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Abstract

I discuss radiative corrections to the BFKL equation for high energy cross sections in perturbative QCD. Due to the gluon Reggeization in the next-to-leading $\ln s$ approximation, the form of the BFKL equation remains unchanged and the corrections to the BFKL kernel are expressed in terms of the two-loop contribution to the gluon Regge trajectory, the one-loop correction to the Reggeon-Reggeon-gluon vertex and the contributions from two-gluon and quark-antiquark production in Reggeon-Reggeon collisions. I present the results of the calculation of the BFKL kernel in the next-to leading logarithmic approximation, the estimate of the Pomeron shift and the next-to-leading contribution to the anomalous dimensions of twist-2 operators near $j = 1$.

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1 Introduction

The BFKL equation [1] is very popular now, mainly due to recent experimental results on deep inelastic scattering of electrons on protons obtained at HERA [2], which show growth of the gluon density in the proton with decreasing of the fraction of the proton momentum carried by gluon. It can be used together with the DGLAP evolution equation [3] for the description of structure functions for the deep inelastic ep scattering at small values of the Bjorken variable x (see, for instance, [4] and references therein). The equation was derived for scattering amplitudes in QCD at high energies $\sqrt{s}$ and fixed momentum transfer $\sqrt{-t}$ in the leading logarithmic approximation (LLA) which means collection of all terms of the type $[\alpha_s \ln s]^n$. This approximation leads to a sharp increase of cross sections with c.m.s. energy $\sqrt{s}$. In fact, calculated in LLA, the total cross section σ_{tot}^{LLA} grows at large c.m.s. energies as a power of s:

$$\sigma_{tot}^{LLA} \sim \frac{s^{\omega_P^B}}{\sqrt{\ln s}}\,, \tag{1}$$

where ω_P^B is the LLA position of the most right singularity in the complex momentum plane of the t-channel partial wave with vacuum quantum numbers (Pomeron singularity), given by

$$\omega_P^B = \frac{g^2}{\pi^2} N \ln 2 \tag{2}$$

for the gauge group SU(N) ($N = 3$ for QCD) with gauge coupling constant g ($\alpha_s = \frac{g^2}{4\pi}$). Therefore, the Froissart bound $\sigma_{tot} < const(\ln s)^2$ is violated in

LLA. The reason of the violation is that the s-channel unitarity constraints for scattering amplitudes are not fulfilled in this approximation. The problem of unitarization of LLA results is extremely important from the theoretical point of view. It is concerned in a lot of papers (see, for example, [5] and references therein).

The violation of the Froissart bound means that LLA can not be applied at asymptotically large energies. But in the region of energies accessible for modern experiments it seems that the most important disadvantage of LLA is that neither the scale of s nor the argument of the running coupling constant α_s are fixed in this approximation. These uncertainties diminish the predictive power of LLA, permitting to change strongly numerical results by changing the scales. From the practical point of view, since the results of LLA are applied to the small x phenomenology, it is extremely important to remove these uncertainties. Another important problem is the determination of the region of energies and momentum transfers where LLA could be applicable. To solve these problems we have to know radiative corrections to LLA.

Therefore, the radiative corrections to the BFKL equation are very important, as they give the possibility to fix the argument of the running coupling, to define the scale of energy and to determine the region of applicability of the results obtained. My talk is devoted to these radiative corrections.

The outline of the talk is the following. In Section 2, I remind the derivation of the BFKL equation in LLA and the solution of this equation. In Section 3, I discuss the general form of corrections in the next-to-leading logarithmic approximation (NLLA) and present the two-loop correction to the gluon Regge trajectory and the contributions to the NLLA kernel from the one-loop correction to the one-gluon production and from the two-gluon and quark-antiquark pair production in the Reggeon-Reggeon collisions. In Section 4, all the contributions are collected together, the cancellation of infrared singularities is performed, the estimate of the shift of the Pomeron intercept and the next-to-leading contributions to anomalous dimensions of twist-2 operators near $j = 1$ are presented.

2 The BFKL equation in LLA

Despite the fact that the BFKL equation was obtained more than 20 years ago, till now a simple derivation of this equation does not exist, though attempts to do it continue (see, for examlpe, [6]). In the original derivation [1] the key role was played by the gluon Reggeization in QCD [7, 8]. In fact, the derivation can be performed without large difficulties if we adopt the

Reggeization. It is worthwhile to stress here, that the gluon Reggeization in QCD means something more than usually assumed. Namely, it means not merely that there is the Reggeon with gluon quantum numbers, negative signature and trajectory

$$j(t) = 1 + \omega(t) \quad (3)$$

passing through 1 at $t = 0$, but also that only this Reggeon gives the leading contribution in each order of the perturbation theory to amplitudes with the gluon quantum numbers in channels with fixed momentum transfers. Due to this property it is not difficult to calculate the leading contributions to the imaginary parts of elastic scattering amplitudes with arbitrary quantum numbers in t-channel at large s and fixed t using the unitarity condition. Full amplitudes are easily restored through their imaginary parts. The BFKL equation emerges from the representation of the amplitudes in the particular case of the forward scattering with vacuum quantum numbers in t - channel.

For the elastic scattering process $A+B \to A'+B'$ the gluon Reggeization means that at large s and fixed t, with

$$s = (p_A + p_B)^2, \; t = q^2, \; q = p_A - p_{A'}, \quad (4)$$

the amplitudes with gluon quantum numbers in t-channel have the factorized form

$$(\mathcal{A}_8)_{AB}^{A'B'} = \Gamma^i_{A'A} \frac{s}{t} \left[\left(\frac{s}{-t} \right)^{\omega(t)} + \left(\frac{-s}{-t} \right)^{\omega(t)} \right] \Gamma^i_{B'B}, \quad (5)$$

where $\Gamma^i_{A'A}$ are the particle-particle-Reggeon (PPR) vertices. In LLA for the deviation of the gluon trajectory from 1 we have [8]

$$\omega(t) = \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{(D-1)}} \frac{N}{2} \int \frac{d^{D-2}k_\perp}{k_\perp^2 (q-k)_\perp^2}, \quad (6)$$

where $t = q^2 \approx q_\perp^2$ and $D = 4 + 2\epsilon$ is the space-time dimension. A non-zero ϵ is introduced to regularize Feynman integrals. The integration in Eq.(6) is performed over $(D-2)$-dimensional momenta orthogonal to the initial particle momentum plane. The PPR vertices can be presented as

$$\Gamma^i_{A'A} = g \langle A'|T^i|A \rangle \Gamma_{A'A}, \quad (7)$$

where $\langle A'|T^i|A \rangle$ stands for a matrix element of the colour group generator in the corresponding representation (i.e. fundamental for quarks and adjoint for gluons). In LLA the helicities λ_P of each of the scattered particles P are conserved, so that in the helicity basis we have

$$\Gamma_{A'A} = \Gamma^{(0)}_{A'A} = \delta_{\lambda_{A'}\lambda_A} . \tag{8}$$

The s - channel unitarity relation for the imaginary part of the elastic scattering amplitude $\mathcal{A}_{AB}^{A'B'}$ can be presented as

$$Im_s \mathcal{A}_{AB}^{A'B'} = \frac{1}{2}\sum_{n=0}^{\infty}\sum_{\{f\}}\int \mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n}\left(\mathcal{A}_{A'B'}^{\tilde{A}\tilde{B}+n}\right)^* d\Phi_{\tilde{A}\tilde{B}+n}, \tag{9}$$

where Im_s stands for the s - channel imaginary part, $\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n}$ is the amplitude of production of n particles with momenta k_i, $i = 1, ..., n$ in the process $A+B \to \tilde{A}+\tilde{B}+n$, $\sum_{\{f\}}$ means sum over the discrete quantum numbers of the final particles in this process, $d\Phi_{\tilde{A},\tilde{B}+n}$ is the element of the final particle phase space. We admit all particles to have non zero masses (reserving the possibility to consider each of them as a group of particles) and use light-cone vectors p_1 and p_2 such that momenta of the initial particles A and B are equal $p_A = p_1 + (m_A^2/s)\, p_2$ and $p_B = p_2 + (m_B^2/s)\, p_1$ respectively and $s = 2(p_1 p_2)$. Using the Sudakov decomposition

$$k_i = \beta_i p_1 + \alpha_i p_2 + k_{i\perp}\, , \quad s\alpha_i\beta_i = k_i^2 - k_{i\perp}^2 = k_i^2 + \vec{k}_i^2, \tag{10}$$

where the vector sign denotes (here and below) transverse momenta, we obtain

$$d\Phi_{\tilde{A},\tilde{B}+n} = \frac{2}{s}(2\pi)^D \delta\Big(1 + \frac{m_A^2}{s} - \sum_{i=0}^{n+1}\alpha_i\Big)\delta\Big(1 + \frac{m_B^2}{s} - \sum_{i=0}^{n+1}\beta_i\Big)$$

$$\times\delta^{(D-2)}\Big(\sum_{i=0}^{n+1}k_{i\perp}\Big)\frac{d\beta_{n+1}}{2\beta_{n+1}}\frac{d\alpha_0}{2\alpha_0}\prod_{i=1}^{n}\frac{d\beta_i}{2\beta_i}\prod_{i=0}^{n+1}\frac{d^{D-2}k_{i\perp}}{(2\pi)^{D-1}}, \tag{11}$$

with the denotations

$$p_{\tilde{A}} = k_0, \; p_{\tilde{B}} = k_{n+1}. \tag{12}$$

In the unitarity condition (9) the contribution of order s, which we are interested in, is given by the region of limited (not growing with s) transverse momenta of produced particles. Only this region is considered in the following. Large logarithms come from integration over longitudinal momenta of the produced particles. Therefore in LLA, where production of each additional particle must give the large logarithm ($\ln s$), they are produced in the multi-Regge kinematics (MRK). By definition, in this kinematics

their transverse momenta are limited and their Sudakov variables α_i and β_i, $i = 0 \div n+1$, are strongly ordered (in another words, the produced particles are strongly ordered in the rapidity space). Let us take, for definiteness, that

$$\alpha_{n+1} \gg \alpha_n \gg \alpha_{n-1} .... \gg \alpha_0, \quad \beta_0 \gg \beta_1 \gg \beta_2 .... \gg \beta_{n+1}. \tag{13}$$

In this case the $\delta-$ functions in Eq. (11) give us

$$\alpha_{n+1} \simeq 1, \; \beta_0 \simeq 1 \tag{14}$$

and therefore

$$\alpha_0 \simeq \frac{\vec{p}_{\tilde{A}}^{\,2} + m_{\tilde{A}}^2}{s}, \quad \beta_{n+1} \simeq \frac{\vec{p}_{\tilde{B}}^{\,2} + m_{\tilde{B}}^2}{s}. \tag{15}$$

In MRK the squared invariant masses $s_{ij} = (k_i + k_j)^2$ of any pair of produced particles i and j are large. In order to obtain the large logarithm from the integration over β_i for each produced particle in the phase space (11), the amplitudes in the r.h.s. of the unitarity relation (9) must not decrease with the growth of the invariant masses. It is possible only in the case where there are exchanges of vector particles (i.e. gluons) in all channels with momentum transfers q_i, $i = 1 \div n+1$, with

$$q_i = p_A - \sum_{j=0}^{i-1}k_j = -\Big(p_B - \sum_{l=i}^{n+1}k_l\Big) \simeq \beta_i p_1 - \alpha_{i-1}p_2 - \sum_{j=0}^{i-1}k_{j\perp}; \quad q_i^2 \simeq q_{i\perp}^2 = -\vec{q}_i^{\,2}. \tag{16}$$

Due to the gluon Reggeization the amplitudes of such processes in LLA have simple multi-Regge form:

$$\mathcal{A}_{AB}^{\tilde{A}\tilde{B}+n} = 2s\Gamma_{\tilde{A}A}^{c_1}\left(\prod_{i=1}^{n}\gamma_{c_ic_{i+1}}^{P_i}(q_i, q_{i+1})\left(\frac{s_i}{s_R}\right)^{\omega(t_i)}\frac{1}{t_i}\right)$$

$$\times\frac{1}{t_{n+1}}\left(\frac{s_{n+1}}{s_R}\right)^{\omega(t_{n+1})}\Gamma_{\tilde{B}B}^{c_{n+1}}; \tag{17}$$

where s_R is some energy scale, which is irrelevant in LLA,

$$s_i = (k_{i-1} + k_i)^2 \simeq s\beta_{i-1}\alpha_i = \frac{\beta_{i-1}}{\beta_i}(\vec{k}_i^{\,2} + k_i^2), \; t_i = q_i^2 \simeq -\vec{q}_i^{\,2}. \tag{18}$$

$\omega(t)$ and $\Gamma_{P'P}^a$ are the gluon Regge trajectory and the PPR vertices given by Eqs.(6) and (7), (8) respectively; $\gamma_{c_ic_{i+1}}^{P_i}(q_i, q_{i+1})$ are the effective vertices of production of particles P_i with momenta $q_i - q_{i+1}$ in collisions of Reggeons (i.e. Reggeized gluons) with momenta q_i and $-q_{i+1}$ and colour indices c_i and c_{i+1} correspondingly. In LLA all produced particles P_i must be gluons; therefore, the masses of produced particles are equal zero. The Reggeon-Reggeon-gluon (RRG) vertex has the form [1]

$$\gamma^{G_i}_{c_i c_{i+1}}(q_i, q_{i+1}) = g T^{d_i}_{c_i c_{i+1}} e^*_\mu(k_i) C^\mu(q_{i+1}, q_i), \tag{19}$$

where $T^{d_i}_{c_i c_{i+1}}$ are the matrix elements of the $SU(N)$ group generators in the adjoint representation, d_i is the colour index of the produced gluon, $e(k_i)$ its polarization vector and $k_i = q_i - q_{i+1}$ its momentum;

$$C^\mu(q_{i+1}, q_i) = -q_i - q_{i+1} + p_1\left(\frac{q_i^2}{k_i p_1} + 2\frac{k_i p_2}{p_1 p_2}\right) - p_2\left(\frac{q_{i+1}^2}{k_i p_2} + 2\frac{k_i p_1}{p_1 p_2}\right). \tag{20}$$

The amplitude $A^{\tilde{A}\tilde{B}+n}_{A'B'}$ entering the unitarity relation (9) can be obtained from Eq. (17) by the substitutions $A \to A'$, $B \to B'$, $q_i \to q_i' \equiv q_i - q$, where $q = p_A - p_{A'} \simeq q_\perp$.

Let us introduce the operators $\hat{\mathcal{P}}_R$ for projection of two-gluon colour states in $t-$ channel in the unitarity condition (9) on the irreducible representations R of the colour group and use the decomposition

$$T^{d_i}_{c_i c_{i+1}} (T^{d_i}_{c_i' c_{i+1}'})^* = \sum_R c_R \langle c_i c_i' | \hat{\mathcal{P}}_R | c_{i+1} c_{i+1}' \rangle. \tag{21}$$

We'll be interested in the singlet (vacuum) and antisymmetrical octet (gluon) representations. For the first of them

$$\langle c_i c_i' | \hat{\mathcal{P}}_0 | c_{i+1} c_{i+1}' \rangle = \frac{\delta_{c_i c_i'} \delta_{c_{i+1} c_{i+1}'}}{N^2 - 1} \tag{22}$$

and for the second

$$\langle c_i c_i' | \hat{\mathcal{P}}_8 | c_{i+1} c_{i+1}' \rangle = \frac{f_{a c_i c_i'} f_{a c_{i+1} c_{i+1}'}}{N}, \tag{23}$$

so that one can easily find

$$c_o = N, \quad c_8 = \frac{N}{2}. \tag{24}$$

Using the decomposition (21) we obtain from (19)

$$\sum_{G_i} \gamma^{G_i}_{c_i c_{i+1}}(q_i, q_{i+1}) (\gamma^{G_i}_{c_i' c_{i+1}'}(q_i, q_{i+1}))^*$$

$$= \sum_R \langle c_i c_i' | \hat{\mathcal{P}}_R | c_{i+1} c_{i+1}' \rangle 2(2\pi)^{D-1} \mathcal{K}_r^{(R)}(\vec{q}_i, \vec{q}_{i+1}; \vec{q}), \tag{25}$$

where the sum is taken over colour and polarization states of the produced gluon and

$$\mathcal{K}_r^{(R)}(\vec{q}_i, \vec{q}_{i+1}; \vec{q}) = -\frac{g^2 c_R}{2(2\pi)^{D-1}} C^\mu(q_{i+1}, q_i) C_\mu(q_{i+1} - q, q_i - q)$$

$$= \frac{g^2 c_R}{(2\pi)^{D-1}} \left(\frac{\vec{q}_i^{\,2} (\vec{q}_{i+1} - \vec{q})^2 + \vec{q}_{i+1}^{\,2} (\vec{q}_i - \vec{q})^2}{(\vec{q}_i - \vec{q}_{i+1})^2} - \vec{q}^{\,2} \right). \tag{26}$$

The decomposition (21) corresponds to the decomposition of the elastic scattering amplitude $A^{A'B'}_{AB}$ in (9):

$$A^{A'B'}_{AB} = \sum_R (A_R)^{A'B'}_{AB}, \tag{27}$$

where A_R is the part of the scattering amplitude corresponding to the definite irreducible representation R of the colour group in t -channel. It is convenient to consider its partial wave $f_R(\omega, \vec{q})^{A'B'}_{AB}$ defined by

$$f_R(\omega, \vec{q})^{A'B'}_{AB} = \int_{s_0}^{\infty} \frac{ds}{s^2} \left(\frac{s}{s_0}\right)^{-\omega} Im_s (A_R)^{A'B'}_{AB}. \tag{28}$$

The amplitude itself is expressed trough the partial wave as

$$(A_R)^{A'B'}_{AB} = \frac{s}{2\pi} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{\sin(\pi\omega)} \left(\left(\frac{-s}{s_0}\right)^\omega - \tau \left(\frac{s}{s_0}\right)^\omega \right) f_R(\omega, \vec{q})^{A'B'}_{AB}, \tag{29}$$

where τ is the signature and coincides with the symmetry of the representation R. For the gluon representation the Born contribution must be added into the r.h.s. of (29). The term with τ takes into account the contribution to the amplitude from the u -channel imaginary part. Pay attention that the only antisymmetrical representation contributing to the decomposition (21) is the representation with the gluon quantum numbers. Therefore, only for this representation the contributions of s- and u- channel imaginary parts do not cancel each other. It means that in each order of perturbation theory the amplitudes with the gluon quantum numbers in t_i -channels are dominant.

Let us calculate the contribution $f_R^{(n)}(\omega;\vec q)_{AB}^{A'B'}$ into the partial wave coming from production of n gluons. Using Eqs.(11), (13)-(15), (18), we have

$$s=\left(\prod_{i=1}^{n+1}s_i\right)\left(\prod_{i=1}^{n}\vec k_i^2\right)^{-1}=\left(\prod_{i=1}^{n+1}\frac{s_i}{\sqrt{\vec k_{i-1}^{\,2}\vec k_i^{\,2}}}\right)\sqrt{\vec q_1^{\,2}\vec q_{n+1}^{\,2}},$$

$$\frac{ds}{s}d\Phi_{\tilde A\tilde B+n}=\frac{2\pi}{s}\prod_{i=1}^{n+1}\frac{ds_i}{2s_i}\frac{d^{D-2}q_{i\perp}}{(2\pi)^{D-1}} \qquad (30)$$

and from Eqs. (28), (9), (17) and (25) we obtain

$$f_R^{(n)}(\omega;\vec q)_{AB}^{A'B'}$$

$$=\frac{1}{(2\pi)^{D-2}}\int\left(\prod_{i=1}^{n+1}\frac{d^{D-2}q_{i\perp}}{\vec q_i^{\,2}(\vec q_i-\vec q)^2}\frac{ds_i}{s_i}\left(\frac{s_i}{s_R}\right)^{\omega(t_i)+\omega(t_i')}\left(\frac{s_i}{\sqrt{\vec k_{i-1}^{\,2}\vec k_i^{\,2}}}\right)^{-\omega}\right)$$

$$\times\left(\frac{s_0}{\sqrt{\vec q_1^{\,2}\vec q_{n+1}^{\,2}}}\right)^{\omega}\sum_\nu I_{A'A}^{(R,\nu)}\left(\prod_{i=1}^{n}\mathcal{K}_r^{(R)}(\vec q_i,\vec q_{i+1};\vec q)\right)I_{B'B}^{(R,\nu)}. \qquad (31)$$

The index ν here enumerates the states in the irreducible representation R, so that

$$\langle c_ic_i'|\hat{\mathcal P}_R|c_{i+1}c_{i+1}'\rangle=\sum_\nu\langle c_ic_i'|\hat{\mathcal P}_R|\nu\rangle\langle\nu|\hat{\mathcal P}_R|c_{i+1}c_{i+1}'\rangle \qquad (32)$$

and

$$I_{A'A}^{(R,\nu)}=\sum_{\tilde A}\Gamma_{\tilde AA}^{c_1}\left(\Gamma_{\tilde AA'}^{c_1'}\right)^*\langle c_1c_1'|\hat{\mathcal P}_R|\nu\rangle,$$

$$I_{B'B}^{(R,\nu)}=\sum_{\tilde B}\Gamma_{\tilde BB}^{c_{n+1}}\left(\Gamma_{\tilde BB'}^{c_{n+1}'}\right)^*\langle\nu|\hat{\mathcal P}_R|c_{n+1}c_{n+1}'\rangle. \qquad (33)$$

The sum here is taken over the discreet quantum numbers of the states $\tilde A$, $\tilde B$.

For the singlet representation the index ν takes only one value, so that we can omit it and have

$$\langle cc'|\hat{\mathcal P}_0|0\rangle=\frac{\delta_{cc'}}{\sqrt{N^2-1}}, \qquad (34)$$

whereas for the antisymmetrical octet (gluon) representation the index ν coincide with gluon colour index and

$$\langle cc'|\hat{\mathcal P}_8|a\rangle=\frac{f_{acc'}}{\sqrt N}, \qquad (35)$$

where f_{abc} are the structure constants of the colour group. Therefore, we obtain

$$I_{P'P}^{(0)}=\frac{g^2C_P}{\sqrt{N^2-1}}\langle P'|P\rangle\delta_{\lambda_{P'}\lambda_P}; \qquad (36)$$

where C_P is the value of the Casimir operator in corresponding representation, i.e. $C_P=\frac{N^2-1}{2N}$ for quarks and $C_P=N$ for gluons;

$$I_{P'P}^{(8,a)}=-ig^2\frac{\sqrt N}{2}\langle P'|T^a|P\rangle\delta_{\lambda_{P'}\lambda_P}=-ig\frac{\sqrt N}{2}\Gamma_{P'P}^{a}. \qquad (37)$$

The integration over s_i in Eq.(31) is performed from some fixed (independent from s) value to infinity. Note, that the essential integration region in Eq. (29) is $\omega\sim(\ln s)^{-1}$, so that in LLA we can omit terms of the type $\omega\ln s_R$, $\omega\ln\vec k_i^{\,2}$, as well as $\omega(t_i)\ln s_R$, $\omega(t_i')\ln s_R$. It corresponds to the statement that in LLA the scale of energy is not fixed. Therefore, independently from the lower limit of the integrations over s_i we obtain

$$f_R^{(n)}(\omega;\vec q)_{AB}^{A'B'}=\frac{1}{(2\pi)^{D-2}}\int\prod_{i=1}^{n+1}\frac{d^{D-2}q_{i\perp}}{(\omega-\omega(t_i)-\omega(t_i'))\vec q_i^{\,2}(\vec q_i-\vec q)^2}$$

$$\times\sum_\nu I_{A'A}^{(R,\nu)}\left(\prod_{i=1}^{n}\mathcal{K}_r^{(R)}(\vec q_i,\vec q_{i+1};\vec q)\right)I_{B'B}^{(R,\nu)}. \qquad (38)$$

Let us present the partial wave in the form

$$f_R(\omega;\vec q)_{AB}^{A'B'}=\frac{1}{(2\pi)^{D-2}}\int\frac{d^{D-2}q_{A\perp}}{\vec q_A^{\,2}(\vec q_A-\vec q)^2}\frac{d^{D-2}q_{B\perp}}{\vec q_B^{\,2}(\vec q_B-\vec q)^2}$$

$$\times\sum_\nu I_{A'A}^{(R,\nu)}G_\omega^{(R)}(\vec q_A,\vec q_B;\vec q)I_{B'B}^{(R,\nu)}. \qquad (39)$$

Then for the function $G_\omega^{(R)}$ (which can be called Green function for scattering of two Reggeized gluons) we obtain

$$G_\omega^{(R)}(\vec q_A,\vec q_B;\vec q)=\sum_{n=0}^{\infty}\int\left(\prod_{i=1}^{n+1}\frac{d^{D-2}q_{i\perp}}{\vec q_i^{\,2}(\vec q_i-\vec q)^2(\omega-\omega(t_i)-\omega(t_i'))}\right)$$

$$\times\left(\prod_{i=1}^{n}\mathcal{K}_r^{(R)}(\vec{q}_i,\vec{q}_{i+1};\vec{q})\right)\vec{q}_A^{\,2}(\vec{q}_A-\vec{q})^2\vec{q}_B^{\,2}(\vec{q}_B-\vec{q})^2$$

$$\times\delta^{(D-2)}(q_{1\perp}-q_{A\perp})\delta^{(D-2)}(q_{n+1\perp}-q_{B\perp}). \tag{40}$$

It is easy to see, that the sum in the r.h.s. is the perturbative solution of the equation

$$\omega G_\omega^{(R)}(\vec{q}_1,\vec{q}_2;\vec{q})=\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2\delta^{(D-2)}(\vec{q}_1-\vec{q}_2)$$

$$+\int\frac{d^{D-2}q_{1\perp}'}{\vec{q}_1^{\,\prime 2}(\vec{q}_1^{\,\prime}-\vec{q})^2}\mathcal{K}^{(R)}(\vec{q}_1,\vec{q}_1^{\,\prime};\vec{q})G_\omega^{(R)}(\vec{q}_1^{\,\prime},\vec{q}_2;\vec{q}), \tag{41}$$

where the kernel

$$\mathcal{K}^{(R)}(\vec{q}_1,\vec{q}_2;\vec{q})=(\omega(q_{1\perp}^2)$$

$$+\omega((q_1-q)_\perp^2))\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2\delta^{(D-2)}(\vec{q}_1-\vec{q}_2)+\mathcal{K}_r^{(R)}(\vec{q}_1,\vec{q}_2;\vec{q}) \tag{42}$$

consists of two parts; the first of them, so called virtual part, is expressed in terms of the gluon Regge trajectory and the second, related with the real particle production, is given by Eq.(26).

Let us consider the partial wave $f_R^{(\nu)}(\omega;\vec{q}_1,\vec{q})_B^{B'}$ for the Reggeized gluon scattering off the particle B defined as

$$f_R^{(\nu)}(\omega;\vec{q}_1,\vec{q})_B^{B'}=\int d^{D-2}q_{B\perp}G_\omega^{(R)}(\vec{q}_1,\vec{q}_B;\vec{q})\frac{I_{B'B}^{(R,\nu)}}{\vec{q}_B^{\,2}(\vec{q}_B-\vec{q})^2}, \tag{43}$$

where the impact factor $I_{B'B}^{(R,\nu)}$ is given by Eq.(33). This partial wave satisfies the equation

$$\omega f_R^{(\nu)}(\omega;\vec{q}_1,\vec{q})_B^{B'}=I_{B'B}^{(R,\nu)}+\int\frac{d^{D-2}q_{1\perp}'}{\vec{q}_1^{\,\prime 2}(\vec{q}_1^{\,\prime}-\vec{q})^2}\mathcal{K}^{(R)}(\vec{q}_1,\vec{q}_1^{\,\prime};\vec{q})f_R^{(\nu)}(\omega;\vec{q}_1^{\,\prime},\vec{q})_B^{B'}. \tag{44}$$

It is easy to see that for the case of the gluon representation the solution of this equation is

$$f_8^a(\omega;\vec{q}_1,\vec{q}))_B^{B'}=\frac{I_{B'B}^{(8,a)}}{\omega-\omega(q_\perp^2)}. \tag{45}$$

Therefore Eqs. (39), (43)-(45) give us

$$f_8(\omega,\vec{q})_{AB}^{A'B'}=I_{A'A}^{(8,a)}\frac{1}{(2\pi)^{D-2}}\int\frac{d^{D-2}q_{1\perp}}{\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^2}I_{B'B}^{(8,a)}\frac{1}{\omega-\omega(q_\perp^2)}$$

$$=\Gamma_{A'A}^a\frac{-\pi\omega(t)}{t(\omega-\omega(t))}\Gamma_{B'B}^a. \tag{46}$$

Exactly the same result one gets substituting Eq.(5) in Eq.(28). So, we have a kind of "bootstrap": we input the existence of the Reggeized gluon and obtained it as the solution of the equation derived from the unitarity.

Strictly speaking, this "bootstrap", although being very impressive and meaningful, can not be considered as a rigorous proof of the gluon Reggeization. For such proof we have to reproduce not only the form (5) of the elastic amplitudes, but also the Reggeized form (17) for all inelastic amplitudes. It was actually done in LLA in Ref. [9], so that in this approximation the proof of the gluon Reggeization really exists.

Let us turn now to the most interesting case - vacuum quantum numbers in t -channel. From now, only this case will be considered, with additional simplifying restriction of the forward scattering, i.e. $A'=A$, $B'=B$, $q=0$. Instead of the imaginary part of the forward scattering amplitude, we'll consider the total cross section $\sigma_{AB}(s)$,

$$\sigma_{AB}(s)=\frac{Im_s\mathcal{A}_{AB}^{AB}}{s}. \tag{47}$$

From Eqs.(29), (39) we obtain

$$\sigma_{AB}(s)=\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\frac{1}{(2\pi)^{D-2}}\int d^{D-2}q_{A\perp}d^{D-2}q_{B\perp}\left(\frac{s}{s_0}\right)^\omega$$

$$\times\frac{\Phi_A(\vec{q}_A)}{\vec{q}_A^{\,2}}G_\omega(\vec{q}_A,\vec{q}_B)\frac{\Phi_B(-\vec{q}_B)}{\vec{q}_B^{\,2}}, \tag{48}$$

where

$$\Phi_A(\vec{q}_A)=I_{AA}^{(0)}=\frac{1}{\sqrt{N^2-1}}\sum_{\tilde{A},c}|\Gamma_{\tilde{A}A}^c|^2;\ \ \vec{q}_A=-\vec{p}_{\tilde{A}},$$

$$\Phi_B(\vec{q}_B)=I_{BB}^{(0)}=\frac{1}{\sqrt{N^2-1}}\sum_{\tilde{B},c}|\Gamma_{\tilde{B}B}^c|^2;\ \ \vec{q}_B=-\vec{p}_{\tilde{B}} \tag{49}$$

and

$$G_\omega(\vec{q}_1,\vec{q}_2)=\frac{G_\omega^{(0)}(\vec{q}_1,\vec{q}_2;0)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}.[B \tag{50}$$

The PPR vertices $\Gamma_{\tilde{P}'P}^c$ are defined in Eqs.(7), (8) and the Green functions $G_\omega^{(R)}(\vec{q}_1,\vec{q}_2;\vec{q})$ in Eqs.(41), (42). Let us note that for quarks and gluons the impact factors $\Phi_P(\vec{q}_P)$ don't depend really on $\vec{q}_P$ in LLA. We indicate this dependence having in mind possible generalizations to the cases of scattering of other objects and corrections to LLA. The Green function $G_\omega(\vec{q}_1;\vec{q}_2)$ satisfies the equation

$$\omega\, G_\omega(\vec{q}_1;\vec{q}_2) = \delta^{(D-2)}(\vec{q}_1-\vec{q}_2) + \int d^{D-2}\tilde{q}\ \mathcal{K}(\vec{q}_1;\vec{q})\, G_\omega(\vec{q};\vec{q}_2) \qquad (51)$$

where

$$\mathcal{K}(\vec{q}_1;\vec{q}_2) = \frac{\mathcal{K}^{(0)}(\vec{q}_1;\vec{q}_2;0)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}} = 2\,\omega(-\vec{q}_1^{\,2})\,\delta^{(D-2)}(\vec{q}_1-\vec{q}_2) + \mathcal{K}_r(\vec{q}_1;\vec{q}_2)\,. \qquad (52)$$

Here $\omega(-\vec{q}^{\,2})$ is the gluon Regge trajectory given by Eq.(6) and the integral kernel $\mathcal{K}_r(\vec{q}_1;\vec{q}_2)$, related with the real particle production, is given by Eq.(26) at $q=0$:

$$\mathcal{K}_r(\vec{q}_1;\vec{q}_2) = \frac{\mathcal{K}_r^{(0)}(\vec{q}_1;\vec{q}_2;0)}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}} = \frac{1}{2(2\pi)^{D-1}(N^2-1)\vec{q}_1^{\,2}\vec{q}_2^{\,2}} \sum_{c_1,c_2,G_1} |\gamma_{c_1c_2}^{G_1}(q_1;q_2)|^2$$

$$= \frac{g^2N}{(2\pi)^{D-1}}\frac{2}{(\vec{q}_1-\vec{q}_2)^2}. \qquad (53)$$

Taken separately, the virtual (6) and real (53) contributions to the kernel (52) lead to infrared singularities. Indeed,

$$\omega^{(1)}(-\vec{q}^{\,2}) = -\frac{g^2N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}\frac{2}{\epsilon}(\vec{q}^{\,2})^{\epsilon}, \qquad (54)$$

whereas the real contribution (53) gives the term of order $1/\epsilon$ after integration around the point $\vec{q}_1-\vec{q}_2=0$. These singularities cancel each other in Eq.(51). Let us demonstrate it for the kernel averaged over the azimuthal angle between $\vec{q}_1$ and $\vec{q}_2$. In fact, only the averaged kernel is relevant until we don't consider spin correlations because in this case the impact factors entering Eq.(48) depend only on squared transverse momenta. It's clear therefore that the high energy behaviour of cross sections is determined by the averaged kernel.

Performing expansion in ϵ and keeping only the terms giving non-vanishing contributions at $\epsilon\to 0$ in (51), we obtain for the averaged kernel

$$\overline{\mathcal{K}_r(\vec{q}_1;\vec{q}_2)} = \frac{g^2N}{(2\pi)^{D-1}}\frac{2}{|\vec{q}_1^{\,2}-\vec{q}_2^{\,2}|}\left(\frac{|\vec{q}_2^{\,2}-\vec{q}_1^{\,2}|}{max(\vec{q}_2^{\,2};\vec{q}_1^{\,2})}\right)^{2\epsilon}. \qquad (55)$$

Instead of the dimensional regularization we can introduce the cut off $|\vec{q}_1^{\,2}-\vec{q}_2^{\,2}|>\lambda^2$ in the kernel $\mathcal{K}_r(\vec{q}_1;\vec{q}_2)$ changing correspondingly the virtual part. We obtain

$$\overline{\mathcal{K}(\vec{q}_1;\vec{q}_2)} = \frac{g^2N}{4\pi^3}\left(-2\ln\frac{\vec{q}_1^{\,2}}{\lambda^2}\delta(\vec{q}_1^{\,2}-\vec{q}_2^{\,2}) + \frac{\theta\left(|\vec{q}_1^{\,2}-\vec{q}_2^{\,2}|-\lambda^2\right)}{|\vec{q}_1^{\,2}-\vec{q}_2^{\,2}|}\right). \qquad (56)$$

The representation (56) permits to find easily such form of the kernel for which the cancellation of the singularities is evident. For this purpose it is enough to present

$$2\ln\frac{\vec{q}_1^{\,2}}{\lambda^2} = \int d\vec{q}_2^{\,2}\frac{\theta\left(|\vec{q}_1^{\,2}-\vec{q}_2^{\,2}|-\lambda^2\right)}{|\vec{q}_1^{\,2}-\vec{q}_2^{\,2}|}\phi\left(\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right) \qquad (57)$$

with $\phi(1)=1$. Evidently, the mentioned form is not unique. The one adopted in [1] and used in literature is

$$\int d^{D-2}q_2\,\mathcal{K}(\vec{q}_1;\vec{q}_2)f(\vec{q}_2^{\,2}) = \frac{N\alpha_s}{\pi}\int d\vec{q}_2^{\,2}\left[\frac{f(\vec{q}_2^{\,2})}{|\vec{q}_2^{\,2}-\vec{q}_1^{\,2}|}\right.$$

$$\left.-f(\vec{q}_1^{\,2})\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\left(\frac{1}{|\vec{q}_2^{\,2}-\vec{q}_1^{\,2}|}-\frac{1}{\sqrt{(\vec{q}_1^{\,2})^2+4(\vec{q}_2^{\,2})^2}}\right)\right]. \qquad (58)$$

In the following we'll use another choice:

$$\int d^{D-2}q_2\,\mathcal{K}(\vec{q}_1;\vec{q}_2)f(\vec{q}_2^{\,2})$$

$$= \frac{N\alpha_s}{\pi}\int\frac{d\vec{q}_2^{\,2}}{|\vec{q}_2^{\,2}-\vec{q}_1^{\,2}|}\left[f(\vec{q}_2^{\,2}) - 2\frac{\min(\vec{q}_2^{\,2};\vec{q}_1^{\,2})}{(\vec{q}_2^{\,2}+\vec{q}_1^{\,2})}f(\vec{q}_1^{\,2})\right]. \qquad (59)$$

Of course, the representations (58) and (59) are equivalent. Both of them make explicit the scale invariance of the kernel, due to which its eigenfunctions are powers of $\vec{q}_2^{\,2}$. We'll take them as $(\vec{q}_2^{\,2})^{\gamma-1}$ and denote the corresponding eigenvalues as $\frac{N\alpha_s}{\pi}\chi^B(\gamma)$:

$$\int d^{D-2}q_2\,\mathcal{K}(\vec{q}_1;\vec{q}_2)(\vec{q}_2^{\,2})^{\gamma-1} = \frac{N\alpha_s}{\pi}\chi^B(\gamma)(\vec{q}_1^{\,2})^{\gamma-1}, \qquad (60)$$

so that [1]

$$\chi^B(\gamma) = 2\psi(1)-\psi(\gamma)-\psi(1-\gamma),\quad \psi(\gamma)=\Gamma'(\gamma)/\Gamma(\gamma)\,. \qquad (61)$$

The set of functions $(\vec{q}_2^{\,2})^{\gamma-1}$ with $\gamma = 1/2 + i\nu$, $-\infty < \nu < \infty$ is complete, so that we have

$$\sigma_{AB}(s) = \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{2\pi i} \int_{-\infty}^{+\infty} \frac{d\nu}{2\pi^2 \left(\omega - \frac{N\alpha_s}{\pi}\chi^B(1/2+i\nu)\right)}$$

$$\times \int \frac{d^2 q_{A\perp}}{2\pi} \int \frac{d^2 q_{B\perp}}{2\pi} \left(\frac{s}{s_0}\right)^{\omega} \Phi_A(\vec{q}_A)(\vec{q}_A^{\,2})^{-i\nu-3/2} \Phi_B(-\vec{q}_B)(\vec{q}_B^{\,2})^{i\nu-3/2}. \tag{62}$$

The cross section exists only if the impact factors possess a good infrared behaviour; otherwise it turns into infinity. In fact, it is infinite for scattering of colour particles, as it should be, and finite for colourless ones, because for them

$$\Phi_P(\vec{q}_P) \sim \vec{q}_P^{\,2} \tag{63}$$

for small $\vec{q}_P$. The maximal value of $\chi^B(\gamma)$ on the integration contour in Eq.(62) is $\chi(1/2) = 4\ln 2$, that corresponds to the maximal eigenvalue of the kernel $\omega_P^B = 4N(\alpha_s/\pi)\ln 2$. At $\omega = \omega_P^B$ the partial wave has the branch point that leads to the growth (1) of the cross section.

3 The Next-to-Leading Approximation

The next-to-leading logarithmic approximation (NLLA) means that all terms of the type $\alpha[\alpha_s \ln(1/x)]^n$ have to be collected. It was argued in Ref. [10] that in this approximation we can use the approach which coincides in the main features with that used in LLA. In general, the programme of the calculations is analogous to that in LLA. The final goal is the elastic scattering amplitude. It has to be restored from its s- and u-channel imaginary parts. The s-channel imaginary part is given by the unitarity relation (9). Evidently, Eqs.(27)-(29) expressing the elastic scattering amplitudes in terms of their s-channel imaginary parts remain unchanged. In the multi-Regge kinematics (MRK) in NLLA, as well as in LLA, only the amplitudes with the gluon quantum numbers in the channels with momentum transfers q_i do contribute. It was mentioned after Eq.(29) that in each order of perturbation theory these amplitudes are dominant, because only for them there is no cancellation between s- and u - channel contributions. Moreover, for the same reason only in these amplitudes the leading terms are real, whereas in other amplitudes they are imaginary. Therefore, the appearance in the r.h.s. of the unitarity relation (9) of amplitudes with quantum numbers in t_i-channels different from the gluon ones leads to loss of at least two large logarithms and therefore can

be ignored in NLLA. Evidently, it is a peculiar property of NLLA. In the approximations next to NLLA such amplitudes do contribute.

As before, the key point in the calculation of the amplitudes contributing in the unitarity relation is the gluon Reggeization. In MRK the real parts of the contributing amplitudes (only these parts are relevant in NLLA because the LLA amplitudes are real) are presented in the same form (29) as in LLA. So, in this kinematics the problem is reduced to the calculation of the two-loop contribution $\omega^{(2)}(t)$ to the gluon Regge trajectory $\omega(t)$ and the corrections to the real parts of the PPR- and RRG- vertices. Let us note here, that the PPR-vertices, as well as in LLA, enter the expressions for the impact factors (see Eq.(33)), but not the expression for the kernel (see Eq.(42),(26)), so that the corrections to these vertices appear at the intermediate steps of the calculations only. The one-loop corrections to the PPR vertices were calculated in Refs. [11, 12]. Though they are necessary for the calculation of the corrections to trajectory and RRG vertex, they don't enter the kernel explicitly, therefore I don't present them here.

But in NLLA, contrary to LLA, MRK is not a single kinematics that contributes in the unitarity relation (9). Since we have a possibility to loose one large logarithm (in comparison with LLA), the limitation of the strong ordering (13) in the rapidity space can not be implied more. Any (but only one) pair of the produced particles can have a fixed (not increasing with s) invariant mass, i.e. components of this pair can have rapidities of the same order. This kinematics was called [10] quasi-multi-Regge kinematics (QMRK). We can treat this kinematics including, together with the one-gluon production, production of more complicated states in the Reggeon-Reggeon (RR) collisions, namely gluon-gluon (GG) and quark-antiquark $Q\bar{Q}$ states, as well as production of "excited" states, i.e. states with larger number of particles, in the Reggeon-particle (RP) collisions in the fragmentation region of one of the initial particles. Therefore, the partial wave (28) can be presented in the same form (39), but with modified impact factors and RR Green function. In the definition of the impact factors (33) we have to include radiative corrections to the PPR vertices and the contribution of the "excited" states in the fragmentation region. The equation (41) for the RR Green function remains unchanged, as well as the representation (42) of the kernel, but the gluon trajectory has to be taken in the two-loop approximation:

$$\omega(t) = \omega^{(1)}(t) + \omega^{(2)}(t), \tag{64}$$

and the part, related with the real particle production, must contain, together with the contribution from the one-gluon production in the RR collisions, contributions from the two-gluon and quark-antiquark productions. The one-gluon contribution must be calculated with the one-loop accuracy, whereas the two-gluon and quark-antiquark contributions have to be taken in the Born approximation. In the following we consider only the case of the forward scattering and present the part of the kernel related with the real production in the form

$$\mathcal{K}_r(\vec{q}_1,\vec{q}_2) = \mathcal{K}_{RRG}^{one-loop}(\vec{q}_1,\vec{q}_2) + \mathcal{K}_{RRGG}^{Born}(\vec{q}_1,\vec{q}_2) + \mathcal{K}_{RRQ\bar{Q}}^{Born}(\vec{q}_1,\vec{q}_2). \tag{65}$$

Here there is a subtle point. Calculating the two-gluon production contribution to the kernel (as well as the contribution to the impact factor from the gluon production in the fragmentation region) we meet divergencies of integrals over invariant masses of the produced particles at upper limits (let us call such divergencies ultraviolet). These divergencies correspond to the uncertainties of the lower limits of integrations over s_i (see Eq. (31)) in MRK, which were not important in LLA, but can not be ignored in NLLA. Of course, the reason for the divergencies is the absence of a natural bound between MRK and QMRK. In order to give a precise meaning to the corresponding contributions and to treat them carefully we introduce an artificial bound (which, of course, disappears in final results). The discussion of the separation of the MRK and QMRK contributions presented in subsection 3.3 is based on the paper [13].

In the next subsections we'll discuss various contributions to the kernel.

3.1 The Reggeized Gluon Trajectory

The two-loop corrections to the gluon trajectory were obtained in Refs. [14, 15]. They were expressed in terms of discontinuities of QCD scattering amplitudes with gluon quantum numbers in t-channel calculated at large energy $\sqrt{s}$ and fixed momentum transfer $\sqrt{-t}$ in the two-loop approximation. The processes of the quark-quark, gluon-gluon and quark-gluon elastic scattering were considered. Independently from the process, it was obtained for the case of n_f massless quark flavours:

$$\omega^{(2)}(t) = \frac{g^2 t}{(2\pi)^{D-1}}\int\frac{d^{(D-2)}q_1}{\vec{q}_1^{\,2}(\vec{q}_1-\vec{q})^{\,2}}\left[F(\vec{q}_1,\vec{q}) - 2F(\vec{q}_1,\vec{q}_1)\right], \tag{66}$$

where $t = q^2 = -\vec{q}^{\,2}$ and

$$F(\vec{q}_1,\vec{q}) = -\frac{g^2N^2\vec{q}^{\,2}}{4(2\pi)^{D-1}}\int\frac{d^{(D-2)}q_2}{\vec{q}_2^{\,2}(\vec{q}_2-\vec{q})^2}\Bigg[\ln\left(\frac{\vec{q}^{\,2}}{(\vec{q}_1-\vec{q}_2)^2}\right) - 2\psi(D-3)$$

$$-\psi\left(3-\frac{D}{2}\right) + 2\psi\left(\frac{D}{2}-2\right) + \psi(1) + \frac{2}{(D-3)(D-4)} + \frac{D-2}{4(D-1)(D-3)}\Bigg]$$
$$+\frac{2g^2Nn_f\Gamma\left(2-\frac{D}{2}\right)\Gamma^2\left(\frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}}\Gamma(D)}\left(\vec{q}^{\,2}\right)^{\left(\frac{D}{2}-2\right)}. \tag{67}$$

Eqs. (66) and (67) give us a closed expression for the two- loop correction to the gluon trajectory. The independence from the properties of the scattered particles, which appears as the result of remarkable cancellations among various terms, sets up a stringent test of the gluon Reggeization beyond the leading logarithmic approximation.

The two-loop correction to the trajectory contains both ultraviolet and infrared divergencies. The former ones can be easily removed by the charge renormalization in the total expression for the trajectory. Since the trajectory itself must not be renormalized, we have only to use the renormalized coupling constant g_μ instead of the bare one g . In the $\overline{MS}$ scheme

$$g = g_\mu\mu^{-\epsilon}\left[1+\left(\frac{11}{3}-\frac{2}{3}\frac{n_f}{N}\right)\frac{\bar{g}_\mu^2}{2\epsilon}\right], \tag{68}$$

where

$$\bar{g}_\mu^2 = \frac{g_\mu^2 N\Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}}, \tag{69}$$

and calculation of the integrals in (66), (67) gives [15]:

$$\omega(t) = -\bar{g}_\mu^2\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^\epsilon\frac{2}{\epsilon}\Bigg\{1+\frac{\bar{g}_\mu^2}{\epsilon}\Bigg[\left(\frac{11}{3}-\frac{2}{3}\frac{n_f}{N}\right)\left(1-\frac{\pi^2}{6}\epsilon^2\right) -$$
$$\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)^\epsilon\left(\frac{11}{6}+\left(\frac{\pi^2}{6}-\frac{67}{18}\right)\epsilon+\left(\frac{202}{27}-\frac{11\pi^2}{18}-\zeta(3)\right)\epsilon^2-\right.$$
$$\left.\frac{n_f}{3N}\left(1-\frac{5}{3}\epsilon+\left(\frac{28}{9}-\frac{\pi^2}{3}\right)\epsilon^2\right)\right)\Bigg]\Bigg\}. \tag{70}$$

The remarkable fact which occurred is the cancellation of the third order poles in ϵ existing in separate contributions to the gluon part of (66). This cancellation is very important for the absence of infrared divergences in the corrections to the BFKL equation. As result of this cancellation, the gluon and quark contributions to $\omega^{(2)}(t)$ have similar infrared behaviour. Moreover, the coefficient of the leading singularity in ϵ is proportional to the coefficient of the one-loop β function. This means that infrared divergences are strongly correlated with ultraviolet ones. The correlation is unique in the sense that it provides the independence of singular contributions to $\omega(t)$ from $\vec{q}^{\,2}$. Indeed, expanding Eq. (70) we have

$$\omega(t) = -\bar{g}_\mu^2 \left(\frac{2}{\epsilon} + 2\ln\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)\right) - \bar{g}_\mu^4 \left[\left(\frac{11}{3} - \frac{2}{3}\frac{n_f}{N}\right)\left(\frac{1}{\epsilon^2} - \ln^2\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)\right)\right.$$

$$\left. + \left(\frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{9}\frac{n_f}{N}\right)\left(\frac{1}{\epsilon} + 2\ln\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)\right) - \frac{404}{27} + 2\zeta(3) + \frac{56}{27}\frac{n_f}{N}\right] \qquad (71)$$

Eq. (71) exhibits explicitly all singularities of the trajectory in the two-loop approximation and gives its finite part in the limit $\epsilon \to 0$. Let us stress that the independence of the singular contributions to the trajectory from $\vec{q}^{\,2}$ is necessary for the cancellation of the infrared divergencies in the BFKL equation.

3.2 One-Gluon Production Contribution

The simplest process for extracting the RRG vertex is production of one gluon with momentum k_1 in MRK:

$$s \gg s_{1,2},\ |t_{1,2}|,\quad s_1 = (k_1 + p_{\tilde{A}})^2,\quad s_2 = (k_1 + p_{\tilde{B}})^2,$$

$$s_1 s_2 \simeq s\vec{k}_1^{\,2},\quad t_i = q_i^2 \simeq -\vec{q}_i^{\,2}. \qquad (72)$$

It is worthwhile to remind, that the MRK amplitudes beyond the LLA have a complicated analytical structure. For the one-particle production amplitude, assuming the Regge behaviour in the sub-channels s_1 and s_2, from general requirements of analiticity, unitarity and crossing symmetry one has (see Refs. [11, 16])

$$A_{AB}^{\tilde{A}\tilde{B}+1} = \frac{s}{4}\Gamma^{c_1}_{\tilde{A}A}\frac{1}{t_1}T^d_{c_1c_2}\frac{1}{t_2}\Gamma^{c_2}_{\tilde{B}B}$$

$$\times\left\{\left[\left(\frac{-s_1}{s_R}\right)^{\omega_1-\omega_2} + \left(\frac{s_1}{s_R}\right)^{\omega_1-\omega_2}\right]\left[\left(\frac{-s}{s_R}\right)^{\omega_2} + \left(\frac{s}{s_R}\right)^{\omega_2}\right]R\right.$$

$$\left. + \left[\left(\frac{-s_2}{s_R}\right)^{\omega_2-\omega_1} + \left(\frac{s_2}{s_R}\right)^{\omega_2-\omega_1}\right]\left[\left(\frac{-s}{s_R}\right)^{\omega_1} + \left(\frac{s}{s_R}\right)^{\omega_1}\right]L\right\}, \qquad (73)$$

where $\omega_i = \omega(t_i)$ and the right and left RRG vertices $R^{(s_R)}$ and $L^{(s_R)}$ are real in all physical channels. Fortunately, as it was explained at the begining of this section, in NLLA only real parts of the production amplitudes do contribute , because only these parts interfere with the LLA amplitudes, which are real. Therefore, for our purposes we can neglect the imaginary parts and present the amplitudes in the form (17) with

$$\gamma^{G_1}_{c_1c_2}(q_1, q_2) = T^d_{c_1c_2}\frac{1}{2}\left\{(R+L)\left(1 - \frac{\omega^{(1)}(t_1) + \omega^{(1)}(t_2)}{2}\ln\left(\frac{\vec{k}_1^2}{s_R}\right)\right)\right.$$

$$\left. + (R-L)\frac{\omega^{(1)}(t_1) - \omega^{(1)}(t_2)}{2}\ln\left(\frac{\vec{k}_1^2}{s_R}\right)\right\} \qquad (74)$$

where $\omega^{(1)}(t)$ is the one-loop contribution to the gluon Regge trajectory.

The right and left RRG vertices R and L were calculated in Refs. [11]. The calculations were performed in the space-time dimension $D \neq 4$, but terms vanishing at $D \to 4$ were omitted in the final expressions. Unfortunately, such terms should be kept, because integration over transverse momenta of the produced gluon leads to divergency at $k_{1\perp} = 0$ for the case $D = 4$. Therefore, in the region $k_{1\perp} \to 0$ we need to know the production amplitude for arbitrary ϵ. The corresponding calculations were performed in [17].

Let us note here, that whereas the dependence of the Regge factors from the energy scale s_R was beyond the accuracy of LLA, in NLLA it has to be taken into account, though we can use an arbitrary scale. It means, that the RRP-vertices, as well as the PPR-vertices become dependent from the energy scale s_R. In the following we'll show explicitly this dependence, denoting the PPG-vertices by $\Gamma^{(s_R)a}_{\tilde{P}P}$ and the RRG-vertices by $\gamma^{(s_R)G_1}_{c_1c_2}(q_1, q_2)$. From the physical requirement of the independence of the production amplitudes from the energy scale we have:

$$\Gamma^{(s_R)a}_{\tilde{P}P} = \Gamma^{(s'_R)a}_{\tilde{P}P}\left(\frac{s_R}{s'_R}\right)^{\frac{1}{2}\omega(t_{P\tilde{P}})} \simeq \Gamma^{(s'_R)a}_{\tilde{P}P}\left(1 + \frac{1}{2}\omega(t_{P\tilde{P}})\ln\left(\frac{s_R}{s'_R}\right)\right), \qquad (75)$$

where $t_{PP'}$ is the squared momentum transfer from P to P', and

$$\gamma^{(s_R)G_1}_{c_1c_2}(q_1, q_2) = \gamma^{(s'_R)G_1}_{c_1c_2}(q_1, q_2)\left(\frac{s_R}{s'_R}\right)^{\frac{1}{2}(\omega_1+\omega_2)}$$

$$\simeq \gamma^{(s'_R)G_1}_{c_1c_2}(q_1, q_2)\left(1 + \frac{1}{2}(\omega_1 + \omega_2)\ln\left(\frac{s_R}{s'_R}\right)\right). \qquad (76)$$

In Refs. [11],[17] the PPR vertices were extracted from the elastic scattering amplitudes assuming the representation (5) for them; therefore, the scales $-t_{P\tilde{P}}$ were used for these vertices. In turn, the production vertices R and L were extracted from the one-gluon production amplitude using the representation (73), with the scale in the Regge factors equal the renormalization scale μ^2, but with the PPR vertices taken for the scales $-t_{P\tilde{P}}$. So, several scales were mixed in the one-gluon production amplitude. Though such mixing of scales is not forbidden, it is quite inconvenient when we consider production of arbitrary number of gluons. Therefore we'll use for definition of all vertices the representation (17) with the single scale s_R. Then, according to Eqs.(75), (76), the vertices $\gamma^{(s_R)G_1}_{c_1c_2}(q_1,q_2)$ differ from given by Eq.(74) with $s_R=\mu^2$ and vertices R and L taken from Refs. [11],[17] by their LLA values (19) multiplied by the factor

$$\frac{1}{2}\left(\omega^{(1)}(t_1)\ln\left(\frac{-t_1 s_R}{\mu^4}\right)+\omega^{(1)}(t_2)\ln\left(\frac{-t_2 s_R}{\mu^4}\right)\right). \quad (77)$$

Therefore we have

$$\gamma^{(s_R)G_1}_{c_1c_2}(q_1,q_2)=T^d_{c_1c_2}g_\mu\mu^{-\epsilon}e^*_\nu(k_1)\Bigg(C^\nu(q_2,q_1)\Bigg[1+\frac{\omega^{(1)}(t_1)+\omega^{(1)}(t_2)}{2}\ln\left(\frac{s_R}{\vec{k}_1^2}\right)$$
$$+\bar{g}^2_\mu\Bigg[\left(\frac{11}{6}-\frac{n_f}{3N}\right)\left(\frac{1}{\epsilon}+\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)-\left(\frac{\vec{k}_1^2}{\mu^2}\right)^\epsilon\left(\frac{1}{\epsilon^2}-\frac{\pi^2}{2}+2\epsilon\zeta(3)\right)$$
$$-\frac{1}{2}\ln^2\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}+\left(1-\frac{n_f}{N}\right)\frac{\vec{k}_1^2}{3}\left(\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}-\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)\Bigg]\Bigg]$$
$$+\left(\frac{p_A}{k_1p_A}-\frac{p_B}{k_1p_B}\right)^\nu\bar{g}^2_\mu\Bigg[\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+\left(1-\frac{n_f}{N}\right)\frac{\vec{k}_1^2}{3}$$
$$\times\left(\frac{(2\vec{k}_1^2-\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{2}\right)+\frac{1}{2}\right)\Bigg]\Bigg). \quad (78)$$

As for the vertices $R^{(s_R)}$ and $L^{(s_R)}$ separately, they can be found taking into account that the combination $R-L$ does not contribute in the LLA (see (74)) and therefore can be taken in the first nonvanishing approximation. We obtain

$$R^{(s_R)}-L^{(s_R)}=R-L,$$

$$R^{(s_R)}+L^{(s_R)}=(R+L)\left[1+\frac{\omega^{(1)}(t_1)}{2}\ln\left(\frac{-t_1}{\mu^2}\right)+\frac{\omega^{(1)}(t_2)}{2}\ln\left(\frac{-t_2}{\mu^2}\right)\right]$$
$$+(R-L)\frac{\omega^{(1)}(t_1)-\omega^{(1)}(t_2)}{2}\ln\left(\frac{s_R}{\mu^2}\right), \quad (79)$$

where R and L are taken from Refs. [11],[17] . So, we have

$$R^{(s_R)}+L^{(s_R)}=2g_\mu\mu^{-\epsilon}e^*_\nu(k_1)\Bigg(C^\nu(q_2,q_1)\Bigg\{1+\bar{g}^2_\mu\Bigg[\left(\frac{11}{6}-\frac{n_f}{3N}\right)$$
$$\times\left(\frac{1}{\epsilon}+\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)-\left(\frac{\vec{k}_1^2}{\mu^2}\right)^\epsilon\left(\left(\frac{1}{\epsilon^2}-\frac{\pi^2}{2}+2\epsilon\zeta(3)-\epsilon\ln\left(\frac{\vec{k}_1^2}{s_R}\right)\right.\right.$$
$$\left.\left.\times\left(\frac{1}{\epsilon^2}-\frac{\pi^2}{6}+2\epsilon\zeta(3)\right)\right)-\frac{1}{2}\ln^2\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}$$
$$+\left(1-\frac{n_f}{N}\right)\frac{\vec{k}_1^2}{3}\left(\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}-\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^3}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)\Bigg]\Bigg\}$$
$$+\left(\frac{p_A}{k_1p_A}-\frac{p_B}{k_1p_B}\right)^\nu\bar{g}^2_\mu\Bigg[\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+\left(1-\frac{n_f}{N}\right)\frac{\vec{k}_{1\perp}^2}{3}$$
$$\times\left(\frac{(2\vec{k}_1^2-\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{2}\right)+\frac{1}{2}\right)\Bigg]\Bigg), \quad (80)$$

$$R^{(s_R)}-L^{(s_R)}=2g_\mu\frac{\mu^{-\epsilon}e^*_\nu(k)C^\nu(q_2,q_1)}{\omega^{(1)}(t_1)-\omega^{(1)}(t_2)}\bar{g}^2_\mu\left(\frac{\vec{k}_1^2}{\mu^2}\right)^\epsilon\left(\frac{-2}{\epsilon}+\frac{\epsilon\pi^2}{3}-4\epsilon^2\zeta(3)\right). \quad (81)$$

One can check that substitution of (80), (81) into (74) gives (78).

It will be shown in the next subsection, that for the appropriate separation between MRK and QMRK the contribution $\mathcal{K}^{one-loop}_{RRG}(\vec{q}_1,\vec{q}_2)$ to the kernel is equal to

$$\mathcal{K}^{one-loop}_{RRG}(\vec{q}_1,\vec{q}_2)=\frac{1}{2(2\pi)^{D-1}(N^2-1)\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\sum_{c_1,c_2,G_1}|\gamma^{(\vec{k}_1^2)G_1}_{c_1c_2}(q_1,q_2)|^2. \quad (82)$$

From Eqs. (78), (20) we obtain

$$\mathcal{K}^{one-loop}_{RRG}(\vec{q}_1,\vec{q}_2)=\frac{\bar{g}^2_\mu\mu^{-2\epsilon}}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{4}{\vec{k}_1^{\,2}}\Bigg(1+\bar{g}^2_\mu\Bigg[-2\left(\frac{\vec{k}_1^{\,2}}{\mu^2}\right)^\epsilon\left(\frac{1}{\epsilon^2}-\frac{\pi^2}{2}\right.$$

$$+2\epsilon\zeta(3)) + \left(\frac{11}{3} - \frac{2n_f}{3N}\right)\frac{1}{\epsilon} + \frac{3\vec{k}_1^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) - \ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)$$

$$+\left(1-\frac{n_f}{N}\right)\Bigg(\frac{\vec{k}^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\left(1-\frac{\vec{k}^{\,2}(\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+4\vec{q}_1\vec{q}_2)}{3(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}\right)\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)$$

$$-\frac{\vec{k}^{\,2}}{6\vec{q}_1^{\,2}\vec{q}_2^{\,2}}(\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+2\vec{q}_1\vec{q}_2) + \frac{(\vec{k}^{\,2})^2(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{6\vec{q}_1^{\,2}\vec{q}_2^{\,2}(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}(\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+4\vec{q}_1\vec{q}_2)\Bigg)\Bigg]\Bigg)\,. \tag{83}$$

3.3 Separation of the MRK and QMRK contributions

Let us rewrite Eq. (28) for the case of the forward scattering:

$$f(\omega,s_0)_{AB} = \int_{s_0}^{\infty}\frac{ds}{s}\left(\frac{s}{s_0}\right)^{-\omega}\sigma_{AB}(s), \tag{84}$$

using relation (47) and introducing s_0 as the argument of the partial wave because in NLLA the partial wave becomes dependent on the energy scale. For the contribution $f^{(n)}_{MRK}(\omega,s_0)_{AB}$ into the partial wave coming from production of n gluons in MRK, repeating the steps leading to Eq. (31) we obtain

$$f^{(n)}_{MRK}(\omega,s_0)_{AB}$$

$$= \frac{1}{(2\pi)^{D-2}}\int\left(\prod_{i=1}^{n+1} d^{D-2}q_{i\perp}\frac{ds_i}{s_i}\left(\frac{s_i}{s_R}\right)^{2\omega_i}\left(\frac{s_i}{\sqrt{\vec{k}_{i-1}^{\,2}\vec{k}_i^{\,2}}}\right)^{-\omega}\theta(s_i-s_\Lambda)\right)$$

$$\times\left(\frac{s_0}{\sqrt{\vec{q}_1^{\,2}\vec{q}_{n+1}^{\,2}}}\right)^{\omega}\frac{\Phi_A^{B+v}(\vec{q}_1;s_R)}{\vec{q}_1^{\,2}}\left(\prod_{i=1}^{n}\mathcal{K}_{RRG}(\vec{q}_i,\vec{q}_{i+1};s_R)\right)\frac{\Phi_B^{B+v}(-\vec{q}_{n+1};s_R)}{\vec{q}_{n+1}^{\,2}}. \tag{85}$$

In order to give the precise meaning to MRK we introduced here the parameter s_Λ. By definition, MRK means that all squared invariant masses are larger than this fixed (not growing with s), but as large as it would be desired parameter. The contribution $f^{(n)}_{MRK}(\omega,s_0)_{AB}$ depends on this parameter although we don't indicate this dependence explicitly. The dependence from this parameter disappears in the final answer for the partial wave. The impact factors (c.f. (49))

$$\Phi_P^{B+v}(\vec{q}_P;s_R) = \frac{1}{\sqrt{N^2-1}}\sum_{\tilde{P},c}|\Gamma_{\tilde{P}P}^{(s_R)c}|^2;\quad \vec{q}_P = -\vec{p}_{\tilde{P}} \tag{86}$$

acquired the upper index $B+v$ for the denotation that they represent the sum of the Born (LLA) contribution and the virtual correction, as well as the additional argument to show their dependence from the scale s_R used in the Regge factors in Eq.(17). Remind, that besides the presented contributions, the total NLA impact factors contain the contributions from the gluon production in the corresponding fragmentation regions as well. The part of the kernel

$$\mathcal{K}_{RRG}(\vec{q}_i,\vec{q}_{i+1};s_R) = \frac{1}{2(2\pi)^{D-1}(N^2-1)\vec{q}_i^{\,2}\vec{q}_{i+1}^{\,2}}\sum_{c_i,c_{i+1},G_i}|\gamma_{c_ic_{i+1}}^{(s_R)G_i}(q_i,q_{i+1})|^2. \tag{87}$$

represents the contribution of the gluon production with the one-loop accuracy and also depends on s_R.

It is convenient to change the argument s_R for $\vec{k}_i^{\,2}$ in $\mathcal{K}_{RRG}(\vec{q}_i,\vec{q}_{i+1};s_R)$ and for s_0 in $\Phi_P^{B+v}(\vec{q}_P;s_R)$ using Eqs.(75), (76). Then after integration over s_i we obtain:

$$f^{(n)}_{MRK}(\omega,s_0)_{AB} = \frac{1}{(2\pi)^{D-2}}\int\left(\prod_{i=1}^{n+1}\frac{d^{D-2}q_{i\perp}}{(\omega-2\omega_i)}\left(\frac{s_\Lambda^2}{\vec{k}_{i-1}^{\,2}\vec{k}_i^{\,2}}\right)^{-\frac{(\omega-2\omega_i)}{2}}\right)$$

$$\times\left(\frac{s_0}{\vec{q}_1^{\,2}}\right)^{\frac{\omega-2\omega_1}{2}}\left(\frac{s_0}{\vec{q}_{n+1}^{\,2}}\right)^{\frac{\omega-2\omega_{n+1}}{2}}\frac{\Phi_A^{B+v}(\vec{q}_1;s_0)}{\vec{q}_1^{\,2}}$$

$$\times\left(\prod_{i=1}^{n}\mathcal{K}_{RRG}(\vec{q}_i,\vec{q}_{i+1};\vec{k}_i^{\,2})\right)\frac{\Phi_B^{B+v}(-\vec{q}_{n+1};s_0)}{\vec{q}_{n+1}^{\,2}}$$

$$\simeq\frac{1}{(2\pi)^{D-2}}\int\left(\prod_{i=1}^{n+1}\frac{d^{D-2}q_{i\perp}}{(\omega-2\omega_i)}\right)\frac{\Phi_A^{B+v}(\vec{q}_1;s_0)}{\vec{q}_1^{\,2}}$$

$$\times\left(\prod_{i=1}^{n}\mathcal{K}_{RRG}(\vec{q}_i,\vec{q}_{i+1};\vec{k}_i^{\,2})\right)\frac{\Phi_B^{B+v}(-\vec{q}_{n+1};s_0)}{\vec{q}_{n+1}^{\,2}}$$

$$\times\Bigg(1-\sum_{i=2}^{n}\frac{(\omega-2\omega_i)}{2}\ln\left(\frac{s_\Lambda^2}{\vec{k}_{i-1}^{\,2}\vec{k}_i^{\,2}}\right)$$

$$-\frac{(\omega-2\omega_1)}{2}\ln\left(\frac{s_\Lambda^2}{\vec{k}_1^{\,2}s_0}\right)-\frac{(\omega-2\omega_{n+1})}{2}\ln\left(\frac{s_\Lambda^2}{\vec{k}_n^{\,2}s_0}\right)\Bigg). \tag{88}$$

In the last equality we performed the expansion in $(\omega - 2\omega_i)$ and have taken into account the first terms of the expansion only, as it should be done in NLLA. Let us note for defineteness that for $n = 1$ in the last factor in Eq.(3.3) only the two last terms remain, whereas for $n = 0$ the whole factor is equal unity, because in this case in Eq. (85) instead of integrations over partial s_i we have an integration over the total s which has to be performed from s_0 to ∞.

Now let us turn to QMRK. Pay attention, that since the contribution of this kinematics is subleading, we can ignore the dependence from the energy scale. In the exact analogy with the PPR and RRG vertices let us introduce the effective vertices $\gamma^{GG}_{c_i c_{i+1}}(q_i, q_{i+1})$ and $\gamma^{Q\bar{Q}}_{c_i c_{i+1}}(q_i, q_{i+1})$ for two-gluon and quark-antiquark productions in the Reggeon-Reggeon collisions as well as the vertices $\Gamma^c_{P^* P}$ for production of the "excited" state P^* in the fragmentation region of the particle P in the process of scattering of this particle off the Reggeon, so that the amplitudes of production of $n+1$ particles in the QMRK are given by Eq. (17) with one of the "old" vertices changed for corresponding "new" vertex. With this definition the contribution $f^{(n+1)}_{QMRK}(\omega)_{AB}$ of the $n+1$ particle production in QMRK is:

$$f^{(n+1)}_{QMRK}(\omega)_{AB} = \frac{1}{(2\pi)^{D-2}} \int \left(\prod_{i=1}^{n+1} \frac{d^{D-2} q_{i\perp}}{(\omega - 2\omega_i)} \right)$$
$$\times \left\{ \frac{\Phi^{(B)}_A(\vec{q}_1)}{\vec{q}_1^{\,2}} \left(\prod_{i=1}^{n} \mathcal{K}^{(B)}_r(\vec{q}_i, \vec{q}_{i+1}) \right) \frac{\Phi^{(r)}_B(-\vec{q}_{n+1})}{\vec{q}_{n+1}^{\,2}} + \frac{\Phi^{(r)}_A(\vec{q}_1)}{\vec{q}_1^{\,2}} \right.$$
$$\times \left(\prod_{i=1}^{n} \mathcal{K}^{(B)}_r(\vec{q}_i, \vec{q}_{i+1}) \right) \frac{\Phi^{(B)}_B(-\vec{q}_{n+1})}{\vec{q}_{n+1}^{\,2}}$$
$$+ \frac{\Phi^{(B)}_A(\vec{q}_1)}{\vec{q}_1^{\,2}} \sum_{j=1}^{n} \left(\prod_{i=1}^{j-1} \mathcal{K}^{(B)}_r(\vec{q}_i, \vec{q}_{i+1}) \right) \mathcal{K}_{QMRK}(\vec{q}_j, \vec{q}_{j+1})$$
$$\left. \times \left(\prod_{i=j+1}^{n} \mathcal{K}^{(B)}_r(\vec{q}_i, \vec{q}_{i+1}) \right) \frac{\Phi^{(B)}_B(-\vec{q}_{n+1})}{\vec{q}_{n+1}^{\,2}} \right\}; \qquad (89)$$

where $\Phi^{(B)}_P(\vec{q}_P)$ and $\mathcal{K}^{(B)}_r(\vec{q}_i, \vec{q}_{i+1})$ denote the Born (LLA) values for the impact factors (49) and the part of the LLA kernel related with the real gluon production (53), $\Phi^{(r)}_P(\vec{q}_P)$ means the corrections to the impact factors due to the production of the "excited" states in the fragmentation regions

and $\mathcal{K}_{QMRK}(\vec{q}_j, \vec{q}_{j+1})$ appears as the contribution to the kernel from QMRK. The last two values depend on the boundary s_Λ between MRK and QMRK, though we don't indicate this dependence. Taking into account that the phase space (11) includes $d^{D-1}k_i/((2\pi)^{D-1}2\epsilon_i)$ and that $d^D k_i = dk_i^2 d^{D-1}k_i/(2\epsilon_i)$, we have

$$\mathcal{K}_{QMRK}(\vec{q}_j, \vec{q}_{j+1}) = \frac{1}{2(N^2-1)\vec{q}_j^{\,2}\vec{q}_{j+1}^{\,2}} \int d\rho_{P_1 P_2} \theta(s_\Lambda - k_j^2)$$
$$\times \sum_{c_j, c_{j+1}, P_1, P_2} |\gamma^{P_1 P_2}_{c_j c_{j+1}}(q_j, q_{j+1})|^2. \qquad (90)$$

Here the sum is taken over colours of Reggeons and all quantum numbers of produced particles. The produced particles $P_1 P_2$ can be quark-antiquark as well as two gluons; denoting their momenta by l_1 and l_2 we have

$$d\rho_{P_1 P_2} = dk_j^2 \delta^{(D)}(k_j - l_1 - l_2) \prod_{i=1}^{2} \frac{d^{D-1} l_i}{(2\pi)^{D-1} 2\epsilon_i}. \qquad (91)$$

Note that in the case of the two-gluon production the gluon identity must be taken into account by the limitation on the integration region. We can represent Eq. (90) as the following:

$$\mathcal{K}_{QMRK}(\vec{q}_j, \vec{q}_{j+1}) = (N^2-1) \int ds_{RR} \frac{2 I_{RR} \sigma_{RR\to 2}(s_{RR}) \theta(s_\Lambda - s_{RR})}{(2\pi)^D 2 \vec{q}_j^{\,2} \vec{q}_{j+1}^{\,2}}, \qquad (92)$$

where $\sigma_{RR\to 2}(k_j^2)$ is the total cross section of the two-particle production in collision of Reggeons with momenta q_j and $-q_{j+1}$, $(q_j - q_{j+1})^2 = s_{RR}$, averaged over colours of the Reggeons, and I_{RR} is the invariant flux:

$$I_{RR} = \sqrt{(s_{RR} - q_j^2 - q_{j+1}^2)^2 - 4 q_j^2 q_{j+1}^2} = 2\sqrt{(q_j q_{j+1})^2 - q_j^2 q_{j+1}^2}; \quad q_i^2 = -\vec{q}_i^{\,2}. \qquad (93)$$

Analogously, the correction $\Phi^{(r)}_P(\vec{q}_R)$ to the impact factor due to the production of the "excited" states in the fragmentation region of the particle P at collision of this particle with momentum p_P and Reggeon with momentum $-q_R$, $(p_P - q_R)^2 = s_{PR}$, is given by

$$\Phi^{(r)}_P(\vec{q}_R) = \frac{(2\pi)^{D-1}}{\sqrt{N^2-1}} \sum_{P^*, c} \int d\rho_{G\tilde{P}} \theta(s_\Lambda - s_{PR}) |\Gamma^c_{G\tilde{A}A}|^2$$
$$= \sqrt{N^2-1} \int ds_{PR} \frac{2 I_{PR} \sigma_{PR\to P^*}(s_{PR}) \theta(s_\Lambda - s_{PR})}{(2\pi) 2s}. \qquad (94)$$

Here we have taken into account that, with the normalization used the matrix element for the $RR \to P_1P_2$ transition coincides with the corresponding effective vertex, whereas for $PR \to P^*$ differs from such vertex by factor $\sqrt{2s}$.

Note, that with the NLLA accuracy the corrections obtained by the expansion in $(\omega - 2\omega_i)$ in Eq. (3.3) for the case of the $(n+1)$-particle production can be presented in the same form as the r.h.s. of Eq. (89) with the substitutions:

$$\frac{\Phi_P^{(r)}(\vec{q}_R)}{\vec{q}_R^{\,2}} \to -\int d^{D-2}\tilde{q}_\perp \frac{\Phi_P^{(B)}(\vec{\tilde{q}})}{\vec{\tilde{q}}^{\,2}} \mathcal{K}_r^{(B)}(\vec{q}_R;\vec{\tilde{q}})\frac{1}{2}\ln\left(\frac{s_\Lambda^2}{(\vec{q}_R-\vec{\tilde{q}})^2 s_0}\right);$$

$$\mathcal{K}_{QMRK}(\vec{q}_j;\vec{q}_{j+1}) \to$$

$$-\int d^{D-2}\tilde{q}_\perp \mathcal{K}_r^{(B)}(\vec{q}_j;\vec{\tilde{q}})\mathcal{K}_r^{(B)}(\vec{\tilde{q}};\vec{q}_{j+1})\frac{1}{2}\ln\left(\frac{s_\Lambda^2}{(\vec{q}_j-\vec{\tilde{q}})^2(\vec{q}_{j+1}-\vec{\tilde{q}})^2}\right). \qquad (95)$$

Therefore from Eqs. (3.3)- (90) we obtain that with the NLLA accuracy the total partial wave can be presented as

$$f(\omega, s_0)_{AB} = \sum_{n=0}^{\infty}\frac{1}{(2\pi)^{D-2}}\int\left(\prod_{i=1}^{n+1}\frac{d^{D-2}q_{i\perp}}{(\omega-2\omega_i)}\right)\frac{\Phi_A(\vec{q}_1;s_0)}{\vec{q}_1^{\,2}}$$

$$\times\left(\prod_{i=1}^{n}\mathcal{K}_r(\vec{q}_i;\vec{q}_{i+1})\right)\frac{\Phi_B(-\vec{q}_{n+1};s_0)}{\vec{q}_{n+1}^{\,2}}, \qquad (96)$$

where $\mathcal{K}_r(\vec{q}_1;\vec{q}_2)$ has the form (65) with $\mathcal{K}_{RRG}^{one-loop}(\vec{q}_1;\vec{q}_2)$ given by Eq.(82);

$$\mathcal{K}_{RRQ\bar{Q}}^{Born}(\vec{q}_j;\vec{q}_{j+1}) = (N^2-1)\int ds_{RR}\frac{2I_{RR}\sigma_{RR\to Q\bar{Q}}(s_{RR})}{(2\pi)^D 2\vec{q}_j^{\,2}\vec{q}_{j+1}^{\,2}}, \qquad (97)$$

$$\mathcal{K}_{RRGG}^{Born}(\vec{q}_j;\vec{q}_{j+1}) = (N^2-1)\int ds_{RR}\frac{2I_{RR}\sigma_{RR\to GG}(s_{RR})\theta(s_\Lambda - s_{RR})}{(2\pi)^D 2\vec{q}_j^{\,2}\vec{q}_{j+1}^{\,2}}$$

$$-\int d^{D-2}\tilde{q}_\perp \mathcal{K}_r^{(B)}(\vec{q}_j;\vec{\tilde{q}})\mathcal{K}_r^{(B)}(\vec{\tilde{q}};\vec{q}_{j+1})\frac{1}{2}\ln\left(\frac{s_\Lambda^2}{(\vec{q}_j-\vec{\tilde{q}})^2(\vec{q}_{j+1}-\vec{\tilde{q}})^2}\right), \qquad (98)$$

and the impact factors have the representation

$$\Phi_P(\vec{q}_R;s_0) = \sqrt{N^2-1}\int ds_{PR}\frac{2I_{PR}\sigma_{PR}(s_{PR})\theta(s_\Lambda - s_{PR})}{(2\pi)2s}$$

$$-\int\frac{d^{D-2}\tilde{q}_\perp}{(2\pi)^{D-1}}\Phi_P^{(B)}(\vec{\tilde{q}})\frac{g^2N\vec{q}_R^{\,2}}{\vec{\tilde{q}}^{\,2}(\vec{q}_R-\vec{\tilde{q}})^2}\ln\left(\frac{s_\Lambda^2}{(\vec{q}_R-\vec{\tilde{q}})^2 s_0}\right). \qquad (99)$$

Here we used the form (53) of $\mathcal{K}_r^{(B)}(\vec{q}_1;\vec{q}_2)$.

In the last two equations the MRK boundary s_Λ have to be taken much larger than typical squared transverse momenta, so that the dependence from s_Λ disappears due to the factorization properties of the Reggeon vertices in the regions of strongly ordered rapidities of produced particles. Namely, the two-gluon production vertex is given by the product of the RRG vertices:

$$\gamma_{c_jc_{j+1}}^{G_1G_2}(q_j;q_{j+1}) \simeq \gamma_{c_j\tilde{c}}^{G_1}(q_j;q_j-l_1)\frac{1}{(q_j-l_1)^2}\gamma_{\tilde{c}c_{j+1}}^{G_2}(q_j-l_1;q_{j+1}) \qquad (100)$$

at $(p_Bl_2) \ll (p_Bl_1)$, $(p_Al_1) \ll (p_Al_2)$, where l_1 and l_2 are momenta of the produced gluons, and the vertices for the gluon production in the fragmentation regions are given by the product of the corresponding vertices without gluon and the RRG vertices:

$$\Gamma_{G\tilde{A}A}^{c} \simeq \Gamma_{\tilde{A}A}^{\tilde{c}}\frac{1}{(q_R+l)^2}\gamma_{\tilde{c}c}^{G}(q_R+l;q_R) \qquad (101)$$

at $(p_Bl) \ll (p_Bp_{\tilde{A}})$, $(p_Ap_{\tilde{A}}) \ll (p_Al)$, l is the gluon momentum, $q_R = p_A - p_{\tilde{A}} - l$;

$$\Gamma_{G\tilde{B}B}^{c} \simeq \Gamma_{\tilde{B}B}^{\tilde{c}}\frac{1}{(q_R-l)^2}\gamma_{c\tilde{c}}^{G}(q_R;q_R-l) \qquad (102)$$

at $(p_Al) \ll (p_Ap_{\tilde{B}})$, $(p_Bp_{\tilde{B}}) \ll (p_Bl)$, $q_R = p_{\tilde{B}} - p_B + l$.

3.4 Two-Particle Production Contributions

Investigation of the two-gluon production contribution was started in Ref.[10]; the next step was done in Ref.[18]. The final result was obtained in Ref.[19] by calculation of differential cross section of the two-gluon production in the RR collisions and integration of this cross section over relative transverse and longitudinal momenta of the produced gluons. In a suitable for integration form we obtain:

$$\mathcal{K}_{RRGG}^{Born}(\vec{q}_1;\vec{q}_2) = \frac{4\pi g^4\mu^{2\epsilon}N^2}{(2\pi)^D\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\int_{\delta_R}^{1-\delta_R}\frac{dx}{x(1-x)}\int\mu^{-2\epsilon}\frac{d^{D-2}l_1}{(2\pi)^{D-1}}\left\{\frac{(\vec{q}_1^{\,2})^2(\vec{q}_2^{\,2})^2}{4\tilde{t}_1\tilde{t}_2\vec{l}_1^{\,2}\vec{l}_2^{\,2}}\right.$$

$$+\frac{1}{2\tilde{t}_1\tilde{t}_2}\left[-2\vec{q}_1^{\,2}\vec{q}_2^{\,2}+\frac{(1+\epsilon)}{2}x(1-x)\left((\vec{k}_1^2-\vec{q}_2^{\,2})^2+(\vec{q}_1^{\,2})^2\right)\right.$$

$$
-4(1+\epsilon)x(1-x)(\vec{l}_1\vec{q}_1)^2\Big] + \frac{x(1-x)\vec{q}_1^{\,2}}{\kappa Z}
$$

$$
\times\Big[2\vec{q}_2^{\,2} + (1+\epsilon)\left(2(1-x)(\vec{l}_1\vec{q}_1) - x(1-x)\vec{q}_1^{\,2} - \vec{l}_2^{\,2}\right) - \epsilon x(1-x)(\vec{k}_1^2 - \vec{q}_2^{\,2})\Big]
$$

$$
+\left(\frac{x(1-x)\vec{q}_1^{\,2}}{Z}\right)^2\left[\frac{(1+\epsilon)}{2} - (3+2\epsilon)x(1-x)\right]
$$

$$
+\left(\frac{x\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{2(1-x)\kappa\vec{l}_1^{\,2}} - \frac{x^2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{2\vec{l}_1^{\,2}Z} - \frac{x\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{l}_1^{\,2}\vec{l}_2^{\,2}}\right) + \frac{1}{\kappa\tilde{t}_1}\Big[-2(1-x)\vec{q}_1^{\,2}\vec{q}_2^{\,2} + (1+\epsilon)(1-x)(\vec{q}_1^{\,2})^2
$$

$$
+\frac{(1+\epsilon)}{2}x(1-x)\left(2(1-x)\vec{q}_1^{\,2}(\vec{k}_1^2 - \vec{q}_2^{\,2}) - x(\vec{q}_1^{\,2})^2 - x(\vec{k}_1^2 - \vec{q}_2^{\,2})^2\right)
$$

$$
-2(1+\epsilon)(2-x)(1-x)\vec{q}_1^{\,2}(\vec{l}_1\vec{q}_1) + 2(1+\epsilon)(1-x)\left((\vec{l}_1\vec{q}_1)^2\right)
$$

$$
+((\vec{l}_1 - x\vec{k}_1)\vec{q}_1)^2 + (1+\epsilon)\vec{q}_1^{\,2}\vec{l}_2^{\,2}\Big]
$$

$$
+\frac{x\vec{q}_1^{\,2}\vec{l}_2^{\,2}\left((1+\epsilon)\vec{l}_2^{\,2} - 2\vec{q}_2^{\,2}\right)}{\kappa Z\tilde{t}_1} + \frac{x\vec{q}_1^{\,2}(\vec{q}_2^{\,2})^2}{2\kappa Z\tilde{t}_1} + \frac{x\vec{q}_2^{\,2}(\vec{q}_1^{\,2})^2}{2\kappa\tilde{t}_1\vec{l}_1^{\,2}}
$$

$$
-\frac{(1-x)(2(\vec{l}_1\vec{q}_1) - \vec{q}_1^{\,2})\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{2\tilde{t}_1\vec{l}_1^{\,2}\vec{l}_2^{\,2}} + (1+\epsilon)(1-x)^2\frac{4(\vec{l}_1\vec{q}_1)^2 + (1-4x)(\vec{q}_1^{\,2})^2}{\tilde{t}_1^{\,2}}\Big\}, \tag{103}
$$

where l_1 and l_2 are momenta of gluons produced in collision of Reggeons with momenta q_1 and $-q_2$; $l_1 + l_2 = q_1 - q_2 = k_1$;

$$
q_1 = \beta_1 p_1 + q_{1\perp};\quad q_2 = -\alpha_2 p_2 + q_{2\perp};\quad t_i = q_{i\perp}^2 = -\vec{q}_i^{\,2};
$$

$$
l_1 = x\beta_1 p_1 + \frac{\vec{l}_1^{\,2}}{x\beta_1 s}p_2 + l_{1\perp};
$$

$$
\tilde{t}_1 = (q_1 - l_1)^2 = -\frac{1}{x}\left((\vec{l}_1 - x\vec{q}_1)^{\,2} + x(1-x)\vec{q}_1^{\,2}\right);
$$

$$
\tilde{t}_2 = (q_1 - l_2)^2 = -\frac{1}{1-x}\left((\vec{l}_2 - (1-x)\vec{q}_1)^{\,2} + x(1-x)\vec{q}_1^{\,2}\right);
$$

$$
\kappa = (l_1 + l_2)^2 = \frac{((1-x)\vec{l}_1 - x\vec{l}_2)^{\,2}}{x(1-x)};\quad Z = -(1-x)\vec{l}_1^{\,2} - x\vec{l}_2^{\,2}. \tag{104}
$$

Here we omitted terms which vanish after integtation. The intermediate parameter δ_R is introduced in (103) to exclude divergencies of separate terms. This parameter should be considered as infinitely small. In the total expression for the integral the dependence on δ_R vanishes.

Note, that taken separately some terms in the r.h.s. of Eq.(103) contain "ultraviolet" divergencies at large $|\vec{l}_1|$; but these divergencies are artificial in the sense that they cancel each other, so that the total integral is convergent at large$|\vec{l}_1|$. In the infrared region for $\epsilon \to 0$ all terms have no more than logarithmic divergency. All divergencies, "ultraviolet" and infrared, can be regularized by the same ϵ. Since in the BFKL equation the kernel has to be integrated over $\vec{q}_2$, or equivalently, over $\vec{k}_1$, and the kernel is singular at $\vec{k}_1 = 0$, we should be careful in performing the expansion in ϵ, in order to keep all terms which give nonvanishing contribution in the physical limit $\epsilon \to 0$ after integration over $\vec{k}_1$. The result of the integration in (103) is:

$$
\mathcal{K}^{Born}_{RRGG}(\vec{q}_1, \vec{q}_2) = \frac{4\bar{g}_\mu^4\mu^{-2\epsilon}}{\pi^{1+\epsilon}\Gamma(1-\epsilon)\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\left\{\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}_1^2}\left(\frac{\vec{k}_1^2}{\mu^2}\right)^\epsilon\right.
$$

$$
\times\left[\frac{1}{\epsilon^2} - \frac{11}{6}\frac{1}{\epsilon} - \frac{2\pi^2}{3} + \frac{67}{18} + \epsilon\left(\frac{11\pi^2}{36} - \frac{202}{27} + 9\zeta(3)\right)\right]
$$

$$
-\frac{(\vec{q}_1^{\,2} + \vec{q}_2^{\,2})(2\vec{q}_1^{\,2}\vec{q}_2^{\,2} - 3(\vec{q}_1\vec{q}_2)^2)}{8\vec{q}_1^{\,2}\vec{q}_2^{\,2}}
$$

$$
-\left(\frac{11}{3}\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})} + \frac{(2\vec{q}_1^{\,2}\vec{q}_2^{\,2} - 3(\vec{q}_1\vec{q}_2)^2)}{16\vec{q}_1^{\,2}\vec{q}_2^{\,2}}(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})\right)
$$

$$
\times\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) - \frac{2}{3}\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})^3}\left[\left(1 - \frac{2(\vec{q}_1\vec{q}_2)^2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\right)\left(\vec{q}_1^{\,4} - \vec{q}_2^{\,4} - 2\vec{q}_1^{\,2}\vec{q}_2^{\,2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)\right.
$$

$$
\left.+(\vec{q}_1\vec{q}_2)\left(2\vec{q}_1^{\,2} - -2\vec{q}_2^{\,2} - (\vec{q}_1^{\,2} + \vec{q}_2^{\,2})\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right)\right]
$$

$$
+\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}(\vec{k}_1(\vec{q}_1 + \vec{q}_2))}{\vec{k}_1^2(\vec{q}_1 + \vec{q}_2)^2}\left[\frac{1}{2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}\vec{k}_1^4}{(\vec{q}_1^{\,2} + \vec{q}_2^{\,2})^4}\right) + L\left(-\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right) - L\left(-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\right]
$$

$$
-\left[4\vec{q}_1^{\,2}\vec{q}_2^{\,2} + \frac{(\vec{q}_1^{\,2} - \vec{q}_2^{\,2})^2}{4} + (2\vec{q}_1^{\,2}\vec{q}_2^{\,2} - 3\vec{q}_1^{\,4} - 3\vec{q}_2^{\,4})\frac{(2\vec{q}_1^{\,2}\vec{q}_2^{\,2} - (\vec{q}_1\vec{q}_2)^2)}{16\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\right]
$$

$$\times \int_0^\infty \frac{dx}{(\vec{q}_1^{\,2}+x^2\vec{q}_2^{\,2})} \ln\left|\frac{1+x}{1-x}\right|$$

$$-\vec{q}_1^{\,2}\vec{q}_2^{\,2}\left(1-\frac{(\vec{k}_1(\vec{q}_1+\vec{q}_2))^2}{\vec{k}_1^2(\vec{q}_1+\vec{q}_2)^2}\right)\left(\int_0^1-\int_1^\infty\right) dz \frac{\ln\left((z\vec{q}_1)^2/\vec{q}_2^{\,2}\right)}{(\vec{q}_2-z\vec{q}_1)^2}\Bigg\}; \qquad (105)$$

with

$$\psi(z)=\frac{\Gamma'(z)}{\Gamma(z)}; \qquad L(z)=\int_0^z \frac{dt}{t}\ln(1-t). \qquad (106)$$

The r.h.s. of Eq.(105) exhibits several terms with unphysical singularities, such as at $\vec{q}_1^{\,2}=\vec{q}_2^{\,2}$ and $(\vec{q}_1+\vec{q}_2)^2=0$. It is not difficult to see, that the only real singularity is at $\vec{k}_1^{\,2}=0$. All others are spurious and singular terms cancel each other. As for the singularity at $\vec{k}_1^{\,2}=0$, Eq. (105) contains all terms which give in the physical case $\epsilon\to 0$ non-zero contributions after the subsequent integration over $\vec{k}_1$. For the terms which are infrared divergent at $D=4$, the region of such small $\vec{k}_1$, that is $\ln(1/\vec{k}_1^{\,2})\sim 1/\epsilon$, gives essential contribution in the integral over $\vec{k}_1$. Therefore, we can not expand $(\vec{k}_1^2/\mu^2)^\epsilon$ in powers of ϵ in such terms. Note, that the calculated contribution is explicitly symmetrical with respect to substitution $\vec{q}_1\leftrightarrow -\vec{q}_2$. It is quite a non-trivial task to notice a reason for this symmetry in the starting expression (103). Nevertheless, this symmetry is hidden there. It is a consequence of the invariance of the expression inside the curly brackets in (103), as well as the phase space element $dx/(x(1-x))d\vec{l}_1$, under the "left-right symmetry" transformation [18], i.e. under the substitution:

$$\vec{q}_1\leftrightarrow-\vec{q}_2;\ \vec{l}_1\leftrightarrow\vec{l}_2;\ \ x\leftrightarrow\frac{x\vec{l}_2^{\,2}}{((1-x)\vec{l}_1^{\,2}+x\vec{l}_2^{\,2})}. \qquad (107)$$

The quark-antiquark pair production contribution was calculated in Refs. [20]. The result is

$$\mathcal{K}^{Born}_{RRQ\overline{Q}}(\vec{q}_1;\vec{q}_2)=\frac{4\bar{g}_\mu^4\mu^{-2\epsilon}n_f}{\pi^{1+\epsilon}\Gamma(1-\epsilon)N^3}\Bigg\{N^2\left[\frac{1}{\vec{k}_1^{\,2}}\left(\frac{\vec{k}_1^{\,2}}{\mu^2}\right)^\epsilon\frac{2}{3}\left(\frac{1}{\epsilon}-\frac{5}{3}+\epsilon(\frac{28}{9}-\frac{\pi^2}{6})\right)\right.$$

$$+\frac{1}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}\left[1-\frac{\vec{k}_1^{\,2}(\vec{q}_1^{\,2}+\vec{q}_2^{\,2}+4\vec{q}_1\vec{q}_2)}{3(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}\right]\ln\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}+\frac{\vec{k}_1^{\,2}}{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}$$

$$\times\left(2-\frac{\vec{k}_1^{\,2}(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})}{3\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\right)+\frac{(2\vec{k}_1^{\,2}-\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}{3\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\Bigg]+\Bigg[-1-\frac{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})^2}{4\vec{q}_1^{\,2}\vec{q}_2^{\,2}}$$

$$+\left(\frac{(\vec{q}_1\vec{q}_2)^2}{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}-2\right)\frac{(2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-3\vec{q}_1^{\,4}-3\vec{q}_2^{\,4})}{16\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\Bigg]\int_0^\infty\frac{dx}{(\vec{q}_1^{\,2}+x^2\vec{q}_2^{\,2})}\ln\left|\frac{1+x}{1-x}\right|$$

$$+\frac{(3(\vec{q}_1\vec{q}_2)^2-2\vec{q}_1^{\,2}\vec{q}_2^{\,2})}{16\vec{q}_1^{\,4}\vec{q}_2^{\,4}}\left[(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+2(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})\right]\Bigg\}. \qquad (108)$$

4 BFKL Pomeron in the next-to-leading approximation

Now we have all contributions to the NLLA kernel. Remind, that it has the same form (52) as the LLA one, where the gluon Regge trajectory in the two-loop approximation is given by (71) and the part $\mathcal{K}_r(\vec{q}_1;\vec{q}_2)$ related with the real particle production (65) contains the contributions from the one-gluon (83), two-gluon (105) and quark-antiquark (108) production in the Reggeon-Reggeon collisions. Representing this part as the sum

$$\mathcal{K}_r(\vec{q}_1;\vec{q}_2)=\mathcal{K}_r^{(B)}(\vec{q}_1;\vec{q}_2)+\mathcal{K}_r^{(1)}(\vec{q}_1;\vec{q}_2);$$

$$\mathcal{K}_r^{(B)}(\vec{q}_1;\vec{q}_2)=\frac{4\bar{g}_\mu^2\mu^{-2\epsilon}}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\frac{1}{(\vec{q}_1-\vec{q}_2)^2} \qquad (109)$$

of the LLA contribution, expressed in terms of the renormalized coupling constant, and the one-loop correction $\mathcal{K}_r^{(1)}(\vec{q}_1;\vec{q}_2)$, we have for this correction [21]:

$$\mathcal{K}_r^{(1)}(\vec{q}_1;\vec{q}_2)$$

$$=\frac{4\bar{g}_\mu^4\mu^{-2\epsilon}}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\Bigg\{\frac{1}{(\vec{q}_1-\vec{q}_2)^2}\Bigg[\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\frac{1}{\epsilon}\left(1-\left(\frac{(\vec{q}_1-\vec{q}_2)^2}{\mu^2}\right)^\epsilon(1-\epsilon^2\frac{\pi^2}{6})\right)$$

$$+\left(\frac{(\vec{q}_1-\vec{q}_2)^2}{\mu^2}\right)^\epsilon\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N}+\epsilon\left(-\frac{404}{27}+14\zeta(3)+\frac{56}{27}\frac{n_f}{N}\right)\right)\Bigg]$$

$$-\left(1+\frac{n_f}{N^3}\right)\frac{2\vec{q}_1^{\,2}\vec{q}_2^{\,2}-3(\vec{q}_1\vec{q}_2)^2}{16\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\left(\frac{2}{\vec{q}_2^{\,2}}+\frac{2}{\vec{q}_1^{\,2}}+(\frac{1}{\vec{q}_2^{\,2}}-\frac{1}{\vec{q}_1^{\,2}})\ln\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)$$

$$-\frac{1}{(\vec{q}_1-\vec{q}_2)^2}\left(\ln\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)^2+\frac{2(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}{(\vec{q}_1-\vec{q}_2)^2(\vec{q}_1+\vec{q}_2)^2}\left(\frac{1}{2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)\right.$$

$$\left.\times\ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}(\vec{q}_1-\vec{q}_2)^4}{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})^4}\right)+L\left(-\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)-L\left(-\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)\right)$$

$$-\left(3+\left(1+\frac{n_f}{N^3}\right)\left(1-\frac{(\vec q_1^{\,2}+\vec q_2^{\,2})^2}{8\vec q_1^{\,2}\vec q_2^{\,2}}-\frac{2\vec q_1^{\,2}\vec q_2^{\,2}-3\vec q_1^{\,4}-3\vec q_2^{\,4}}{16\vec q_1^{\,4}\vec q_2^{\,4}}(\vec q_1\vec q_2)^2\right)\right)$$

$$\times\int_0^\infty \frac{dx\,\ln\left|\frac{1+x}{1-x}\right|}{\vec q_1^{\,2}+x^2\vec q_2^{\,2}} - \left(1-\frac{(\vec q_1^{\,2}-\vec q_2^{\,2})^2}{(\vec q_1-\vec q_2)^2(\vec q_1+\vec q_2)^2}\right)\left(\int_0^1-\int_1^\infty\right)\frac{dz\,\ln\frac{(z\vec q_1)^2}{(\vec q_2)^2}}{(\vec q_2-z\vec q_1)^2}\Bigg\}; \qquad (110)$$

where

$$L(z)=\int_0^z\frac{dt}{t}\ln(1-t)\,;\ \zeta(n)=\sum_{k=1}^{\infty}k^{-n} \qquad (111)$$

The remarkable fact exhibited by Eq.(110) is the cancellation of the infrared singularities [22] (remind, that separate terms in (65) are singular as $1/\epsilon^2$) at fixed $\vec k_1=\vec q_1-\vec q_2$, where we can expand $(\vec k_1^2/\mu^2)^\epsilon$ in powers of ϵ. This expansion is not performed in (110) because for the terms having singularity at $\vec k_1^{\,2}=0$ the region of such small $\vec k_1$, that is $\ln(1/\vec k_1^{\,2})\sim 1/\epsilon$, does contribute in the integral over $\vec k_1$. The singular contributions given by this region are canceled, in turn, in the BFKL equation by the singular terms in the gluon trajectory [22].

As well as in LLA, only the kernel averaged over the angle between the momenta $\vec q_1$ and $\vec q_2$ is relevant until we don't consider spin correlations. For the averaged one-loop correction we have:

$$\overline{K_r^{(1)}(\vec q_1,\vec q_2)} = \frac{4\bar g_\mu^4\mu^{-2\epsilon}}{\pi^{1+\epsilon}\Gamma(1-\epsilon)}\left\{\frac{1}{|\vec q_2^{\,2}-\vec q_1^{\,2}|}\right.$$

$$\times\left[\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\frac{1}{\epsilon}\left(\left(\frac{|\vec q_2^{\,2}-\vec q_1^{\,2}|}{(\max(\vec q_1^{\,2},\vec q_2^{\,2}))}\right)^{2\epsilon}\left(1+\epsilon^2\frac{\pi^2}{3}\right)\right.\right.$$

$$\left.-\left(\frac{(\vec q_2^{\,2}-\vec q_1^{\,2})^4}{\mu^2(\max(\vec q_1^{\,2},\vec q_2^{\,2}))^3}\right)^\epsilon\left(1+\epsilon^2\frac{5\pi^2}{6}\right)\right)$$

$$+\left(\frac{|\vec q_2^{\,2}-\vec q_1^{\,2}|^4}{\mu^2(\max(\vec q_1^{\,2};\vec q_2^{\,2}))^3}\right)^\epsilon\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N}\right.$$

$$\left.\left.+\epsilon\left(-\frac{404}{27}+14\zeta(3)+\frac{56}{27}\frac{n_f}{N}\right)\right)\right]$$

$$-\frac{1}{32}\left(1+\frac{n_f}{N^3}\right)\left(\frac{2}{\vec q_2^{\,2}}+\frac{2}{\vec q_1^{\,2}}+\left(\frac{1}{\vec q_2^{\,2}}-\frac{1}{\vec q_1^{\,2}}\right)\ln\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right)$$

$$-\frac{1}{|\vec q_1^{\,2}-\vec q_2^{\,2}|}\left(\ln\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right)^2-\left(3+\left(1+\frac{n_f}{N^3}\right)\left(\frac34-\frac{(\vec q_1^{\,2}+\vec q_2^{\,2})^2}{32\vec q_1^{\,2}\vec q_2^{\,2}}\right)\right)$$

$$\times\int_0^\infty\frac{dx}{\vec q_1^{\,2}+x^2\vec q_2^{\,2}}\ln\left|\frac{1+x}{1-x}\right|+\frac{1}{\vec q_2^{\,2}+\vec q_1^{\,2}}\left(\frac{\pi^2}{3}-4L\left(\min\left(\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}};\frac{\vec q_2^{\,2}}{\vec q_1^{\,2}}\right)\right)\right)\Bigg\}. \qquad (112)$$

Instead of the dimensional regularization we can use the cut off $|\vec q_1^{\,2}-\vec q_2^{\,2}|>\lambda^2$ changing correspondingly the virtual part. Using Eqs. (52), (71), (109) it is possible to verify that the averaged NLLA kernel at $\epsilon\to 0$ is equivalent to the expression

$$\overline{K(\vec q_1,\vec q_2)} = -2\frac{\alpha_s(\mu^2)N}{\pi^2}\left\{\ln\frac{\vec q_1^{\,2}}{\lambda^2}+\frac{\alpha_s(\mu^2)N}{4\pi}\left[\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\left(\ln\frac{\vec q_1^{\,2}}{\lambda^2}\ln\frac{\mu^2}{\lambda^2}+\frac{\pi^2}{12}\right)+\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N}\right)\ln\frac{\vec q_1^{\,2}}{\lambda^2}-3\zeta(3)\right]\right\}$$

$$\times\delta(\vec q_1^{\,2}-\vec q_2^{\,2})+\frac{\alpha_s(\mu^2)N}{\pi^2}\frac{\theta(|\vec q_1^{\,2}-\vec q_2^{\,2}|-\lambda^2)}{|\vec q_1^{\,2}-\vec q_2^{\,2}|}$$

$$\times\left\{1-\frac{\alpha_s(\mu^2)N}{4\pi}\left[\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\ln\left(\frac{|\vec q_1^{\,2}-\vec q_2^{\,2}|^2}{\max(\vec q_1^{\,2},\vec q_2^{\,2})\mu^2}\right)-\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N}\right)\right]\right\}$$

$$-\frac{\alpha_s^2(\mu^2)N^2}{4\pi^3}\left\{\frac{1}{32}\left(1+\frac{n_f}{N^3}\right)\left(\frac{2}{\vec q_2^{\,2}}+\frac{2}{\vec q_1^{\,2}}+\left(\frac{1}{\vec q_2^{\,2}}-\frac{1}{\vec q_1^{\,2}}\right)\ln\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right)\right.$$

$$+\frac{1}{|\vec q_1^{\,2}-\vec q_2^{\,2}|}\left(\ln\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}}\right)^2+\left(3+\left(1+\frac{n_f}{N^3}\right)\left(\frac34-\frac{(\vec q_1^{\,2}+\vec q_2^{\,2})^2}{32\vec q_1^{\,2}\vec q_2^{\,2}}\right)\right)$$

$$\times\int_0^\infty\frac{dx}{\vec q_1^{\,2}+x^2\vec q_2^{\,2}}\ln\left|\frac{1+x}{1-x}\right|-\frac{1}{\vec q_2^{\,2}+\vec q_1^{\,2}}\left(\frac{\pi^2}{3}-4L\left(\min\left(\frac{\vec q_1^{\,2}}{\vec q_2^{\,2}};\frac{\vec q_2^{\,2}}{\vec q_1^{\,2}}\right)\right)\right)\Bigg\}. \qquad (113)$$

in the two-dimensional transverse space, with $\lambda\to 0$. Of course, the dependence from λ disappears when the kernel acts on a function. Moreover, the representation (113) permits to find such form of the kernel for which this cancellation is evident, just in the same way as it was done at transition from (56) to (59). We obtain:

$$\overline{K(\vec q_1,\vec q_2)} = \frac{\alpha_s(\mu^2)N}{\pi^2}\int d\vec q^{\,2}\frac{1}{|\vec q_1^{\,2}-\vec q^{\,2}|}$$

$$\times\left(\delta(\vec{q}^{\,2}-\vec{q}_2^{\,2})-2\frac{\min(\vec{q}_1^{\,2};\vec{q}^{\,2})}{(\vec{q}_1^{\,2}+\vec{q}^{\,2})}\delta(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})\right)$$

$$\times\left[1-\frac{\alpha_s(\mu^2)N}{4\pi}\left(\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\ln\left(\frac{|\vec{q}_1^{\,2}-\vec{q}^{\,2}|^2}{\max(\vec{q}_1^{\,2};\vec{q}^{\,2})\mu^2}\right)\right.\right.$$

$$\left.\left.-\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N}\right)\right)\right]-\frac{\alpha_s^2(\mu^2)N^2}{4\pi^3}\left[\frac{1}{32}\left(1+\frac{n_f}{N^3}\right)\right.$$

$$\times\left(\frac{2}{\vec{q}_2^{\,2}}+\frac{2}{\vec{q}_1^{\,2}}+\left(\frac{1}{\vec{q}_2^{\,2}}-\frac{1}{\vec{q}_1^{\,2}}\right)\ln\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+\frac{\left(\ln(\vec{q}_1^{\,2}/\vec{q}_2^{\,2})\right)^2}{|\vec{q}_1^{\,2}-\vec{q}_2^{\,2}|}$$

$$+\left(3+(1+\frac{n_f}{N^3})\left(\frac{3}{4}-\frac{(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})^2}{32\vec{q}_1^{\,2}\vec{q}_2^{\,2}}\right)\right)\int_0^\infty\frac{dx}{\vec{q}_1^{\,2}+x^2\vec{q}_2^{\,2}}\ln\left|\frac{1+x}{1-x}\right|$$

$$\left.-\frac{1}{\vec{q}_2^{\,2}+\vec{q}_1^{\,2}}\left(\frac{\pi^2}{3}-4L(\min(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}};\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}))\right)\right]$$

$$+\frac{\alpha_s^2(\mu^2)N^2}{4\pi^3}\left(6\zeta(3)-\frac{5\pi^2}{12}\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\right)\delta(\vec{q}_1^{\,2}-\vec{q}_2^{\,2}). \tag{114}$$

The μ - dependence in the right hand side of this equality leads to the violation of the scale invariance and is related with running the QCD coupling constant.

The form (114) is very convenient for finding the action of the kernel on the eigenfunctions $\vec{q}_2^{\,2(\gamma-1)}$ of the Born kernel:

$$\int d^{D-2}q_2\;K(\vec{q}_1;\vec{q}_2)\left(\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)^{\gamma-1}=\frac{\alpha_s(\vec{q}_1^{\,2})\,N}{\pi}\left(\chi^B(\gamma)+\frac{\alpha_s(\vec{q}_1^{\,2})N}{\pi}\chi^{(1)}(\gamma)\right), \tag{115}$$

were within our accuracy we expressed the result in terms of the running coupling constant

$$\alpha_s(\vec{q}^{\,2})=\frac{\alpha_s(\mu^2)}{1+\frac{\alpha_s(\mu^2)N}{4\pi}\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\ln\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)}$$

$$\simeq\alpha_s(\mu^2)\left(1-\frac{\alpha_s(\mu^2)N}{4\pi}\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\ln\left(\frac{\vec{q}^{\,2}}{\mu^2}\right)\right); \tag{116}$$

$\chi^B(\gamma)$ is given by (61) and and the correction $\chi^{(1)}(\gamma)$ is:

$$\chi^{(1)}(\gamma)=-\frac{1}{4}\left[\left(\frac{11}{3}-\frac{2n_f}{3N}\right)\frac{1}{2}\left(\chi^2(\gamma)-\psi'(\gamma)+\psi'(1-\gamma)\right)\right.$$

$$-6\zeta(3)+\frac{\pi^2\cos(\pi\gamma)}{\sin^2(\pi\gamma)(1-2\gamma)}\left(3+\left(1+\frac{n_f}{N^3}\right)\frac{2+3\gamma(1-\gamma)}{(3-2\gamma)(1+2\gamma)}\right)$$

$$\left.-\left(\frac{67}{9}-\frac{\pi^2}{3}-\frac{10}{9}\frac{n_f}{N}\right)\chi(\gamma)-\psi''(\gamma)-\psi''(1-\gamma)-\frac{\pi^3}{\sin(\pi\gamma)}+4\phi(\gamma)\right] \tag{117}$$

with

$$\phi(\gamma)=-\int_0^1\frac{dx}{1+x}\left(x^{\gamma-1}+x^{-\gamma}\right)\int_x^1\frac{dt}{t}\ln(1-t)$$

$$=\sum_{n=0}^{\infty}(-1)^n\left[\frac{\psi(n+1+\gamma)-\psi(1)}{(n+\gamma)^2}+\frac{\psi(n+2-\gamma)-\psi(1)}{(n+1-\gamma)^2}\right]. \tag{118}$$

For the relative correction $r(\gamma)$ defined as $\chi^{(1)}(\gamma)=-r(\gamma)\chi^B(\gamma)$ in the symmetrical point $\gamma=1/2$, corresponding to the eigenfunction of the LLA kernel with the largest eigenvalue, we have [21]

$$r\left(\frac{1}{2}\right)=\left(\frac{11}{6}-\frac{n_f}{3N}\right)\ln 2-\frac{67}{36}+\frac{\pi^2}{12}+\frac{5}{18}\frac{n_f}{N}+\frac{1}{\ln 2}\left[\int_0^1\arctan(\sqrt{t})\ln(\frac{1}{1-t})\frac{dt}{t}\right.$$

$$\left.+\frac{11}{8}\zeta(3)+\frac{\pi^3}{32}\left(\frac{27}{16}+\frac{11}{16}\frac{n_f}{N^3}\right)\right]\simeq 6,46+0.05\frac{n_f}{N}+2.66\frac{n_f}{N^3}. \tag{119}$$

It shows that the correction is very large.

In some sense, the large value of the correction is natural and is a consequence of the large value of the LLA Pomeron intercept $\omega_P^B=4N\ln 2\;\alpha_s(q^2)/\pi$. If we express the corrected intercept ω_P in terms of the Born one, we obtain

$$\omega_P=\omega_P^B(1-\frac{r\left(\frac{1}{2}\right)}{4\ln 2}\omega_P^B)\simeq\omega_P^B(1-2.4\omega_P^B). \tag{120}$$

The coefficient 2.4 does not look very large. Moreover, it corresponds to the rapidity interval where correlations become important in the hadron production processes.

Nevertheless, these numerical estimates show, that in the kinematical region of HERA probably it is not enough to take into account only the next-to-leading correction. For example, if $\alpha_s(\vec{q}^{\,2})=0.15$, where the Born intercept is $\omega_P^B=4N\alpha_s(\vec{q}^{\,2})/\pi\ln 2=.39714$, the relative correction for $n_f=0$ is very big:

$$\frac{\omega_P}{\omega_P^B}=1-r\left(\frac{1}{2}\right)\frac{\alpha_s(\vec{q}^{\,2})}{\pi}N=0.0747. \tag{121}$$

The maximal value of $\omega_P \simeq 0.1$ is obtained for $\alpha_s(\vec{q}^{\,2}) \simeq 0.08$.

But it is necessary to realize that, firstly, the above estimates are quite straightforward and do not take into account neither the influence of the running coupling on the eigenfunctions nor the nonperturbative effects [23]; secondly, the value of the correction strongly depends on its representation. For example, if one takes into account the next-to-leading correction by the corresponding increase of the argument of the running QCD coupling constant, ω_P at $\alpha_s(q^2) = 0.15$ turns out to be only two times smaller, than its Born value.

The above results can be applied for the calculation of anomalous dimensions of the local operators in the vicinity of the point $\omega = J - 1 = 0$. The deep-inelastic moments $\mathcal{F}^{gB}_{\omega}(q^2)$ defined as follows

$$\mathcal{F}^{gB}_{\omega}(q^2) = \int_{q^2}^{\infty} \frac{ds}{s} \left(\frac{s}{q^2}\right)^{-\omega} \sigma^{gB}(q^2, s), \qquad (122)$$

where

$$\sigma^{gB}(q^2, s) = \int \frac{d^2q'}{2\pi q'^2} \Phi_B(\vec{q}\,') \int_{a-i\infty}^{a+i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{q\, q'}\right)^{\omega} G_{\omega}(\vec{q}, \vec{q}\,'), \qquad (123)$$

obey the integral equation of the type (51) with the inhomogeneous term equal to $\Phi_B(\vec{q})/(2\pi \vec{q}^{\,2})$ and the kernel

$$\widetilde{K}(\vec{q}_1, \vec{q}_2) = K(\vec{q}_1, \vec{q}_2) - \frac{1}{2} \int d^{D-2}q \; K^B(\vec{q}_1, \vec{q}) \ln \frac{\vec{q}^{\,2}}{\vec{q}_1^{\,2}} K^B(\vec{q}, \vec{q}_2). \qquad (124)$$

The action of the modified kernel on the Born eigenfunctions $\vec{q}_2^{\,2(\gamma-1)}$ can be calculated easily:

$$\int d^{D-2}q_2 \; \widetilde{K}(\vec{q}_1, \vec{q}_2) \left(\frac{\vec{q}_2^{\,2}}{\vec{q}_1^{\,2}}\right)^{\gamma-1} = \frac{\alpha_s(\vec{q}_1^{\,2})\, N}{\pi} \left(\chi^B(\gamma) + \frac{\alpha_s(\vec{q}_1^{\,2}) N}{\pi} \widetilde{\chi}^{(1)}(\gamma)\right), \qquad (125)$$

where

$$\widetilde{\chi}^{(1)}(\gamma) = \chi^{(1)}(\gamma) - \frac{1}{2} \chi(\gamma)\, \chi'(\gamma). \qquad (126)$$

The anomalous dimensions $\gamma_\omega(\alpha_s) = \gamma_0(\alpha_s/\omega) + \alpha_s \gamma_1(\alpha_s/\omega)$ of the twist-2 operators near point $\omega = 0$ are determined from the solution of the equation

$$\omega = \frac{\alpha_s N}{\pi} \left(\chi^B(\gamma) + \frac{\alpha_s N}{\pi} \widetilde{\chi}^{(1)}(\gamma)\right) \simeq \frac{\alpha_s N}{\pi} \frac{1}{\gamma} - \frac{\alpha_s^2 N^2}{4\pi^2} \left(\frac{11 + 2n_f/N^3}{3\gamma^2} + \frac{n_f(10 + 13/N^2)}{9\gamma N} + \frac{395}{27} - 2\zeta(3) - \frac{11}{3}\frac{\pi^2}{6} + \frac{n_f}{N^3}\left(\frac{71}{27} - \frac{\pi^2}{9}\right)\right) \qquad (127)$$

for $\gamma \to 0$. For the low orders of the perturbation theory we reproduce the known results and predict the higher loop correction for $\omega \to 0$:

$$\gamma \simeq \frac{\alpha_s N}{\pi} \left(\frac{1}{\omega} - \frac{11}{12} - \frac{n_f}{6N^3}\right) - \left(\frac{\alpha_s}{\pi}\right)^2 \frac{n_f N}{6\omega} \left(\frac{5}{3} + \frac{13}{6N^2}\right)$$
$$- \frac{1}{4\omega^2} \left(\frac{\alpha_s N}{\pi}\right)^3 \left(\frac{395}{27} - 2\zeta(3) - \frac{11}{3}\frac{\pi^2}{6} + \frac{n_f}{N^3}\left(\frac{71}{27} - \frac{\pi^2}{9}\right)\right). \qquad (128)$$

The results presented in this Section were obtained in Ref.[21]. They differ from the corresponding results of Ref.[24] because the results of [24] were obtained for so called "irreducible part" of the kernel and therefore are incomplete. After appearance of Ref.[21] the results obtained there were confirmed in Ref. [25].

Finally, let us estimate the change of the maximal value of the anomalous dimension $\gamma_\omega(\alpha_s)$ which is determined by the position of the extremum of the function $\omega(\gamma)$ (127). In LLA this extremum is reached at $\gamma^B_{max} = 1/2$ independently from α_s. In NLLA, assuming the validity of the perturbative expansion, we obtain

$$\gamma_{max} \simeq \frac{1}{2} - \frac{N\alpha_s}{\pi} \frac{(\widetilde{\chi}^{(1)}(\frac{1}{2}))'}{(\chi^B(\frac{1}{2}))''} = \frac{1}{2} + \frac{N\alpha_s}{4\pi}\left(\frac{11}{6} - \frac{n_f}{3N} + 8\ln 2\right). \qquad (129)$$

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