imaginary part of the scattering amplitude of the particle P with momentum p_P off the Reggeon with momentum $-q_R$, $\vec{q}$ being the momentum transfer. The argument s_0 in the impact factor and in the amplitude shows that these two quantities depend on the energy scale s_0 of the Mellin transformation. Of course, physical quantities do not depend on this artificial parameter. It can be shown that with the NLLA accuracy the R.H.S. of Eq. (15) with the impact factors defined by Eq. (19) and Eq. (27) below, does not depend on s_0. The Born (LLA) impact factors $\Phi_{P'P}^{(\mathcal{R},\nu)B}$ are given by the first term in the R.H.S. of Eq. (19) taken in the Born approximation. Note that for the Born case the integral over s_{PR} in Eq. (19), as well as over s_{RR} in Eq. (18), is convergent, so that

the parameter s_A does not play any role. In the NLLA the independence of the impact factors on s_A is supplied by the factorization properties (12) and (13).

The imaginary parts of the Reggeon-Reggeon and particle-Reggeon scattering amplitudes, entering Eqs. (18) and (19) respectively, can be expressed in terms of the corresponding vertices, with the help of the operators $\hat{\mathcal{P}}_\mathcal{R}$ for the projection of two-gluon colour states in the t-channel on the irreducible representations $\mathcal{R}$ of the colour group. We have

$$\mathcal{I}m\mathcal{A}^{(\mathcal{R})}_{RR}\,(q_1,q_2;\vec{q})$$

$$= \frac{\langle c_1c_1'|\hat{\mathcal{P}}_\mathcal{R}|c_2c_2'\rangle}{2n_\mathcal{R}} \sum_{\{f\}} \int \gamma^{\{f\}}_{c_1c_2}\,(q_1,q_2)\left(\gamma^{\{f\}}_{c_1'c_2'}\,(q_1',q_2')\right)^* d\rho_f\ , \qquad (20)$$

where $n_\mathcal{R}$ is the number of the states in the representation $\mathcal{R}$, $\gamma^{\{f\}}_{c_1c_2}\,(q_1,q_2)$ is the effective vertex for the production of the particles $\{f\}$ in Reggeon-Reggeon collisions, $d\rho_f$ is their phase space element,

$$d\rho_f = (2\pi)^D\,\delta^{(D)}(q_1-q_2-\sum_{\{f\}} l_f)\prod_{\{f\}}\frac{d^{D-1}l_f}{(2\pi)^{D-1}\,2\epsilon_f}\ , \qquad (21)$$

and $q_i' = q_i - q$. The sum over $\{f\}$ in Eq. (20) is performed over all the contributing particles $\{f\}$ and over all their discreet quantum numbers. In the LLA only one-gluon production does contribute; in the NLLA the contributing states include also the two-gluon and the quark-antiquark states. The normalization of the corresponding vertices is defined by Eq. (11).

For us the most interesting representations $\mathcal{R}$ are the colour singlet (vacuum) and antisymmetric colour octet (gluon) ones. We have for the singlet case

$$\langle c_1c_1'|\hat{\mathcal{P}}_0|c_2c_2'\rangle = \frac{\delta_{c_1c_1'}\delta_{c_2c_2'}}{N^2-1}\ ; \quad n_0 = 1\ , \qquad (22)$$

and for the octet case

$$\langle c_1c_1'|\hat{\mathcal{P}}_8|c_2c_2'\rangle = \frac{f_{c_1c_1'c}f_{c_2c_2'c}}{N}\ ; \quad n_8 = 8\ , \qquad (23)$$

where f_{abc} are the structure constants of the colour group. The above matrix elements can be decomposed as

$$\sum_\nu \langle c_1c_1'|\hat{\mathcal{P}}_\mathcal{R}|\nu\rangle\langle\nu|\hat{\mathcal{P}}_\mathcal{R}|c_2c_2'\rangle \qquad (24)$$

with

$$\langle cc'|\hat{\mathcal{P}}_0|0\rangle = \frac{\delta_{cc'}}{\sqrt{N^2-1}}\ , \qquad (25)$$

$$\langle cc'|\hat{\mathcal{P}}_8|a\rangle = \frac{f_{acc'}}{\sqrt{N}}\ . \qquad (26)$$

This decomposition allows to write the imaginary part of the scattering amplitude of the particle P off the Reggeon in the form

$$\mathcal{I}m\mathcal{A}^{(\mathcal{R},\nu)}_{P'P}\,(p_P,q_R;\vec{q};s_0) = \langle cc'|\hat{\mathcal{P}}_\mathcal{R}|\nu\rangle$$

$$\times s\sum_{\{f\}}\int \Gamma^c_{\{f\}P}(q_R)\left(\Gamma^{c'}_{\{f\}P'}(q_R')\right)^*\left(\frac{s_0}{\vec{q}_R^{\,2}}\right)^{\frac{1}{2}\omega(-\vec{q}_R^{\,2})}\left(\frac{s_0}{\vec{q}_R^{\,\prime 2}}\right)^{\frac{1}{2}\omega(-\vec{q}_R^{\,\prime 2})} d\rho_f\ , \qquad (27)$$

where $\Gamma^c_{\{f\}P}$ are the effective vertices for the production of the states $\{f\}$. Their normalization is fixed by Eq. (11). At the parton level the contributing vertices are the PPR vertices $\Gamma^c_{P'P}$ which have to be taken in the one-loop approximation [10,11], the vertices $\Gamma^c_{G\hat{P}P}$ for the gluon emission in the fragmentation region and the vertices $\Gamma^c_{Q\bar{Q}G}$ for the gluon $\to$ quark-antiquark transition [12]. It is worthwhile to stress that for the case of the singlet representation $\mathcal{R}$ the expression (19) for the impact factors, together with Eq. (27), is valid for colorless objects (at the hadron level) as well. For small size objects (such as a photon with large virtuality) the impact factors can be calculated in the perturbation theory.

4 The bootstrap condition

Let us compare the s-channel imaginary part of the amplitude (3) with the imaginary part given by Eq. (15) in the case of the gluon representation in the t-channel. In the LLA from Eq. (3) we get

$$\mathcal{I}m_s(\mathcal{A}_8)^{A'B'}_{AB} = \Gamma^{c(B)}_{A'A}\left(\frac{s}{|t|}\right)^{1+\omega^{(1)}(t)}\pi\omega^{(1)}\,(t)\,\Gamma^{c(B)}_{B'B}\ , \qquad (28)$$

where the index (B) denotes the Born (LLA) expression and $\omega^{(1)}$ stands for the gluon trajectory calculated with the one-loop accuracy:

$$\omega^{(1)}(t) = \frac{g^2t}{(2\pi)^{(D-1)}}\frac{N}{2}\int\frac{d^{D-2}k}{\vec{k}^2(\vec{q}-\vec{k})^2}\ . \qquad (29)$$

The R.H.S. of Eq. (28) coincides in the LLA with that one of Eq. (15) due to the properties of the LLA impact factors and the Green function:

$$\Phi^{(8,c)B}_{P'P} = -ig\frac{\sqrt{N}}{2}\Gamma^{c(B)}_{P'P} \; , \tag{30}$$

independently of $\vec{q}_1$, $\vec{q}_2$, and

$$\int G^{(8)(B)}_{\omega}\left(\vec{q}_1,\vec{q}_2,\vec{q}\right)\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}\left(\vec{q}_2-\vec{q}\right)^2} = \frac{1}{\omega-\omega^{(1)}(t)} \; . \tag{31}$$

Applying these properties the imaginary part shown in Eq. (15) in the NLLA becomes

$$\begin{aligned}
\mathcal{I}m_s\left(\mathcal{A}_R\right)^{A'B'}_{AB} &= \pi\left(\frac{s}{|t|}\right)^{1+\omega^{(1)}(t)}\left\{\Gamma^{c(B)}_{A'A}\left[\omega^{(1)}(t)\left(1+\omega^{(1)}(t)\ln\left(\frac{|t|}{s_0}\right)\right)\right.\right. \\
&+\frac{g^2Nt}{2\left(2\pi\right)^{D-1}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}\left(\vec{q}_1-\vec{q}\right)^2}\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}\left(\vec{q}_2-\vec{q}\right)^2}\mathcal{K}^{(8)(1)}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)\ln\left(s\right)\Big]\Gamma^{c(B)}_{B'B} \\
&+ig\frac{\sqrt{Nt}}{\left(2\pi\right)^{D-1}}\int\frac{d^{D-2}q'}{\vec{q}\,'^2\left(\vec{q}\,'-\vec{q}\right)^2}\left[\Phi^{(8,c)(1)}_{A'A}\left(\vec{q}\,';\vec{q};s_0\right)\Gamma^{c(B)}_{B'B}\right. \\
&\left.\left.+\Gamma^{c(B)}_{A'A}\Phi^{(8,c)(1)}_{A'A}\left(\vec{q}\,';\vec{q};s_0\right)\right]\right\} \; ,
\end{aligned} \tag{32}$$

where $\mathcal{K}^{(8)(1)}$ and $\Phi^{(8,c)(1)}_{P'P}$ are the next-to-leading contributions to the kernel and to the impact factors respectively for the gluon quantum numbers in the t-channel. If now we require that this expression coincides with the imaginary part of the Regge form (3) in the NLLA:

$$\begin{aligned}
&\mathcal{I}m_s\left(\mathcal{A}_R\right)^{A'B'}_{AB} \\
&= \pi\left(\frac{s}{|t|}\right)^{1+\omega^{(1)}(t)}\left\{\Gamma^{c(B)}_{A'A}\left[\omega^{(1)}(t)+\omega^{(2)}(t)+\omega^{(1)}(t)\,\omega^{(2)}(t)\,\ln\left(s\right)\right]\Gamma^{c(B)}_{B'B}\right. \\
&\left.+\,\omega^{(1)}(t)\left[\Gamma^{c(1)}_{A'A}\Gamma^{c(B)}_{B'B}+\Gamma^{c(B)}_{A'A}\Gamma^{c(1)}_{B'B}\right]\right\} \; ,
\end{aligned} \tag{33}$$

where $\Gamma^{c(1)}_{P'P}$ is the one-loop correction to the PPR vertex and $\omega^{(2)}(t)$ is the two-loop contribution to the trajectory, we arrive at the following bootstrap equations:

$$\frac{g^2Nt}{2\left(2\pi\right)^{D-1}}\int\frac{d^{D-2}q_1}{\vec{q}_1^{\,2}\left(\vec{q}_1-\vec{q}\right)^2}\int\frac{d^{D-2}q_2}{\vec{q}_2^{\,2}\left(\vec{q}_2-\vec{q}\right)^2}\mathcal{K}^{(8)(1)}\left(\vec{q}_1,\vec{q}_2;\vec{q}\right)$$

$$=\omega^{(1)}(t)\,\omega^{(2)}(t) \tag{34}$$

and

$$\begin{aligned}
&ig\frac{\sqrt{Nt}}{\left(2\pi\right)^{D-1}}\int\frac{d^{D-2}q'}{\vec{q}\,'^2\left(\vec{q}\,'-\vec{q}\right)^2}\Phi^{(8,c)(1)}_{P'P}\left(\vec{q}\,';\vec{q};s_0\right) \\
&=\Gamma^{c(1)}_{P'P}\omega^{(1)}(t)+\Gamma^{c(B)}_{P'P}\frac{1}{2}\left[\omega^{(2)}(t)-\left(\omega^{(1)}(t)\right)^2\ln\left(\frac{\vec{q}^{\,2}}{s_0}\right)\right] \; .
\end{aligned} \tag{35}$$

5 Summary

In this paper we have considered the scattering amplitudes in QCD at high energies $\sqrt{s}$ and fixed momentum transfer $\sqrt{-t}$ in the next-to-leading approximation in $\ln s$. Due to analyticity and crossing properties Eqs. (14) and (15) define the non-forward scattering amplitudes in the NLLA in terms of the impact factors and the Green function for the Reggeized gluon scattering. The impact factors and the kernel of the equation for the Green function are given in terms of the gluon Regge trajectory and the Reggeon-particle vertices which were used also in the derivation of the BFKL equation for the forward scattering in the NLLA [16] and are known.

The requirement of the self consistency of the derivation of the BFKL equation, based on the gluon Reggeization, is expressed by Eqs. (34) and (35). Since the BFKL equation is very important for the theory of Regge processes at high energy $\sqrt{s}$ in perturbative QCD, these equations should be checked. All quantities entering these equations are unambiguously defined. The gluon Regge trajectory $\omega(t)$ is known with the two-loop accuracy [12]. The one-loop correction $\mathcal{K}^{(8)(1)}$ to the kernel is given by Eqs. (17), (18) and (20) in terms of the trajectory and the effective vertices for the particle production in Reggeon-Reggeon collisions. All vertices entering these equations were calculated with the required accuracy: the Reggeon-Reggeon-gluon vertex in Ref. [10], the vertices for the two-gluon production in Refs. [9,13] and the vertices for the quark-antiquark production in Refs. [14,15]. The PPR vertices entering the second bootstrap equation, i.e. Eq. (35), were obtained with the one-loop accuracy in Refs. [10,11]. Finally, the one loop correction $\Phi^{(8,c)(1)}_{P'P}$ to the impact factors is expressed by Eqs. (19) and (27) in terms of the PPR vertices and the vertices for the production of the "excited" states in the fragmentation regions. For the cases where the initial particles are quarks and gluons these vertices can be found in Ref. [12].

The explicit check of the validity of Eqs. (34) and (35) will be the subject of subsequent publications.

Acknowledgment. One of us (V.S.F.) thanks the Dipartimento di Fisica dell'Università di Milano, the Dipartimento di Fisica dell'Università della Calabria and the Istituto Nazionale di Fisica Nucleare - sezione di Milano and gruppo collegato di Cosenza for their warm hospitality during his stay in Italy. This work was supported in part by the Ministero italiano dell'Università e della Ricerca Scientifica e Tecnologica, in part by INTAS and in part by the Russian Fund of Basic Researches. Fruitful discussions with G. Marchesini and M. Ciafaloni were very helpful.

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Budker INP 98-54

Ответственный за выпуск А.М. Кудрявцев
Работа поступила 1.07.1998 г.

Сдано в набор 10.07.1998 г.
Подписано в печать 10.07.1998 г.
Формат бумаги 60×90 1/16 Объем 1.3 печ.л., 1.0 уч.-изд.л.
Тираж 105 экз. Бесплатно. Заказ № 54

Обработано на IBM PC и отпечатано на
ротапринте ИЯФ им. Г.И. Будкера СО РАН
Новосибирск, 630090, пр. академика Лаврентьева, 11.