Siberian Branch of Russian Academy of Science
BUDKER INSTITUTE OF NUCLEAR PHYSICS

R.N. Lee, A.I. Milstein, V.M. Strakhovenko

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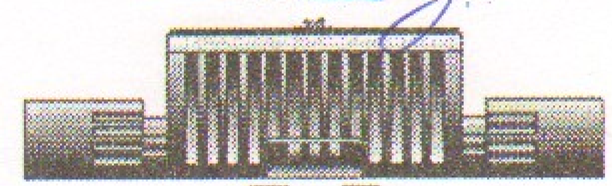

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Simple analytical representation
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R.N. Lee, A.I. Milstein, V.M. Strakhovenko

Budker Institute of Nuclear Physics 630090 Novosibirsk, Russia

Abstract

Using a new representation for the quasiclassical Green function of the Dirac equation in a Coulomb field, analytical expressions for the high-energy small-angle Delbrück scattering amplitudes are obtained exactly in the parameter $Z\alpha$. Magnitudes of the amplitudes coincide with the previous results. However, the structure of the expressions obtained is much more simple, which considerably facilitates numerical calculations.

The coherent scattering of photons in the electric fields of atoms via virtual electron-positron pairs (Delbrück scattering [1]) has been intensively investigated both theoretically and experimentally [2]. From the theoretical point of view this process is interesting since the higher orders of the perturbation theory in the parameter $Z\alpha$ ($Z|e|$ is the nucleus charge, $\alpha = e^2 = 1/137$ is the fine-structure constant, e is the electron charge, $\hbar = c = 1$) drastically change the amplitudes as compared to the first (Born) approximation. Delbrück scattering at high photon energy $\omega \gg m$ (m is the electron mass) is the most favorable to compare the theoretical and experimental results. Recently, the most accurate Delbrück scattering cross section measurements have been performed in the Budker Institute of Nuclear Physics [3] at photon energies $140 - 450$ MeV and scattering angles $2.6 - 16.6$ mrad. The Delbrück scattering amplitudes exact in the parameter $Z\alpha$ at high photon energies and small scattering angle were obtained in [4, 5, 6] by summing in a definite approximation the Feynman diagrams with an arbitrary number of photons exchanged with a Coulomb center. Another representation of these amplitudes was derived in [7, 8] using the quasiclassical Green function of the Dirac equation in a Coulomb field. Such a Green function in an arbitrary spherically symmetrical external field was obtained in [9, 10] where Delbrück scattering in a screened Coulomb field was investigated as well.

In recent papers [12, 13, 14] the new approach was developed, which significantly simplifies the consideration of high-energy processes in a spherically symmetrical external field. It was used to obtain the exact in $Z\alpha$ photon splitting amplitudes. These results are used in the data processing of the first successful experiment on the high-energy photon splitting in the electric fields of atoms recently performed in the Budker INP (the preliminary experimental results are published in [11]).

In the present paper the approach developed in [12, 13, 14] is applied to the calculation of the high-energy small-angle Delbrück scattering amplitudes. Numerically, the new amplitudes coincide with the previously known results. However, the numerical calculations by means of the new formulas, having actually only double integral, are significantly easier.

As shown in [9], it is convenient to express the Delbrück scattering amplitude via the Green function of the squared Dirac equation $D(\mathbf{r},\mathbf{r}'|\varepsilon)$:

$$D(\mathbf{r}_1,\mathbf{r}_2|\varepsilon) = \langle \mathbf{r}_1|1/(\hat{\mathcal{P}}^2 - m^2 + i0)|\mathbf{r}_2\rangle ,$$

where $\hat{\mathcal{P}} = \gamma^0(\varepsilon + Z\alpha/r) - \boldsymbol{\gamma}\,\mathbf{p}$, $\mathbf{p} = -i\boldsymbol{\nabla}$.

In terms of the function $D(\mathbf{r},\mathbf{r}'|\varepsilon)$ the amplitude of the process reads ([9]):

$$\begin{aligned} M = i\alpha \int d\mathbf{r}_1 d\mathbf{r}_2 \exp[i(\mathbf{k}_1\mathbf{r}_1 - \mathbf{k}_2\mathbf{r}_2)] \int d\varepsilon \times \\ \times \mathrm{Sp}\left[(2\mathbf{e}_2^*\mathbf{p}_2 - \hat{e}_2^*\hat{k}_2)D(\mathbf{r}_2,\mathbf{r}_1|\omega-\varepsilon)\right]\left[(2\mathbf{e}_1\mathbf{p}_1 + \hat{e}_1\hat{k}_1)D(\mathbf{r}_1,\mathbf{r}_2|-\varepsilon)\right] + \\ +2i\alpha\mathbf{e}_2^*\mathbf{e}_1 \int d\mathbf{r} \exp[i(\mathbf{k}_1 - \mathbf{k}_2)\mathbf{r}] \int d\varepsilon\, \mathrm{Sp}D(\mathbf{r},\mathbf{r}|\varepsilon) , \end{aligned} \tag{1}$$

where e_1, k_1 (e_2, k_2) are the polarization vector and 4-momentum of the initial (final) photon, $\mathbf{p}_{1,2} = -i\boldsymbol{\nabla}_{1,2}$. By definition, the Delbrück scattering amplitude (1) should vanish at $Z = 0$. Therefore, one should subtract from the integrand in (1) its value at $Z = 0$. We perform this subtraction in the explicit form below. At $\omega \gg m$ the main contribution to the cross section comes from the momentum transfer region $\Delta \sim m$ corresponding to the small scattering angles. Then we can neglect the last term in (1) since it depends only on the momentum transfer $\boldsymbol{\Delta} = \mathbf{k}_2 - \mathbf{k}_1$, while the amplitude at $\omega \gg \Delta$ is proportional to ω (see, e.g., [2]).

According to the uncertainty relation, the lifetime of the virtual electron-positron pair is $\tau \sim |\mathbf{r}_2 - \mathbf{r}_1| \sim \omega/(m^2 + \Delta^2)$ and $\rho \sim 1/\Delta$ for the characteristic impact parameter. For $\omega \gg \Delta \gg m^2/\omega$ the main contribution to the integral in (1) is given by small angles between the vectors $\mathbf{r}_2$, $-\mathbf{r}_1$, and $\mathbf{k}_1$. Then the characteristic angular momentum is $l \sim \omega\rho \sim \omega/\Delta \gg 1$ and the quasiclassical approximation is valid. Note that the screening should be taken into account only at $\Delta \sim r_c^{-1} \ll m$, where r_c is the screening radius ($r_c \sim (m\alpha)^{-1}Z^{-1/3}$ in the Thomas-Fermi model). In the

present paper we consider the momentum transfer region $\Delta \gg m^2/\omega$, r_c^{-1}, which gives the main contribution to the total cross section of the process. We emphasize that the contribution of higher orders of perturbation theory in $Z\alpha$ (Coulomb corrections) is not affected by screening and the corresponding expressions are valid for any $\Delta \ll \omega$. The modification of the Born contribution at $\Delta \sim m^2/\omega$ was studied in [5], the effects of screening were discussed in detail in [9].

In papers [12, 13], the convenient representation for the quasiclassical Green function $D(\mathbf{r}_1,\mathbf{r}_2|\varepsilon)$ of the squared Dirac equation in a Coulomb field was obtained. For small angles between $\mathbf{r}_2$, $-\mathbf{r}_1$ and the z axis, we have

$$\begin{aligned} D(\mathbf{r}_1,\mathbf{r}_2|\varepsilon) = \frac{i\kappa}{8\pi^2 r_1 r_2} \mathrm{e}^{i\kappa(r_1+r_2)} \int d\mathbf{q} \left[1 + Z\alpha\frac{\boldsymbol{\alpha}\mathbf{q}}{\kappa q^2}\right] \times \\ \times \exp\left[i\kappa\frac{q^2(r_1+r_2)}{2r_1r_2} + i\kappa\mathbf{q}(\boldsymbol{\theta}_1 + \boldsymbol{\theta}_2)\right]\left(\frac{4r_1r_2}{q^2}\right)^{iZ\alpha\lambda} , \end{aligned} \tag{2}$$

where $\boldsymbol{\alpha} = \gamma^0\boldsymbol{\gamma}$, $\kappa^2 = \varepsilon^2 - m^2$, $\lambda = \varepsilon/\kappa$, $\mathbf{q}$, $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$ are two-dimensional vectors in the xy plane, $\boldsymbol{\theta}_1 = \mathbf{r}_{1\perp}/r_1$, $\boldsymbol{\theta}_2 = \mathbf{r}_{2\perp}/r_2$. Expression (2) contains only elementary functions, and the angles $\boldsymbol{\theta}_1$ и $\boldsymbol{\theta}_2$ appear only in the factor $\exp[i\mathbf{q}(\boldsymbol{\theta}_1 + \boldsymbol{\theta}_2)]$. Therefore, representation (2) for the Green function is very convenient for calculations.

We direct the z axis along $\mathbf{k}_1$. The z component of the polarization vector $\mathbf{e}_2$ can be eliminated owing to the relation $\mathbf{e}_2\,\mathbf{k}_2 = 0$, which leads to $(\mathbf{e}_2)_z = -\mathbf{e}_2\boldsymbol{\Delta}/\omega$. After that, within the small-angle approximation one can neglect the difference between the vector $(\mathbf{e}_2)_\perp$ and the polarization vector of a photon, propagating along the z-axis and having the same helicity. So, the amplitudes of the process are expressed via the transverse polarization vectors $\mathbf{e}$ and $\mathbf{e}^*$, corresponding to the positive and negative helicities, respectively. It is sufficient to calculate two amplitudes, for instance, M_{++} and M_{+-}. Other amplitudes (M_{--} and M_{-+}) can be obtained by the substitution $\mathbf{e} \leftrightarrow \mathbf{e}^*$.

Substituting (2) into (1), we expand the amplitudes at small angles when $d\mathbf{r}_1 d\mathbf{r}_2 \approx r_1^2 r_2^2 dr_1 dr_2 d\boldsymbol{\theta}_1 d\boldsymbol{\theta}_2$. Taking the trace and performing the elementary integration over the angles $\boldsymbol{\theta}_1$ and $\boldsymbol{\theta}_2$, we obtain

$$M = -\frac{i\alpha}{\omega^2}\int_0^\omega d\varepsilon\,\varepsilon\kappa \int_0^\infty \frac{dr_1}{r_1}\int_0^\infty \frac{dr_2}{r_2} \iint \frac{d\mathbf{q}_1\,d\mathbf{q}_2}{(2\pi)^2}\left[\left(\frac{q_1}{q_2}\right)^{2iZ\alpha} - 1\right]\mathrm{e}^{i\Phi}T . \tag{3}$$

Here

$$\Phi = \frac{1}{2}\left[\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\mathbf{Q}^2 + \frac{\varepsilon-\kappa}{\omega}\mathbf{Q}\Delta + \mathbf{q}\Delta - m^2(r_1+r_2)\right], \tag{4}$$

the function T for different polarizations has the form

$$T_{++} = \frac{2\mathbf{Q}^2}{r_1 r_2} - \frac{\omega^2}{2\varepsilon\kappa}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\left[\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\mathbf{Q}^2 - 2i\right],\; T_{+-} = \frac{4}{r_1 r_2}(\mathbf{e}\mathbf{Q})^2 \tag{5}$$

and following notation is introduced $\kappa = \omega - \varepsilon$, $\mathbf{Q} = \mathbf{q}_1 + \mathbf{q}_2$ и $\mathbf{q} = \mathbf{q}_1 - \mathbf{q}_2$. When deriving (3) , we integrate by parts over $\mathbf{q}_1$, $\mathbf{q}_2$ so that the integrand contains the parameter $Z\alpha$ only in the factor $[(q_1/q_2)^{2iZ\alpha} - 1]$. Besides, we make the substitution $r_{1,2} \to (\varepsilon\kappa/\omega)r_{1,2}$. The expression T_{++} in (5) can be simplified if we pass temporarily in (3) from the variables r_1, r_2 to $R = r_1 r_2/(r_1+r_2)$, $t = r_1/r_2$ and integrate by parts over R the term proportional to $\mathbf{Q}^2$ in square brackets in (5) . As a result, we get

$$T_{++} = \frac{2\mathbf{Q}^2}{r_1 r_2} + \frac{\omega^2 m^2}{2\varepsilon\kappa r_1 r_2}(r_1+r_2)^2 \tag{6}$$

Let us pass from the variables $\mathbf{q}_1$, $\mathbf{q}_2$ to $\mathbf{q}$ and $\mathbf{Q}$. Then the integral over $\mathbf{q}$ acquires the form:

$$J = \int\frac{d\mathbf{q}}{\mathbf{Q}^2}\left[\left(\frac{|\mathbf{q}+\mathbf{Q}|}{|\mathbf{q}-\mathbf{Q}|}\right)^{2iZ\alpha} - 1\right]\exp(-\frac{i}{2}\mathbf{q}\Delta)\,. \tag{7}$$

As shown in [12], this integral is equal to

$$J = \int\frac{d\mathbf{q}}{\Delta^2}\left[\left(\frac{|\mathbf{q}+\Delta|}{|\mathbf{q}-\Delta|}\right)^{2iZ\alpha} - 1\right]\exp(-\frac{i}{2}\mathbf{q}\mathbf{Q})\,. \tag{8}$$

Using this representation and the parametrization

$$\exp(i\frac{\mathbf{Q}^2}{2r_1}) = ir_1\int\frac{d\mathbf{x}}{2\pi}\exp(-i\frac{r_1\mathbf{x}^2}{2} - i\mathbf{Q}\mathbf{x})\,, \tag{9}$$

where $\mathbf{x}$ is a two-dimensional vector lying in the same plane as $\mathbf{Q}$, we take the integrals in (3) first over r_1, then over $\mathbf{Q}$ and over r_2. Finally, we have

$$\left\{\begin{matrix} M_{++} \\ M_{+-}\end{matrix}\right\} = -\frac{i\alpha m^2}{\pi^2\Delta^2\omega^2}\int_0^\omega d\varepsilon\int d\mathbf{q}\left[\left(\frac{q_+}{q_-}\right)^{2iZ\alpha} - 1\right]\times$$
$$\times\int\frac{d\mathbf{x}}{(\mathbf{x}^2+m^2)^2(\mathbf{v}^2+m^2)^2}\left\{\begin{matrix} m^2(\varepsilon^2+\kappa^2)+\omega^2\mathbf{x}\mathbf{v} \\ 4\varepsilon\kappa(\mathbf{e}\mathbf{v})^2\end{matrix}\right\}, \tag{10}$$

where $q_\pm = |\mathbf{q}_\pm|$, $\mathbf{q}_\pm = \mathbf{q} \pm \Delta$, $\mathbf{v} = \mathbf{x} + \mathbf{q}/2 + \Delta(\varepsilon - \kappa)/2\omega$.

For the further integration it is convenient to rewrite the expression (10) in another form. Using the identities

$$\frac{m^2}{(\mathbf{v}^2+m^2)^2} = \frac{1}{2}\nabla_{\mathbf{v}}\frac{\mathbf{v}}{\mathbf{v}^2+m^2} = \nabla_{\mathbf{q}}\frac{\mathbf{v}}{\mathbf{v}^2+m^2}\,,$$
$$\frac{\mathbf{v}}{(\mathbf{v}^2+m^2)^2} = -\frac{1}{2}\nabla_{\mathbf{v}}\frac{1}{\mathbf{v}^2+m^2} = -\nabla_{\mathbf{q}}\frac{1}{\mathbf{v}^2+m^2}$$

and integrating by parts over $\mathbf{q}$, we find

$$\left\{\begin{matrix} M_{++} \\ M_{+-}\end{matrix}\right\} = \frac{2\alpha(Z\alpha)m^2}{\pi^2\Delta^2\omega^2}\int_0^\omega d\varepsilon\int d\mathbf{q}\left(\frac{q_+}{q_-}\right)^{2iZ\alpha}\int\frac{d\mathbf{x}}{(\mathbf{x}^2+m^2)^2(\mathbf{v}^2+m^2)}\times$$
$$\times\left(\frac{\mathbf{q}_+}{q_+^2} - \frac{\mathbf{q}_-}{q_-^2}\right)\left\{\begin{matrix} \omega^2\mathbf{x} - (\varepsilon^2+\kappa^2)\mathbf{v} \\ 4\varepsilon\kappa(\mathbf{e}\mathbf{v})\mathbf{e}\end{matrix}\right\} \tag{11}$$

Applying the Feynman parametrization for the denominators, we take the integral over $\mathbf{x}$ and pass from the variable ε to $s = 2\varepsilon/\omega - 1$. We obtain

$$\left\{\begin{matrix} M_{++} \\ M_{+-}\end{matrix}\right\} = \frac{4\alpha(Z\alpha)m^2\omega}{\pi\Delta^2}\int_{-1}^1 ds\int d\mathbf{q}\left(\frac{q_+}{q_-}\right)^{2iZ\alpha}\times \tag{12}$$
$$\times\int_0^1\frac{t\,dt}{[t(1-t)(\mathbf{q}+s\Delta)^2+4m^2]^2}\left(\frac{\mathbf{q}_+}{q_+^2} - \frac{\mathbf{q}_-}{q_-^2}\right)\left\{\begin{matrix} [t(1-s^2)-2](\mathbf{q}+s\Delta) \\ 2t(1-s^2)(\mathbf{e},\mathbf{q}+s\Delta)\mathbf{e}\end{matrix}\right\}$$

We perform the integration over $\mathbf{q}$ by means of a trick used in [13, 14]. Let us multiply the integrand in (12) by

$$1 \equiv \int_{-1}^1 dy\,\delta\left(y - \frac{2\mathbf{q}\Delta}{\mathbf{q}^2+\Delta^2}\right)$$
$$= (\mathbf{q}^2+\Delta^2)\int_{-1}^1\frac{dy}{|y|}\delta((\mathbf{q}-\Delta/y)^2 - \Delta^2(1/y^2-1)),$$

change the order of integration over $\mathbf{q}$ and y, and make the shift $\mathbf{q} \to \mathbf{q} + \Delta/y$. After that the integration over $\mathbf{q}$ can be done easily. By changing the variables $y = \mathrm{th}(\tau - \tau_0)$, where

$$\tau_0 = \frac{1}{2}\ln\left(\frac{B+(1+s)^2}{B+(1-s)^2}\right),\; B = \frac{4m^2}{\Delta^2 t(1-t)}\,, \tag{13}$$

we express the integral over τ in terms of two functions, the same as in [14]:

$$\mathcal{F}_1 = a^2 \int_0^\infty d\tau \frac{\mathrm{ch}\,\tau\,\cos(2Z\alpha\tau)}{(\mathrm{sh}^2\tau + a^2)^{3/2}} = \frac{2\pi a^2}{\mathrm{sh}(\pi Z\alpha)} \mathrm{Im}\, P'_{iZ\alpha}(2a^2-1)\,, \tag{14}$$

$$\mathcal{F}_2 = a^2 \int_0^\infty d\tau \frac{\mathrm{sh}\,\tau\,\sin(2Z\alpha\tau)}{(\mathrm{sh}^2\tau + a^2)^{3/2}} = -\frac{2\pi a^2}{\mathrm{sh}(\pi Z\alpha)} \mathrm{Re}\, P'_{iZ\alpha}(2a^2-1)\,,$$

where $a^2 = 4B/[(B+(1+s)^2)(B+(1-s)^2)]$, $P'_\nu(x)$ is the derivative of the Legendre function. Note that $a^2 \le 1$ for any s and $B > 0$.

The final result for the Delbrück scattering amplitudes reads:

$$M_{++} = i\frac{\alpha(Z\alpha)\omega}{8m^2}\int_0^1 ds \int_0^1 dt\, a^2 t[2 - t(1-s^2)] \times \tag{15}$$
$$\times \left[4sB\sin(2Z\alpha\tau_0)\mathcal{F}_1 + [B^2 - (s^2-1)^2]\cos(2Z\alpha\tau_0)\mathcal{F}_2\right]\,,$$

$$M_{+-} = i\frac{\alpha(Z\alpha)\omega(\mathbf{e}\Delta)^2}{4m^2\Delta^2}\int_0^1 ds \int_0^1 dt\, a^2 t(s^2-1)\left[4sB(1-t)\sin(2Z\alpha\tau_0)\mathcal{F}_1 + \right.$$
$$\left. + [B^2(2-3t) + 2B(s^2+1)(1-2t) - (s^2-1)^2 t]\cos(2Z\alpha\tau_0)\mathcal{F}_2\right]\,.$$

Let us derive now the asymptotics of the amplitudes (15) at $\Delta \ll m$ and $\Delta \gg m$. In the case $\Delta \ll m$, when $B \sim m^2/\Delta^2 \gg 1$, $a^2 \approx 4/B \ll 1$, and $\tau_0 \sim 1/B \ll 1$, the calculations are especially easy. Then the functions $\mathcal{F}_{1,2}$ have the asymptotic form:

$$\mathcal{F}_1 \approx 1\,,\ \mathcal{F}_2 = -2Z\alpha\, a^2[\ln a + C + \mathrm{Re}\,\psi(1+iZ\alpha)]\,,$$

where $C = 0.577\ldots$ is the Euler constant, $\psi(x) = d\ln\Gamma(x)/dx$. Substituting the asymptotics of the functions $\mathcal{F}_{1,2}$ into (15), we find

$$\left\{\begin{array}{c} M_{++} \\ M_{+-} \end{array}\right\} = i\frac{4\alpha(Z\alpha)^2\omega}{m^2}\int_0^1 ds \int_0^1 t\,dt \times \tag{16}$$
$$\times\left[\frac{1}{2}\ln\frac{m^2}{t(1-t)\Delta^2} - C - \mathrm{Re}\,\psi(1+iZ\alpha)\right]\left\{\begin{array}{c} [2 - t(1-s^2)] \\ 2(1-s^2)(3t-2)(\mathbf{e}\Delta)^2/\Delta^2 \end{array}\right\}$$

Taking here the trivial integrals, we obtain at $m^2/\omega \ll \Delta \ll m$:

$$M_{++} = i\frac{28\alpha(Z\alpha)^2\omega}{9m^2}[\ln(m/\Delta) + \frac{41}{42} - C - \mathrm{Re}\,\psi(1+iZ\alpha)]\,,$$

$$M_{+-} = i\frac{4\alpha(Z\alpha)^2\omega(\mathbf{e}\Delta)^2}{9m^2\Delta^2}\,. \tag{17}$$

To obtain the asymptotics at $\Delta \gg m$, it is convenient to start from (12). The main contribution to the integral over t comes from the region $1 - t \sim m^2/\Delta^2 \ll 1$. Taking in (12) the integral over t in this approximation, we have:

$$\left\{\begin{array}{c} M_{++} \\ M_{+-} \end{array}\right\} = \frac{\alpha(Z\alpha)\omega}{\pi\Delta^2}\int_{-1}^{1} ds \int \frac{d\mathbf{q}}{(\mathbf{q}+s\Delta)^2}\left(\frac{q_+}{q_-}\right)^{2iZ\alpha} \times$$
$$\times\left(\frac{\mathbf{q}_+}{q_+^2} - \frac{\mathbf{q}_-}{q_-^2}\right)\left\{\begin{array}{c} -(1+s^2)(\mathbf{q}+s\Delta) \\ 2(1-s^2)(\mathbf{e},\mathbf{q}+s\Delta)\mathbf{e} \end{array}\right\}\,; \tag{18}$$

Substituting (13) into this formula, we take the elementary integral over $\mathbf{q}$, and then over s. After that we perform the substitution $y = \mathrm{th}\,\tau$ and obtain at $\Delta \gg m$:

$$M_{++} = i\frac{4\alpha(Z\alpha)\omega}{3\Delta^2}\int_0^\infty d\tau\,\sin(2Z\alpha\tau)\,[4 - 3\,\mathrm{th}(\tau/2) - \mathrm{th}^3(\tau/2)] =$$
$$= i\frac{8\alpha\omega}{3\Delta^2}\left\{1 - \frac{2\pi Z\alpha}{\mathrm{sh}(2\pi Z\alpha)}[1 - (Z\alpha)^2]\right\}\,, \tag{19}$$
$$M_{+-} = i\frac{16\alpha(Z\alpha)\omega(\mathbf{e}\Delta)^2}{\Delta^4}\int_0^\infty \frac{d\tau\,\sin(2Z\alpha\tau)}{\mathrm{sh}^2\tau}(\tau\,\mathrm{cth}\,\tau - 1) =$$
$$= i\frac{16\alpha(Z\alpha)^2\omega(\mathbf{e}\Delta)^2}{\Delta^4}[1 - Z\alpha\,\mathrm{Im}\,\psi'(1 - iZ\alpha)]\,.$$

The asymptotics (17) and (19) coincide with the results of [6, 8]. Additionally, we checked numerically for different Z that the magnitudes of the amplitudes (15), as should be, coincide with the results of [6, 7, 8] at intermediate values of the momentum transfer Δ. The expression (15) for the Delbrück scattering amplitudes is a double integral, being essentially simpler than the known representations [6, 7, 8]. Therefore, the formula (15) is very convenient for numerical calculations.

The calculation of the Delbrück scattering amplitudes, performed in the present paper, demonstrate again the advantage of our approach in the consideration of different high-energy QED processes in a Coulomb field.

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R.N. Lee, A.I. Milstein, V.M. Strakhovenko

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Р.Н. Ли, А.И. Мильштейн, В.М. Страховенко

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