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THE COMPATIBILITY
OF THE GLUON REGGEIZATION
WITH THE s -CHANNEL UNITARITY

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Abstract

Recently the non-forward BFKL kernel for interaction of two Reggeized gluons in the antisymmetric colour octet state in the t -channel was obtained in the next-to-leading order. It gives the possibility to check in this order the bootstrap condition for this kernel, appearing as the requirement of the compatibility of gluon Reggeization with the s -channel unitarity.

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Introduction

Reggeization of elementary particles in the non-Abelian gauge theories [1], [2], [3], particularly in QCD, seems to be one of the most intriguing properties of these theories. Origin and consequences of this property are not completely understood, but its role in the high energy behaviour of scattering amplitudes hardly can be overestimated. The gluon Reggeization determines the high energy behaviour of non-decreasing with energy cross sections in perturbative QCD. In particular, it appears to be the basis of the BFKL approach [4].

Originally the BFKL equation was derived in the leading logarithm approximation (LLA), that means summation of all terms of perturbation theory, where smallness of the coupling constant α_s is compensated by the large logarithm $\ln s$ of the squared c.m.s. energy. Two years ago the first radiative correction to the kernel of the BFKL equation for the forward scattering, i.e. $t = 0$ and colour singlet state in the t -channel, was found [5]. It gives the possibility to sum also the next-to-leading order (NLO) terms. The correction was derived assuming the validity of the gluon Reggeization in the NLO, which is not proved, and therefore must be carefully tested. That can be done by checking the bootstrap equations [6] appearing as the conditions of the compatibility of the gluon Reggeization with the s -channel unitarity. The second, not less important reason for this check is that it gives the over-crossed test of the calculations of almost all ingredients used in the BFKL approach. In this approach the high energy scattering amplitudes are given by the convolution of the impact factors of scattered particles and the Green function for the Reggeon-Reggeon scattering. The impact factors and the kernel of the BFKL equation for the Green function, in turn, are expressed through the effective vertices for the Reggeon-particle interactions and the gluon Regge trajectory. The

antisymmetric colour octet state of the two Reggeized gluons in the t -channel $\mathcal{K}^{(8)}(\vec{q}_1, \vec{q}_2; \vec{q})$ with the gluon trajectory $j = 1 + \omega(t)$. In the NLO the equation takes the form

$$\frac{g^2 N t}{2 (2\pi)^{D-1}} \int \frac{d^{D-2} q_1}{\vec{q}_1^2 \vec{q}'_1{}^2} \int \frac{d^{D-2} q_2}{\vec{q}_2^2 \vec{q}'_2{}^2} \mathcal{K}^{(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q}) = \omega^{(1)}(t) \omega^{(2)}(t) \quad (1)$$

where g is the coupling constant, N is the number of colours ($N = 3$ for QCD), $D = 4 + 2\epsilon$ is the space-time dimension taken different from 4 for regularization of infrared divergences. $\mathcal{K}^{(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q})$ is the first radiative correction to the octet kernel, the vector sign is used for denoting component momenta transverse to the plane of momenta of initial particles, \vec{q} is the momentum transfer in the scattering process. For the sake of brevity we put $\vec{q}'_i = \vec{q}_i - \vec{q}$, $i = 1, 2$; \vec{q}'_1 and $-\vec{q}'_1$ (\vec{q}'_2 and $-\vec{q}'_2$) being the transverse momenta of the initial (final) Reggeized gluons in the t -channel.

The second bootstrap equation connects the colour octet impact factor of any particle and its effective interaction vertex with the Reggeized gluon. This equation was checked for the gluon [7] and quark [8] impact factors. It was proved that the equation is fulfilled both for helicity conserving and non-conserving parts of the impact factors at arbitrary space-time dimension D . As for the first bootstrap equation, Eq. (1), it was checked (and was found to be satisfied also at arbitrary D) only in the quark part of this equation [9]. The most important (and much more complicated) gluon part of this equation remained unchecked till now. Recently, the gluon part of the radiative correction to $\mathcal{K}^{(8)}$ was obtained [10], that gives a possibility to analyze Eq. (1) in this part.

In this paper we check the first bootstrap condition (1). Since the fulfillment of this equation for the quark part is established [9], we consider pure gluodynamics. In the next section we present the explicit expressions for the octet BFKL kernel and the gluon trajectory, and analyse Eq. (1). The proof of its fulfillment is given in Section 3. In the last section we discuss possible generalizations.

Let us start with the gluon trajectory. Its one-loop expression [2] is well known:

$$\omega^{(1)}(t) = \frac{g^2 N t}{2 (2\pi)^{D-1}} \int \frac{d^{D-2} k}{\vec{k}^2 (\vec{q} - \vec{k})^2} . \quad (2)$$

The integral can be easily calculated at arbitrary D and one has

$$\omega^{(1)}(t) = -\bar{g}^2 (\vec{q}^2)^\epsilon \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} , \quad (3)$$

where Γ is the Euler gamma-function and

$$\bar{g}^2 = \frac{g^2 N \Gamma(1 - \epsilon)}{(4\pi)^{D/2}} . \quad (4)$$

We stress that everywhere in this paper we use the unrenormalized coupling constant g .

In the two-loop approximation the integral representation for the trajectory is [11]

$$\omega^{(2)}(t) = \frac{g^2 t}{(2\pi)^{D-1}} \int \frac{d^{D-2} q_1}{\vec{q}_1^2 \vec{q}_1'^2} [F_G(\vec{q}_1, \vec{q}) - 2F_G(\vec{q}_1, \vec{q}_1)] , \quad (5)$$

where

$$F_G(\vec{q}_1, \vec{q}) = -\frac{g^2 N^2 \vec{q}^2}{4 (2\pi)^{D-1}} \int \frac{d^{D-2} q_2}{\vec{q}_2^2 (\vec{q}_2 - \vec{q})^2} \left[\ln \left(\frac{\vec{q}^2}{(\vec{q}_1 - \vec{q}_2)^2} \right) - 2\psi(1 + 2\epsilon) \right. \\ \left. - \psi(1 - \epsilon) + 2\psi(\epsilon) + \psi(1) + \frac{1}{1 + 2\epsilon} \left(\frac{1}{\epsilon} + \frac{1 + \epsilon}{2(3 + 2\epsilon)} \right) \right] , \quad (6)$$

and $\psi(x) = \Gamma'(x)/\Gamma(x)$. The integral (5) can be expressed in terms of elementary functions only for $\epsilon \rightarrow 0$. The answer is [12]

$$\omega^{(2)}(t) \simeq \left(\frac{\bar{g}^2 (\vec{q}^2)^\epsilon}{\epsilon} \right)^2 \left[\frac{11}{3} + \left(2\psi'(1) - \frac{67}{9} \right) \epsilon \right. \\ \left. + \left(\frac{404}{27} + \psi''(1) - \frac{22}{3} \psi'(1) \right) \epsilon^2 \right] . \quad (7)$$

of two Reggeized gluons in the t -channel is given by the sum of the “virtual” part, defined by the gluon trajectory, and the “real” part $\mathcal{K}_r^{(\mathcal{R})}$, related to the real particle production:

$$\mathcal{K}^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) \quad (8)$$

$$= \left(\omega(-\vec{q}_1^2) + \omega(-\vec{q}_1'^2) \right) \vec{q}_1^2 \vec{q}_1'^2 \delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + \mathcal{K}_r^{(\mathcal{R})}(\vec{q}_1, \vec{q}_2; \vec{q}) .$$

The octet kernel in the Born approximation differs only by the coefficient 1/2 from the singlet (Pomeron) kernel:

$$\mathcal{K}_r^{(8)(B)}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{g^2 N}{2 (2\pi)^{D-1}} f_B , \quad f_B = \frac{\vec{q}_1^2 \vec{q}_2'^2 + \vec{q}_2^2 \vec{q}_1'^2}{\vec{k}^2} - \vec{q}^2 , \quad (9)$$

where $\vec{k} = \vec{q}_1 - \vec{q}_2$. The radiative correction to this kernel is known only in the limit $\epsilon \rightarrow 0$ [10]. It can be presented as the sum of three contributions:

$$\mathcal{K}_r^{(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{\vec{q}^4}{\pi^{1+\epsilon} \Gamma(1-\epsilon)} (\mathcal{K}_1 + \mathcal{K}_2 + \mathcal{K}_3) . \quad (10)$$

The first contribution is proportional to the Born kernel (9):

$$\begin{aligned} \mathcal{K}_1 = & -f_B \frac{(\vec{k}^2)^\epsilon}{\epsilon} \left[\frac{11}{3} + \left(2\psi'(1) - \frac{67}{9} \right) \epsilon \right. \\ & \left. + \left(\frac{404}{27} + 7\psi''(1) - \frac{11}{3}\psi'(1) \right) \epsilon^2 \right] . \end{aligned} \quad (11)$$

Note that this part of the kernel contains all explicit singularities in ϵ , as it should be, because of factorization of singularities in QCD amplitudes. Note also that in Eq. (11) the term $(\vec{k}^2)^\epsilon$ is not expanded in ϵ and the terms of order ϵ are kept in the coefficient. It is done because the kernel is singular at $\vec{k} = 0$, so that, at subsequent integrations of the kernel in the BFKL equation, the region of arbitrary small, for $\epsilon \rightarrow 0$, values of \vec{k}^2 , where $\epsilon |\ln \vec{k}^2| \sim 1$, does contribute and, moreover, leads to the appearance of an extra $1/\epsilon$ factor. Consequently, the

terms after the integration. The last two terms in Eq. (10) are known for $\epsilon \rightarrow 0$ only. To stress this circumstance we shall indicate them with the superscript (0) . The second contribution can be expressed in terms of logarithms:

$$\begin{aligned} \mathcal{K}_2^{(0)} = & \left\{ \vec{q}^2 \left[\frac{11}{6} \ln \left(\frac{\vec{q}_1^2 \vec{q}_2^2}{\vec{q}^2 \vec{k}^2} \right) + \frac{1}{4} \ln \left(\frac{\vec{q}_1^2}{\vec{q}^2} \right) \ln \left(\frac{\vec{q}_1'^2}{\vec{q}^2} \right) \right. \right. \\ & + \frac{1}{4} \ln \left(\frac{\vec{q}_2^2}{\vec{q}^2} \right) \ln \left(\frac{\vec{q}_2'^2}{\vec{q}^2} \right) + \frac{1}{4} \ln^2 \left(\frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \left. \right] - \frac{\vec{q}_1^2 \vec{q}_2'^2 + \vec{q}_2^2 \vec{q}_1'^2}{2\vec{k}^2} \ln^2 \left(\frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \\ & \left. + \frac{\vec{q}_1^2 \vec{q}_2'^2 - \vec{q}_2^2 \vec{q}_1'^2}{\vec{k}^2} \ln \left(\frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \left(\frac{11}{6} - \frac{1}{4} \ln \left(\frac{\vec{q}_1^2 \vec{q}_2^2}{\vec{k}^4} \right) \right) \right\} + \{ \vec{q}_i \leftrightarrow \vec{q}_i' \} \end{aligned} \quad (12)$$

while the last cannot be written through elementary functions. It has the following integral representation:

$$\begin{aligned} \mathcal{K}_3^{(0)} = & \left\{ \frac{1}{2} \left[\vec{q}^2 \left(\vec{k}^2 - \vec{q}_1^2 - \vec{q}_2^2 \right) + 2\vec{q}_1^2 \vec{q}_2^2 - \vec{q}_1^2 \vec{q}_2'^2 \right. \right. \\ & \left. \left. - \vec{q}_2^2 \vec{q}_1'^2 + \frac{\vec{q}_1^2 \vec{q}_2'^2 - \vec{q}_2^2 \vec{q}_1'^2}{\vec{k}^2} \left(\vec{q}_1^2 - \vec{q}_2^2 \right) \right] \right. \\ & \left. \times \int_0^1 \frac{dx}{(\vec{q}_1(1-x) + \vec{q}_2 x)^2} \ln \left(\frac{\vec{q}_1^2(1-x) + \vec{q}_2^2 x}{\vec{k}^2 x(1-x)} \right) \right\} + \{ \vec{q}_i \leftrightarrow \vec{q}_i' \} . \quad (13) \end{aligned}$$

Knowing the “real” kernel only in the limit $\epsilon \rightarrow 0$, we can think about the check by the bootstrap equation of only the terms in the gluon trajectory non-vanishing for $\epsilon \rightarrow 0$. But we shall see that even for this purpose the knowledge of the kernel (10) is not sufficient. The reason comes from the singular behaviour of the integration measure in the bootstrap condition (1). Let us note that terms with the trajectory contribution in the formula (8) for the kernel can be integrated in Eq. (1) in a general form, even without the knowledge of an explicit expression for the trajectory. The matter is that the dependence of

so that

$$\omega^{(2)}(t) = \bar{g}^4 (\vec{q}^2)^{2\epsilon} N_\epsilon, \quad (14)$$

where N_ϵ is some coefficient depending on ϵ (whose expression in the limit $\epsilon \rightarrow 0$ is given by Eq. (7)). Using the generic integral

$$\begin{aligned} & \frac{1}{(2\pi)^{D-1}} \int \frac{d^{D-2}k}{(\vec{k}^2)^{1-\delta_1} ((\vec{k} - \vec{q})^2)^{1-\delta_2}} = \\ & \frac{2(\vec{q}^2)^{\epsilon+\delta_1+\delta_2-1}}{(4\pi)^{2+\epsilon}} \frac{B(\epsilon + \delta_1, \epsilon + \delta_2) \Gamma(1 - \delta_1 - \delta_2 - \epsilon)}{\Gamma(1 - \delta_1) \Gamma(1 - \delta_2)}, \end{aligned} \quad (15)$$

where $B(x, y)$ is the Euler beta-function, putting $\delta_1 = 2\epsilon$, $\delta_2 = 0$, from Eqs. (1), (3) and (8) we obtain

$$\begin{aligned} & (\vec{q}^2)^{1-3\epsilon} \int \frac{d^{D-2}q_1}{\vec{q}_1^2 \vec{q}_1'^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^2 \vec{q}_2'^2} \frac{\mathcal{K}_r^{(8)(1)}(\vec{q}_1, \vec{q}_2; \vec{q})}{\bar{g}^4 \pi^{1+\epsilon} \Gamma(1 - \epsilon)} = \\ & N_\epsilon B(\epsilon, \epsilon) \left[1 + \frac{B(\epsilon, 3\epsilon)}{\epsilon B(1 - \epsilon, -2\epsilon) B(\epsilon, \epsilon)} \right]. \end{aligned} \quad (16)$$

The integration measure in Eq. (16) is singular at zero momenta of scattered Reggeons. Fortunately, the kernel (10) turns into zero at this points [10], as it can be checked by direct inspection of Eqs. (11) - (13), so that at first sight these points could not bring additional singularities in ϵ . It would be so if integration over only one momentum was performed. But in the region where the momenta of two Reggeons (let us say, \vec{q}_1 and \vec{q}_2) turn into zero simultaneously, being of the same order, the kernel does not vanish (as it can be easily observed in the example of the Born kernel (9)). Therefore, these regions can give additional singularities in ϵ ; moreover, since integration over these regions leads to singularities, an expansion of, let us say, $(\vec{q}_1^2)^\epsilon$ and $(\vec{q}_2^2)^\epsilon$ is not possible anymore, so that we need to know about the kernel more than that is given by Eqs. (10) - (13).

In the limit $\epsilon \rightarrow 0$ the coefficient N_ϵ Eq. (14) is defined from Eq. (7). Our aim is to check the non-vanishing, for $\epsilon \rightarrow 0$, term in N_ϵ . Using the explicit expression for N_ϵ we can rewrite Eq. (16) as

$$(\vec{q}^2)^{1-3\epsilon} \int \frac{d^{D-2}q_1}{\vec{q}_1^2 \vec{q}_1'^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^2 \vec{q}_2'^2} \frac{\mathcal{K}_1 + \mathcal{K}_2 + \mathcal{K}_3}{(\pi^{1+\epsilon} \Gamma(1-\epsilon))^2} \simeq -\frac{2}{3\epsilon^3} \left[\frac{11}{3} + \left(2\psi'(1) - \frac{67}{9} \right) \epsilon + \left(\frac{404}{27} - 11\psi'(1) + \psi''(1) \right) \epsilon^2 \right], \quad (17)$$

where in the R.H.S. we have kept only terms singular in ϵ (that corresponds to non-vanishing terms in $\omega^{(2)}$). We need to calculate the L.H.S. of Eq. (17) with the same accuracy. Let us consider where the terms singular in ϵ can come from. In general, there are two kinds of singularities: the singularity already existing in the kernel (namely, that in the expression (11) for \mathcal{K}_1), and the singularities appearing as the results of integration in Eq. (17). The last singularities, in turn, can come from two kinds of integration regions: one is the region $|\vec{k}| \rightarrow 0$, where the kernel (again \mathcal{K}_1 , and only this part of the kernel) is singular, and the other consists in the regions where the transverse momenta of the two Reggeons simultaneously tend to zero, as it was explained at the end of the preceding section. These last regions do not overlap each others, but overlap with the region $|\vec{k}| \rightarrow 0$. It is clear that the most dangerous is the term \mathcal{K}_1 of the kernel, proportional to the Born kernel, which gives the singularities of all kinds. Nevertheless, it is not very difficult to calculate the contribution of this part in Eq. (17). The main point here is that we know this part of the kernel sufficiently well in the singular regions.

First of all, let us understand, in which of the regions, where the momenta of the two Reggeons are simultaneously tending to zero, the part \mathcal{K}_1 can contribute. Such regions are four, in general:

- 1) $|\vec{q}_1| \sim |\vec{q}_2| \rightarrow 0; \quad \vec{q}_1' \simeq \vec{q}_2' \simeq -\vec{q}, \quad |\vec{k}| \rightarrow 0,$
- 2) $|\vec{q}_1'| \sim |\vec{q}_2'| \rightarrow 0; \quad \vec{q}_1 \simeq \vec{q}_2 \simeq \vec{q}, \quad |\vec{k}| \rightarrow 0,$
- 3) $|\vec{q}_1| \sim |\vec{q}_2'| \rightarrow 0; \quad \vec{q}_1' \simeq -\vec{q}_2 \simeq \vec{k} \simeq -\vec{q},$
- 4) $|\vec{q}_1'| \sim |\vec{q}_2| \rightarrow 0; \quad \vec{q}_1 \simeq -\vec{q}_2' \simeq \vec{k} \simeq \vec{q}.$

(18)

consider only the first and third regions. One can easily see that in the third region $\mathcal{K}_1 \rightarrow 0$, so that only the first region remains. However, this is the region of small $|\vec{k}|$, where we know the kernel sufficiently well, so that for the calculation we do not need more information than that shown in Eq. (11).

The calculation could be done (and it was done) straightforwardly, but a more sophisticated way of calculation leads to our aim through a much more easy way. Let us first of all put

$$d\rho = \frac{1}{\pi^{2\epsilon+2}\Gamma^2(1-\epsilon)} \frac{d^{D-2}q_1}{\vec{q}_1^2 \vec{q}_1'^2} \frac{d^{D-2}q_2}{\vec{q}_2^2 \vec{q}_2'^2} . \quad (19)$$

We need to calculate the terms non-vanishing with ϵ in the integral

$$\begin{aligned} J = \int d\rho f_B \left(\frac{\vec{k}^2}{\vec{q}^2} \right)^\epsilon &\equiv \int d\rho f_B + \int d\rho f_B \left(\left(\frac{\vec{k}^2}{\vec{q}^2} \right)^\epsilon - 1 \right) \theta(\mu^2 - \vec{k}^2) \\ &+ \int d\rho f_B \left(\left(\frac{\vec{k}^2}{\vec{q}^2} \right)^\epsilon - 1 \right) \theta(\vec{k}^2 - \mu^2) , \end{aligned} \quad (20)$$

where $\theta(x)$ is the usual step function and μ is chosen in such a way that the ratio $\mu^2/\vec{q}^2 \sim \epsilon^n \ll 1$, n being arbitrary fixed integer number. Then last integral is of order ϵ (since we can expand here $(\vec{k}^2/\vec{q}^2)^\epsilon$ and the regions of singularities are excluded by the θ -function) and can be neglected, whereas the first one can be easily calculated with the help of Eq. (15) and gives

$$\int d\rho f_B \simeq (\vec{q}^2)^{2\epsilon-1} \left(\frac{2}{\epsilon^2} - 4\psi'(1) \right) . \quad (21)$$

In the second integral we can perform the first integration over \vec{q}_1 keeping \vec{k} fixed. In terms of these variables one can write

$$\frac{f_B}{\vec{q}_1^2 \vec{q}_1'^2 \vec{q}_2^2 \vec{q}_2'^2} = \frac{1}{\vec{k}^2} \left(\frac{1}{(\vec{q}_1 - \vec{q})^2 (\vec{q}_1 - \vec{k})^2} + \frac{1}{\vec{q}_1^2 (\vec{q}_1 - \vec{k} - \vec{q})^2} \right)$$

$$\frac{\vec{q}_1^2 (\vec{q}_1 - \vec{q})^2 (\vec{q}_1 - \vec{k})^2 (\vec{q}_1 - \vec{k} - \vec{q})^2}{\vec{q}_1^2 \vec{q}_1'^2 \vec{q}_2^2 \vec{q}_2'^2} \quad (22)$$

The first two terms can be easily integrated over \vec{q}_1 ; as for the last term, two non-overlapping regions do contribute to the integral at small $|\vec{k}|$: $|\vec{q}_1| \sim |\vec{k}|$ and $|\vec{q}_1 - \vec{q}| \sim |\vec{k}|$. They give equal contributions, so it is sufficient considering the first of them, where we can put

$$\frac{\vec{q}^2}{\vec{q}_1^2 \vec{q}_1'^2 \vec{q}_2^2 \vec{q}_2'^2} \simeq \frac{1}{\vec{q}^2 \vec{q}_1^2 (\vec{q}_1 - \vec{k})^2}, \quad (23)$$

and then expand the integration region at all \vec{q}_1 , due to the convergence of the integral. In such a way we obtain

$$\begin{aligned} & \int d\rho f_B \left(\left(\frac{\vec{k}^2}{\vec{q}^2} \right)^\epsilon - 1 \right) \theta(\mu^2 - \vec{k}^2) \simeq \\ & \frac{2}{\vec{q}^2} \int \frac{d^{D-2}k}{\pi^{\epsilon+1} \Gamma(1-\epsilon)} \frac{1}{\vec{k}^2} \left(\left(\frac{\vec{k}^2}{\vec{q}^2} \right)^\epsilon - 1 \right) \\ & \times \theta(\mu^2 - \vec{k}^2) B(\epsilon, \epsilon) \left((\vec{q}^2)^\epsilon - (\vec{k}^2)^\epsilon \right). \end{aligned} \quad (24)$$

Using the relation

$$\frac{d^{D-2}k}{\pi^{\epsilon+1} \Gamma(1-\epsilon)} = \frac{(\vec{k}^2)^\epsilon d\vec{k}^2}{\Gamma(1-\epsilon)\Gamma(1+\epsilon)} \quad (25)$$

we obtain

$$\int d\rho f_B \left(\left(\frac{\vec{k}^2}{\vec{q}^2} \right)^\epsilon - 1 \right) \theta(\mu^2 - \vec{k}^2) \simeq (\vec{q}^2)^{2\epsilon-1} \left[-\frac{4}{3\epsilon^2} + \frac{8}{3}\psi'(1) \right]. \quad (26)$$

This result, together with Eq. (21) gives

$$J \simeq (\vec{q}^2)^{2\epsilon-1} \left[\frac{2}{3\epsilon^2} - \frac{4}{3}\psi'(1) \right], \quad (27)$$

$$\begin{aligned}
& (\vec{q}^2)^{1-3\epsilon} \int \frac{d^{D-2}q_1}{\vec{q}_1^2 \vec{q}_1'^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^2 \vec{q}_2'^2} \frac{\mathcal{K}_1}{(\pi^{1+\epsilon} \Gamma(1-\epsilon))^2} \simeq \\
& -\frac{2}{3\epsilon^3} \left[\frac{11}{3} + \left(2\psi'(1) - \frac{67}{9} \right) \epsilon + \left(\frac{404}{27} - 11\psi'(1) + 7\psi''(1) \right) \epsilon^2 \right]. \quad (28)
\end{aligned}$$

This result coincides with that of the straightforward calculation which was also performed. Comparing with Eq. (17) we see that the contribution of the term proportional to the Born kernel almost ‘‘saturates’’ the bootstrap condition; the only difference is that, instead of the term $7\psi''(1)$ of Eq. (28), in the R.H.S. of Eq. (17) there is $\psi''(1)$.

We now turn to the remaining contributions to the kernel, \mathcal{K}_2 and \mathcal{K}_3 . They have neither explicit singularities in ϵ , nor singular behaviour for $|\vec{k}| \rightarrow 0$. Therefore, terms of order $1/\epsilon$ can be obtained only from the regions 1) - 4) (see Eq. (18)), where the momenta of the two Reggeons tend to zero being of the same order. It is easy to see from Eqs. (12) and (13) that in the regions 3) and 4) both $\mathcal{K}_2^{(0)}$ and $\mathcal{K}_3^{(0)}$ turn into zero. Using the symmetry of the kernel with respect to $\vec{q}_i \leftrightarrow \vec{q}_i'$, we can consider only the first region. Here we have

$$\begin{aligned}
\mathcal{K}_2^{(0)} & \simeq \vec{q}^2 \left[\frac{11}{6} \left(\ln \left(\frac{\vec{q}_1^2 \vec{q}_2^2}{\vec{k}^4} \right) + \frac{\vec{q}_1^2 - \vec{q}_2^2}{\vec{k}^2} \ln \left(\frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \right) \right. \\
& \left. + \left(\frac{1}{4} - \frac{\vec{q}_1^2 + \vec{q}_2^2}{2\vec{k}^2} \right) \ln^2 \left(\frac{\vec{q}_1^2}{\vec{q}_2^2} \right) - \frac{1}{4} \frac{\vec{q}_1^2 - \vec{q}_2^2}{\vec{k}^2} \ln \left(\frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \ln \left(\frac{\vec{q}_1^2 \vec{q}_2^2}{\vec{k}^4} \right) \right], \quad (29)
\end{aligned}$$

$$\mathcal{K}_3^{(0)} \simeq \frac{\vec{q}^2}{2} \left[\vec{k}^2 - \vec{q}_1^2 - \vec{q}_2^2 + \frac{2(\vec{q}_1^2(\vec{q}_2 \vec{k}) - \vec{q}_2^2(\vec{q}_1 \vec{k}))}{\vec{k}^2} \right] I, \quad (30)$$

where

$$I = \int_0^1 \frac{dx}{(\vec{q}_1(1-x) + \vec{q}_2 x)^2} \ln \left(\frac{\vec{q}_1^2(1-x) + \vec{q}_2^2 x}{\vec{k}^2 x(1-x)} \right). \quad (31)$$

at $|\vec{q}_1| \sim |\vec{k}|$ since

$$d\rho \simeq \frac{1}{\pi^{2\epsilon+2}\Gamma^2(1-\epsilon)} \frac{d^{D-2}k}{\vec{q}^4} \frac{d^{D-2}q_1}{\vec{q}_1^2(\vec{q}_1 - \vec{k})^2}. \quad (32)$$

Therefore with such order of integration at fixed small \vec{k} we can integrate over all \vec{q}_1 and nevertheless use the expressions (29) - (31).

All integrals of the terms entering into Eq. (29) for $\mathcal{K}_2^{(0)}$ can be taken using Eq. (15) and its derivatives with respect to δ_1 and δ_2 . All the integrals of this part can be calculated for arbitrary ϵ . We are interested in the limit $\epsilon \rightarrow 0$, but in the region of arbitrary small \vec{k} , so that we cannot expand $(\vec{k}^2)^\epsilon$ in powers of ϵ . In this limit we obtain

$$\int \frac{d^{D-2}q_1}{\vec{q}_1^2(\vec{q}_1 - \vec{k})^2} \frac{\mathcal{K}_2^{(0)}}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \simeq \frac{\vec{q}^2}{\vec{k}^2} (\vec{k}^2)^\epsilon 3\psi''(1). \quad (33)$$

The integration of Eq. (30) for $\mathcal{K}_3^{(0)}$ is not straightforward. We use the representation

$$I = \int_0^1 dx \int_1^\infty \frac{dz}{z} \frac{1}{(\vec{q}_1 - x\vec{k})^2 + zx(1-x)\vec{k}^2} =$$

$$\int_0^1 dx \int_1^\infty \frac{dz}{z} \frac{1}{x(\vec{q}_1 - \vec{k})^2 + (1-x)\vec{q}_1^2 + (z-1)x(1-x)\vec{k}^2}. \quad (34)$$

After this the integration over \vec{q}_1 is reduced to integrations of terms with two denominators, which are performed using the standard Feynman parametrization. Remaining integrations over x , z and the Feynman parameter are rather tedious, although not very complicated. The result for $\epsilon \rightarrow 0$ is the same as for $\mathcal{K}_2^{(0)}$:

$$\int \frac{d^{D-2}q_1}{\vec{q}_1^2(\vec{q}_1 - \vec{k})^2} \frac{\mathcal{K}_3^{(0)}}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \simeq \frac{\vec{q}^2}{\vec{k}^2} (\vec{k}^2)^\epsilon 3\psi''(1). \quad (35)$$

Again we do not expand $(\vec{k}^2)^\epsilon$, because at the subsequent integration over \vec{k} the region where $\epsilon \left| \ln \vec{k}^2 \right| \sim 1$ does contribute. Note that

appearance of the factors $(\epsilon^{-1})^\epsilon$ in Eqs. (33) and (35) is evident without calculations: they are dimensional factors and in the integrals (33) and (35) we have only one dimensional parameter.

We see that scale factors can be important even in calculation of terms singular in ϵ which come from the nonsingular contributions \mathcal{K}_2 and \mathcal{K}_3 in Eq. (17). But we know \mathcal{K}_2 and \mathcal{K}_3 only in the limit $\epsilon \rightarrow 0$ at fixed Reggeon momenta, where scale factors are put equal to 1 (see Eqs. (12) and (13)). The problem therefore is to restore the scale factors. Fortunately, we need to know the scale factors only in the first of regions (18), where we have only two essentially different scales since we have $|\vec{q}_1| \sim |\vec{q}_2| \sim |\vec{k}|$ from one side and $|\vec{q}_1'| \simeq |\vec{q}_2'| \simeq |\vec{q}|$ from the other. Even without calculations it is clear that the relevant scale should be $|\vec{k}|$, since the scale appears as a result of integration over transverse momenta (transverse momenta of gluons produced in the Reggeon-Reggeon collision for the two-gluon contribution to the kernel, transverse momenta of virtual gluons in the radiative correction to the Reggeon-Reggeon-gluon (RRG) vertex for the one-gluon contribution), and for $|\vec{q}_1| \sim |\vec{q}_2| \sim |\vec{k}|$ in the essential region of the integration these momenta should be of the same order. This conclusion is confirmed by direct inspection of the kernel. Fortunately, for the two-gluon contribution this inspection can be done straightforwardly, since there is the explicit expression for this contribution for arbitrary D [10]. The one-gluon contribution consists of two pieces: in one of them radiative corrections are contained in the RRG vertex with momenta q_1 and q_2 , in the other with momenta q_1' and q_2' . It follows from the kinematical structure of the RRG vertex that in the region 1) only the first piece does contribute to \mathcal{K}_2 (\mathcal{K}_3 comes totally from the two-gluon production). The evident scale for this piece is $|\vec{k}|$. Therefore, in the first of regions (18) we have

$$\mathcal{K}_2 = \left(\vec{k}^{-2}\right)^\epsilon \mathcal{K}_2^{(0)}, \quad \mathcal{K}_3 = \left(\vec{k}^{-2}\right)^\epsilon \mathcal{K}_3^{(0)}. \quad (36)$$

Using Eqs. (33) and (35), and taking into account the region 2) by doubling the results we obtain

$$\vec{q}^2 \int \frac{d^{D-2}q_1}{\vec{q}_1^2 \vec{q}_1'^2} \int \frac{d^{D-2}q_2}{\vec{q}_2^2 \vec{q}_2'^2} \frac{\mathcal{K}_2 + \mathcal{K}_3}{(\pi^{1+\epsilon} \Gamma(1-\epsilon))^2} \simeq$$

$$12\psi''(1) \int \frac{d^{D-2}k}{\pi^{1+\epsilon} \Gamma(1-\epsilon)} (\vec{k}^2)^{2\epsilon-1} \theta(\vec{q}^2 - \vec{k}^2) \simeq \frac{4\psi''(1)}{\epsilon}. \quad (37)$$

Putting Eqs. (28) and (37) into Eq. (17) we see that the bootstrap condition is satisfied.

3 Conclusions

We have proved the fulfillment of the first bootstrap condition in the form of Eq. (1) in the limit of the space-time dimension D tending to its physical value $D = 4$. All terms in the two-loop contribution to the gluon trajectory and in the NLO correction to the octet BFKL kernel, non-vanishing in this limit, are involved in this bootstrap. Therefore, the performed verification of the bootstrap gives us not only a powerful confirmation of the gluon Reggeization, but at the same time a stringent test of the calculations of the trajectory and the kernel.

Now we have practically no doubts on the gluon Reggeization in the NLO, as well as on the calculations of gluon trajectory and kernel. Nevertheless it would be interesting to verify if the first bootstrap condition is satisfied for arbitrary space-time dimension D . That is known to be true for the quark part of the kernel and for the second bootstrap condition in the cases of quark and gluon impact factors. It will become possible to do this verification after the calculation of the one-loop correction to the Reggeon-Reggeon-gluon vertex for arbitrary D , which is in progress now [13]. Another interesting possibility is to check the so-called “first strong bootstrap condition” for the kernel suggested by Braun and Vacca [14]. This condition is derived from the requirement that the particle-Reggeon scattering amplitudes have a Reggeized form and it is satisfied for the quark part of the kernel, as well as the analogous condition for impact factors in the quark and gluon cases, although the role of the strong bootstrap conditions in the BFKL approach is not completely understood.

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