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Abstract

Radiative effects are considered for an electron moving in a medium in the presence of an external electromagnetic field. Anomalous magnetic moment (AMM) of the electron in α-order is calculated under these conditions in the form of two-dimensional integral. Behavior of AMM of high energy electron under influence of multiple scattering in a medium is analyzed. Both mentioned effects lead to reduction of AMM.

1 Introduction

The contributions of higher orders of the perturbation theory over the interaction with an electromagnetic field give the electromagnetic radiative corrections to the electron mass and lead to appearance of the anomalous magnetic moment (AMM) of the electron [1]. It is known that under influence of an external electromagnetic field these effects in particular the AMM of electron are changed essentially [2], [3]. Here we consider how the corrections to the electron mass and the AMM of electron are modified in a medium.

The correction to the electron mass is described by the amplitude defined by the diagram of the electron self-energy. We use the operator quasiclassical method (see [3], [4], [5]). In this method the mentioned amplitude is described by the diagram where the electron first radiates a photon passing to a virtual state and then absorbs it. This corresponds to use of the non-covariant perturbation theory where in the high energy region only the contribution of this diagram survives. For the electron with energy $\varepsilon \gg m$ (m is the electron mass) this process occurs in a rather long time (or at a rather long distance) known as the lifetime of the virtual state

$$l_f = \frac{2\varepsilon}{q_c^2}, \tag{1.1}$$

where $q_c \geq m$ is the characteristic transverse momentum of the process, the system $\hbar = c = 1$ is used. When the virtual electron is moving in a medium it scatters on atoms. The mean square of momentum transfer to the electron from a medium on the distance l_f

is

$$q_s^2 = 4\pi Z^2\alpha^2 n_a \ln(q_c^2 a^2) l_f, \tag{1.2}$$

where $\alpha = e^2 = 1/137$, Z is the charge of nucleus, n_a is the number density of atoms in the medium, a is the screening radius of atom.

In the case of weak scattering $q_s \equiv \sqrt{q_s^2} \ll m$ the influence of a medium is weak, in this case $q_c = m$. At high energy it is possible that $q_s \geq m$.In this case the characteristic value of the momentum transfer (giving the main contribution into the spectral probability) is defined by the value of q_c. The self-consistency condition is

$$q_c^2 = q_s^2 = \frac{8\pi\varepsilon Z^2\alpha^2 n_a \ln(q_c^2 a^2)}{q_c^2} \geq m. \tag{1.3}$$

With q_c increase the lifetime of the virtual state (1.1) decreases and correspondingly the corrections to the electron mass and AMM of the electron are suppressed.

In the present paper we consider a motion of the charged particle in a medium in presence of an external electromagnetic field. In Sec.2 the general expressions for the correction to the electron mass and for AMM of the electron are derived to order α of the perturbation theory. In Sec.3 we analyze the influence of medium on AMM of the electron in detail. In Sec.4 the spin correction to the radiation intensity is given. In Sec.5 the possibility of observation of discovered effect is discussed.

2 Radiative corrections to the electron mass

In the frame of the operator quasiclassical method the probability of photon emission which will be used below is

$$dw = \frac{\alpha}{(2\pi)^2}\frac{d^3k}{\omega}\int dt_1 dt_2 R^*(t_2)R(t_1)\exp\left\{-i\frac{\varepsilon}{\varepsilon-\omega}[kx(t_2) - kx(t_1)]\right\}, \tag{2.1}$$

where $k = (\omega, \mathbf{k})$ is the four-momentum of photon, $k^2 = 0$, $x(t) = (t, \mathbf{r}(t))$, $\mathbf{r}(t))$ is the trajectory of particle, $R(t)$ is the matrix element depending on particle spin. To calculate the combination $R^*(t_2)R(t_1)$ it is convenient to start from Eq.(7.59) of [5]. Summing over the final electron polarization we find that the combination $R\ (t_2)R(t_1)$ in the expression for the probability of radiation (2.1) has the form

$$R^*(t_2)R(t_1) = \frac{\varepsilon}{2\varepsilon'\gamma^2}\left\{\frac{\omega^2}{\varepsilon\varepsilon'} + \left(\frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}\right)\mathbf{p}\mathbf{p}' + i\frac{\omega}{\varepsilon}((\mathbf{p}'-\mathbf{p})\times\mathbf{v})\boldsymbol{\zeta}\right\}, \tag{2.2}$$

where $\boldsymbol{\zeta}$ is the vector describing the initial polarization of the electron (in its rest frame), $\mathbf{p} = \gamma\boldsymbol{\vartheta}(t_1)$ and $\mathbf{p}' = \gamma\boldsymbol{\vartheta}(t_2)$. The first two terms coincide with Eq.(7.60) of [5] (unpolarized electrons), the third term depends on electron polarization.

In the paper [6](see also [7]) we derived the following expression for the spectral distribution of the probability of radiation per unit time for unpolarized electron

$$\begin{aligned} \frac{dW}{d\omega} &= \frac{2\alpha m^2}{\varepsilon^2}\mathrm{Im}T, \\ T &= \left\langle 0\left|R_1\left(G^{-1} - G_0^{-1}\right) + R_2\mathbf{p}\left(G^{-1} - G_0^{-1}\right)\mathbf{p}\right|0\right\rangle, \end{aligned} \tag{2.3}$$

where

$$\begin{aligned} &W = \frac{dw}{dt}, \quad R_1 = \frac{\omega^2}{\varepsilon\varepsilon'}, \quad R_2 = \frac{\varepsilon}{\varepsilon'} + \frac{\varepsilon'}{\varepsilon}, \quad \varepsilon' = \varepsilon - \omega; \\ &G_0 = \mathcal{H}_0 + 1,\ \mathcal{H}_0 = \mathbf{p}^2,\ \mathbf{p} = -i\boldsymbol{\nabla}_{\boldsymbol{\varrho}},\ G = \mathcal{H} + 1, \quad \mathcal{H} = \mathbf{p}^2 - iV(\boldsymbol{\varrho}), \\ &V(\boldsymbol{\varrho}) = Q\boldsymbol{\varrho}^2\left(L_1 + \ln\frac{4}{\boldsymbol{\varrho}^2} - 2C\right), \quad Q = \frac{2\pi Z^2\alpha^2\varepsilon\varepsilon' n_a}{m^4\omega}, \quad L_1 = \ln\frac{a_{s2}^2}{\lambda_c^2}, \\ &\frac{a_{s2}}{\lambda_c} = 183Z^{-1/3}\mathrm{e}^{-f}, \quad f = f(Z\alpha) = (Z\alpha)^2\sum_{k=1}^{\infty}\frac{1}{k(k^2 + (Z\alpha)^2)}, \end{aligned} \tag{2.4}$$

here $C = 0.577216\ldots$ is Euler's constant, n_a is the number density of atoms in the medium, $\boldsymbol{\varrho}$ is the coordinate in the two-dimensional space measured in the Compton wavelength λ_c, which is conjugate to the space of the transverse momentum transfers measured in the electron mass m.

The total probability of radiation W is connected with imaginary part of the radiative correction to the electron mass according to

$$m\Delta m = \varepsilon\Delta\varepsilon, \quad -2\mathrm{Im}\Delta\varepsilon = W, \quad \mathrm{Im}\ \Delta m = -\frac{\varepsilon}{2m}W. \tag{2.5}$$

Since the amplitude T is the analytic function of the potential V we have that

$$\Delta m = -\alpha m \int_0^{\varepsilon} \frac{d\omega}{\varepsilon} T. \tag{2.6}$$

One can derive this formula considering the self-energy diagram and the corresponding amplitude of forward scattering of electron (see Eqs.(12.18) and (12.39) in [3]). Here it is helpful also to use the approach formulated in [8].

In the presence of a homogeneous external field the Hamiltonian (2.4) acquires the linear over coordinate term. One can find the explicit form of this term using the Eq.(7.84) of [5] and carrying out the scale transformation of variables according to Eq.(2.7) of [6]:

$$\Delta \mathcal{H} = \frac{2}{u} \boldsymbol{\chi} \boldsymbol{\varrho}, \tag{2.7}$$

where

$$\boldsymbol{\chi} = \frac{\varepsilon}{m^3} \mathbf{F}, \quad \mathbf{F} = e(\mathbf{E}_\perp + \mathbf{v} \times \mathbf{H}), \quad u = \frac{\omega}{\varepsilon'}, \tag{2.8}$$

here $\mathbf{E}_\perp$ is the electric field strength transverse to the velocity of particle $\mathbf{v}$, $\mathbf{H}$ is the magnetic field strength, $\chi = |\boldsymbol{\chi}|$ is the known parameter characterizing quantum effects in the radiation process in an external field.

We split the potential $V(\boldsymbol{\varrho})$ in the same way as in Eq.(2.17) of [6]:

$$\begin{aligned} &V(\boldsymbol{\varrho}) = V_c(\boldsymbol{\varrho}) + v(\boldsymbol{\varrho}), \quad V_c(\boldsymbol{\varrho}) = q\boldsymbol{\varrho}^2, \quad q = QL_c, \\ &L_c \equiv L(\varrho_c) = \ln \frac{a_{s2}^2}{\lambda_c^2 \varrho_c^2}, \quad v(\boldsymbol{\varrho}) = -\frac{q\boldsymbol{\varrho}^2}{L_c}\left(\ln \frac{\boldsymbol{\varrho}^2}{4\varrho_c^2} + 2C\right). \end{aligned} \tag{2.9}$$

The definition of the parameter ϱ_c will be considered below. According to this splitting and taking into account the addition to the Hamiltonian (2.7) we present the propagators in Eq.(2.3) as

$$G^{-1} - G_0^{-1} = G^{-1} - G_F^{-1} + G_F^{-1} - G_0^{-1}, \tag{2.10}$$

where

$$\begin{aligned} &G_F = \mathcal{H}_F + 1, \quad G = G_F - iv, \\ &\mathcal{H}_F = \mathbf{p}^2 + V_F, \quad V_F = -iV_c + \frac{2}{u}\boldsymbol{\chi}\boldsymbol{\varrho}. \end{aligned} \tag{2.11}$$

The representation of the propagator G permits to carry out its decomposition over the "perturbation" v

$$G^{-1} - G_F^{-1} = G_F^{-1} iv G_F^{-1} + G_F^{-1} iv G_F^{-1} iv G_F^{-1} + \dots \tag{2.12}$$

The procedure of matrix elements calculation in this decomposition was formulated in [6], see Eqs.(2.30), (2.31). Here the basic matrix element is $< \boldsymbol{\varrho}_1 | G_F^{-1} | \boldsymbol{\varrho}_2 >$. The matrix element for the case when an external field is absent was calculated in [6], Eqs.(2.20)-(2.27):

$$\begin{aligned}
< \boldsymbol{\varrho}_1 | G_c^{-1} | \boldsymbol{\varrho}_2 > &= i \int_0^\infty dt \exp(-it) K_c(\boldsymbol{\varrho}_1, \boldsymbol{\varrho}_2, t), \ G_c = \mathbf{p}^2 + 1 - iq\boldsymbol{\varrho}^2, \\
K_c(\boldsymbol{\varrho}_1, \boldsymbol{\varrho}_2, t) &= \frac{\nu}{4\pi i \sinh \nu t} \exp\left\{ \frac{i\nu}{4} \left[(\boldsymbol{\varrho}_1^2 + \boldsymbol{\varrho}_2^2) \coth \nu t \right.\right. \\
&- \left.\left. \frac{2}{\sinh \nu t} \boldsymbol{\varrho}_1 \boldsymbol{\varrho}_2 \right] \right\},
\end{aligned} \tag{2.13}$$

where $\nu = 2\sqrt{iq}$.

The matrix element of propagator G_F^{-1} can be obtained from (2.13) by transformation of the Hamiltonian $\mathcal{H}_F$ to the quadratic form over coordinate

$$\mathcal{H}_F = \mathbf{p}^2 - iq\boldsymbol{\varrho}^2 + \frac{2}{u}\boldsymbol{\chi}\boldsymbol{\varrho} = \mathbf{p}^2 - iq\boldsymbol{\varrho}'^2 - i\frac{\boldsymbol{\chi}^2}{qu^2}, \quad \boldsymbol{\varrho}' = \boldsymbol{\varrho} + i\frac{\boldsymbol{\chi}}{qu} \tag{2.14}$$

In the new variables the Hamiltonian (2.14) has the form similar to (2.11) and one can use (2.13). Substituting into (2.13) $\boldsymbol{\varrho}_{1,2} \to \boldsymbol{\varrho}'_{1,2}$ and taking into account the constant term in the Hamiltonian (2.14) we obtain

$$\begin{aligned}
K_F(\boldsymbol{\varrho}_1, \boldsymbol{\varrho}_2, t) &= K_c(\boldsymbol{\varrho}_1, \boldsymbol{\varrho}_2, t) K_\chi(\boldsymbol{\varrho}_1, \boldsymbol{\varrho}_2, t), \\
K_\chi(\boldsymbol{\varrho}_1, \boldsymbol{\varrho}_2, t) &= \exp\left[-\frac{4i\boldsymbol{\chi}^2 t}{u^2\nu^2} \left(1 - \frac{2}{\nu t} \tanh \frac{\nu t}{2} \right) \right. \\
&- \left. \frac{2i}{u\nu} \boldsymbol{\chi}(\boldsymbol{\varrho}_1 + \boldsymbol{\varrho}_2) \tanh \frac{\nu t}{2} \right].
\end{aligned} \tag{2.15}$$

The presence in the problem under consideration the axial vector

$$\mathbf{H}_R = \gamma(\mathbf{H}_\perp + \mathbf{E} \times \mathbf{v}) \tag{2.16}$$

leads to appearance in Eqs.(2.3) and (2.6) additional terms depending on the electron polarization $\boldsymbol{\zeta}$ (see Eq.(2.2)). According to analysis performed in Subsec.7.4 of [5] and in [6] the vector $\mathbf{p}'$ in (2.2) gets over to the operator $\mathbf{p}$ standing in the formula (2.3) for T from the right of the propagator G^{-1} and the vector $\mathbf{p}$ in (2.2) gets over to the operator $\mathbf{p}$ standing in the formula (2.3) for T from the left of the propagator G^{-1}. As a result we have for the addition to the function T in Eqs.(2.3) and (2.6) the term depending on the spin vector $\boldsymbol{\zeta}$:

$$T \to T + T_{\boldsymbol{\zeta}}, \quad T_{\boldsymbol{\zeta}} = i\frac{\omega}{\varepsilon}\left(< 0|(G^{-1}\mathbf{p} - \mathbf{p}G^{-1})|0 > \times \mathbf{v}\right)\boldsymbol{\zeta}. \qquad (2.17)$$

For the radiative correction to the electron mass we have respectively

$$\Delta m \to \Delta M = \Delta m + \Delta m_{\boldsymbol{\zeta}}. \qquad (2.18)$$

It should be noted that expressions for T (2.3) and (2.17) have universal form while the specific of the particle motion is contained in the propagator G^{-1} through the effective potential

$$V_{eff}(\boldsymbol{\varrho}) = -iV(\boldsymbol{\varrho}) + \frac{2}{u}\boldsymbol{\chi}\boldsymbol{\varrho}. \qquad (2.19)$$

In the present paper we restrict ourselves to the main term in the decomposition (2.10). This means that result will have the logarithmic accuracy over the scattering (but not over an external field). With regard for an external field the parameter ϱ_c in Eq.(2.9) is defined by a set of equations:

$$\varrho_c = 1 \quad \text{for} \quad 4\frac{\chi^2}{u^2} + 4QL_1 \leq 1;$$
$$\varrho_c^4\left[4\frac{\chi^2}{u^2}\varrho_c^2 + |\nu(\varrho_c)|^2\right] = 1 \quad \text{for} \quad 4\frac{\chi^2}{u^2} + 4QL_1 \geq 1, \qquad (2.20)$$

where $\nu = 2\sqrt{iq}$, $q = QL_c$, L_1 and Q are defined in Eq.(2.4), L_c is defined in Eq.(2.9).

The matrix elements entering into the mass correction have in the used approximation the following form

$$\left\langle 0|G_F^{-1} - G_0^{-1}|0\right\rangle = \frac{1}{4\pi}\int_0^\infty \exp(-it)\left(\frac{\nu}{\sinh \nu t}\varphi - \frac{1}{t}\right)dt,$$

$$
\left\langle 0|\mathbf{p}(G_F^{-1} - G_0^{-1})\mathbf{p}|0\right\rangle = -\frac{}{4\pi}\int_0 \ \exp(-it)\left[\frac{}{\sinh \nu t}\left(\frac{\chi}{u^2\nu^2}\tanh^2\frac{}{2}\right.\right.
$$

$$
\left.\left. + \frac{i\nu}{\sinh \nu t}\right)\varphi - \frac{i}{t^2}\right]dt,
$$

$$
\left\langle 0|(G_F^{-1}\mathbf{p} - \mathbf{p}G_F^{-1})|0\right\rangle = \frac{\boldsymbol{\chi}}{2\pi u}\int_0^\infty \exp(-it)\frac{\varphi}{\cosh^2\frac{\nu t}{2}}dt,
$$

$$
\varphi \equiv \varphi(\chi, \nu, t) = \exp\left[-\frac{4i\chi^2 t}{\nu^2 u^2}\left(1 - \frac{2}{\nu t}\tanh\frac{\nu t}{2}\right)\right]. \tag{2.21}
$$

The two first equations are consistent with obtained in [9]. Thus, in the used approximation ($G = G_F$) we have for $T_{\boldsymbol{\zeta}}$ Eq.(2.17)

$$
T_{\boldsymbol{\zeta}} = \frac{i}{2\pi u}\frac{\omega}{\varepsilon}\int_0^\infty \exp(-it)\frac{\varphi}{\cosh^2\frac{\nu t}{2}}dt(\boldsymbol{\zeta}\boldsymbol{\chi}\mathbf{v}), \tag{2.22}
$$

where $(\boldsymbol{\zeta}\boldsymbol{\chi}\mathbf{v}) = (\boldsymbol{\zeta}\cdot(\boldsymbol{\chi}\times\mathbf{v}))$.

3 Anomalous magnetic moment of electron

Within relativistic accuracy (up to terms of higher order over $1/\gamma$) the combination of vectors entering in Eq.(2.22) can be written as (see Eq.(12.19) in [3] and [11])

$$
(\boldsymbol{\zeta}\boldsymbol{\chi}\mathbf{v}) = 2\mu_0\frac{\boldsymbol{\zeta}\mathbf{H}_R}{m}, \tag{3.1}
$$

where $\mathbf{H}_R$ defined in Eq.(2.16) is the magnetic field in the electron rest frame. Here we use that $\boldsymbol{\chi} = \chi\mathbf{s}$, where $\mathbf{s}$ is the unit vector in the direction of acceleration (this vector is used in [3]).

In the electron rest frame one can consider the value Re $\Delta m_{\boldsymbol{\zeta}}$ depending on the electron spin as the energy of interaction of the AMM of electron with the magnetic field $\mathbf{H}_R$ (see Eq.(12.23) in [3] and [11])

$$
\text{Re}\ \Delta m_{\boldsymbol{\zeta}} = -\mu'\boldsymbol{\zeta}\mathbf{H}_R, \tag{3.2}
$$

Taking into account Eqs.(2.6), (2.17), (2.18), (2.22), (3.1), (3.2) we obtain the following general expression for the AMM of electron moving in a medium in the presence of an external electromagnetic field:

$$\frac{\mu'}{\mu_0} = -\frac{\alpha}{\pi}\text{Im} \int_0^\infty \frac{du}{(1+u)^3} \int_0^\infty \exp(-it)\frac{\varphi}{\cosh^2\frac{\nu t}{2}}dt \qquad (3.3)$$

In the absence of scattering ($\nu \to 0$) the expression (3.3) gets over into the formula for the AMM of electron in external field (see Eq.(12.24) in [3] and [10], [11])

$$\frac{\mu'}{\mu_0} = -\frac{\alpha}{\pi}\text{Im} \int_0^\infty \frac{du}{(1+u)^3} \int_0^\infty \exp\left[-it\left(1+\frac{\chi^2 t^2}{3u^2}\right)\right] dt \qquad (3.4)$$

In the weak external field ($\chi \ll 1, \varphi \simeq 1$) we obtain the formula for the AMM of electron under influence of multiple scattering

$$\frac{\mu'}{\mu_0} = -\frac{\alpha}{\pi}\text{Im} \int_0^\infty \frac{du}{(1+u)^3} \int_0^\infty \exp(-it)\frac{1}{\cosh^2\frac{\nu t}{2}}dt = \frac{\alpha}{2\pi}r, \qquad (3.5)$$

where

$$\begin{aligned} r = \text{Re}J, \quad J &= 2i\int_0^\infty \frac{du}{(1+u)^3} \int_0^\infty \exp(-it)\frac{1}{\cosh^2\frac{\nu t}{2}}dt \\ &= 4i\int_0^\infty \frac{du}{(1+u)^3}\frac{1}{\nu}\left[\frac{2i}{\nu}\beta\left(\frac{i}{\nu}\right)-1\right], \\ \beta &= \frac{1}{2}\left[\psi\left(\frac{1+x}{2}\right)-\psi\left(\frac{x}{2}\right)\right], \end{aligned} \qquad (3.6)$$

where $\psi(x)$ is the logarithmic derivative of the gamma function.

The dependence of the AMM of electron on its energy ε in gold is shown in Fig. It is seen that at energy ε = 500 GeV value of AMM is 0.85 part of standard quantity of AMM (SQ):$\frac{\mu'}{\mu_0} = \frac{\alpha}{2\pi}$, at energy ε = 1 TeV it is 0.77 part of SQ and at energy ε = 5.5 TeV it is 0.5 part of SQ. Actually in all heavy elements the behavior of AMM of electron will be quite similar, i.e. the scale of energy where the AMM of electron deviates from SQ is of order of TeV.

In the case of weak effect of multiple scattering

$$\nu^2 = \frac{i}{u}\nu_b^2, \quad \nu_b^2 = \frac{\varepsilon}{\varepsilon_e}, \quad \varepsilon_e = m\left(8\pi Z^2\alpha^2 n_a \lambda_c^3 L_1\right)^{-1}, \qquad (3.7)$$

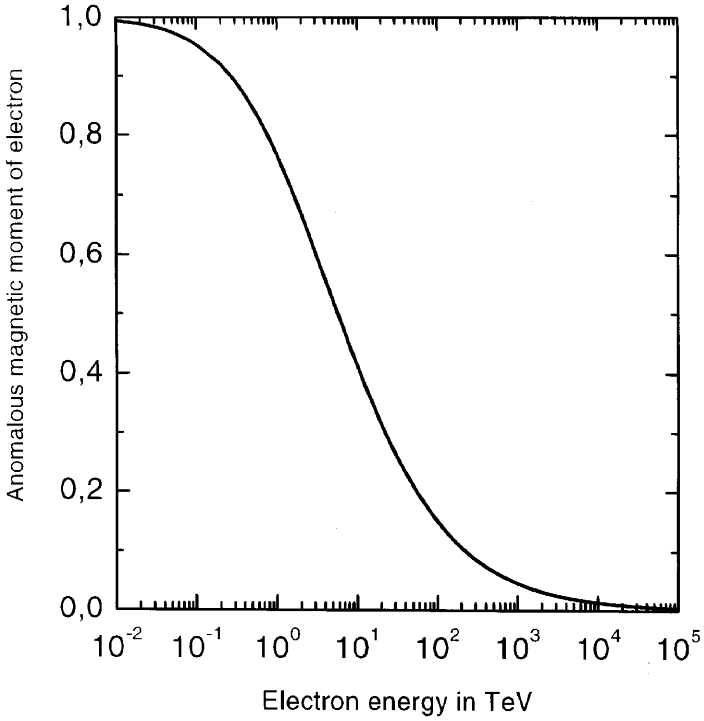

Figure 1: Anomalous magnetic moment(AMM) of electron in units $\alpha/2\pi$ (r in Eq.(3.6)) in gold vs electron energy in TeV.

where ε_e is the characteristic electron energy starting with the multiple scattering distorted the whole spectrum of radiation ($\varepsilon_e = 2.6$ TeV in gold and it has a similar value for heavy elements). Integrating in Eq.(3.6) by parts over the variable t we find

$$J = 1 + f, \quad f = f(\nu_b) = -2\int_0^\infty \frac{du}{(1+u)^3}\int_0^\infty \exp(-it)\frac{\tanh\frac{\nu t}{2}}{\cosh^2\frac{\nu t}{2}}dt. \tag{3.8}$$

The asymptotic behavior of the function f at $\nu_b \ll 1$ ($\varepsilon \ll \varepsilon_e$) calculated in Appendix:

$$f(\nu_b) = \nu_b^2\left[-\frac{\pi}{2} + 2i\left(\ln\frac{2}{\nu_b} + C - \frac{7}{4} + A\right)\right], \tag{3.9}$$

where

$$A = \int_0^\infty \left(3\ln t \tanh t + \frac{\sinh t - t}{t^2} \right) \frac{dt}{\cosh^3 t} = -0.364291\ldots \quad (3.10)$$

Substituting this result into (3.5) we obtain for AMM of electron at $\varepsilon \ll \varepsilon_e$

$$\frac{\mu'}{\mu_0} = \frac{\alpha}{2\pi}\left(1 - \frac{\pi}{2}\frac{\varepsilon}{\varepsilon_e}\right). \quad (3.11)$$

At very high energy $\varepsilon \gg \varepsilon_e$ the effect of multiple scattering becomes strong. In this case (see Eq.(3.3) in [7])

$$\nu^2 = \frac{i\varepsilon}{u\varepsilon_e}\left(1 + \frac{1}{2L_1}\ln\frac{\varepsilon}{\varepsilon_e u}\right), \quad (3.12)$$

the main contribution into the integral (3.6) give the regions $u \sim 1$ and $t \sim 1/|\nu| \ll 1$. Making the substitution of variable as in (A.8) and expanding the exponent as in Appendix we have from (A.11) the asymptotic expansion of AMM of electron:

$$r = \frac{2\pi}{\alpha}\frac{\mu'}{\mu_0} = \frac{\pi}{2\sqrt{2}}\sqrt{\frac{\varepsilon_c}{\varepsilon}}\left(1 - \frac{\pi^2}{2}\frac{\varepsilon_c}{\varepsilon}\right), \quad \varepsilon_c = \varepsilon_e\frac{L_1}{L_0}, \quad L_0 = L_1 + \frac{1}{2}\ln\frac{\varepsilon}{\varepsilon_e}. \quad (3.13)$$

Redefinition of the characteristic energy $\varepsilon_e \to \varepsilon_c$ is connected with enlargement of the radiation cone (in comparison with $1/\gamma$) or in another terms with increasing of the characteristic transverse momentum transfers due to the multiple scattering.

4 Spin correction to the intensity of radiation

To obtain the additional term in the radiation intensity depending on the spin of the radiating electron one has to substitute the formula (2.22) into expression for the spectral distribution of the probability of radiation per unit time Eq.(2.3) and to multiply it on the photon energy ω. As a result we obtain the mentioned term

$$I_{\boldsymbol{\zeta}} = (\boldsymbol{\zeta}\boldsymbol{\chi}\mathbf{v})\frac{\alpha m^2}{\pi}\mathrm{Re}\int_0^\infty \frac{u du}{(1+u)^4}\int_0^\infty \exp(-it)\frac{\varphi}{\cosh^2\frac{\nu t}{2}}dt. \quad (4.1)$$

So for polarized electrons the total intensity of radiation is the sum of Eq.(3.8) of [7] and $I_{\boldsymbol{\zeta}} L_{rad}^0/\varepsilon$ (4.1).

At $\varepsilon \ll \varepsilon_e$ one can decompose $\cosh^{-2} \nu t/2 \simeq 1 - \nu^2 t^2/4$ in integral over t in (4.1). We find

$$I_{\boldsymbol{\zeta}} \simeq (\boldsymbol{\zeta}\boldsymbol{\chi}\mathbf{v}) \frac{\alpha m^2}{6\pi} \frac{\varepsilon}{\varepsilon_e}, \quad \frac{I_{\boldsymbol{\zeta}}}{I} \simeq \frac{2}{3}(\boldsymbol{\zeta}\boldsymbol{\chi}\mathbf{v}), \tag{4.2}$$

Where I is given by Eq.(3.9) of [7].

In the opposite case of large energies $\varepsilon \gg \varepsilon_e$ the main contribution into integrals in (4.1) gives the region $u \sim 1,\ t \ll 1$. Then substituting the exponent by unity we find

$$I_{\boldsymbol{\zeta}} \simeq (\boldsymbol{\zeta}\boldsymbol{\chi}\mathbf{v}) \frac{\alpha m^2}{\pi} \sqrt{\frac{2\varepsilon_c}{\varepsilon}} \int_0^\infty \int_0^\infty \frac{u^{3/2} du}{(1+u)^4} = (\boldsymbol{\zeta}\boldsymbol{\chi}\mathbf{v}) \frac{\alpha m^2}{8\sqrt{2}} \sqrt{\frac{\varepsilon_c}{\varepsilon}}$$

$$\frac{I_{\boldsymbol{\zeta}}}{I} \simeq \frac{4}{9} \frac{\varepsilon_c}{\varepsilon} (\boldsymbol{\zeta}\boldsymbol{\chi}\mathbf{v}), \tag{4.3}$$

where I is given by Eq.(3.11) of [7].

5 Conclusion

Here we discuss the possibility of experimental observation of influence of medium on the AMM of electron. At a very high energy where the observation becomes feasible the rotation angle φ of the spin vector $\boldsymbol{\zeta}$ in the transverse to the particle velocity $\mathbf{v}$ magnetic field $\mathbf{H}$ depends only on the value of AMM and doesn't depend on the particle energy:

$$\varphi = \left(\frac{m}{\varepsilon} + \frac{\mu'}{\mu_0}\right) \frac{eH}{m} l \simeq r \frac{\alpha}{2\pi} \frac{H}{H_0} \frac{l}{\lambda_c}, \tag{5.1}$$

where l is the path of electron in the field, $H_0 = m^2/e = 4.41 \cdot 10^{13}$ Oe. The dependence of r on energy ε is found in Sec.3 and shown in Fig.

Since at radiation of hard photons in a medium the picture is quite complicated:energy losses, cascade processes, spin flip and depolarization, it is desirable to measure the particles which don't radiate photons on the path l. The number of such particles N is determined by the total probability of radiation in a medium found in [7], Eqs.(3.12) (3.14):

$$N = N_0 \exp(-\psi(\varepsilon)),\ \ \psi(\varepsilon) = W(\varepsilon)l = \frac{k(\varepsilon)\varphi_{SQ}}{2\chi(\varepsilon_e)};\ \ k(\varepsilon) = W(\varepsilon)L_{rad}^0,$$

$$(L_{rad}^0)^{-1} = \frac{\alpha}{4\pi}\frac{m^2}{\varepsilon_e}, \quad \chi(\varepsilon_e) = \frac{\varepsilon_e}{m}\frac{H}{H_0}, \tag{5.2}$$

where φ_{SQ} is the rotation angle for standard value of AMM in QED ($r=1$), N_0 is the number of initial electrons. With energy increase the function $k(\varepsilon)$ decreases [7]

$$k(\varepsilon \ll \varepsilon_e) \simeq \frac{4}{3}\left(\ln\frac{\varepsilon_e}{\varepsilon} + 1.96\right), \quad k(\varepsilon = \varepsilon_e) \simeq 3.56,$$

$$k(\varepsilon \gg \varepsilon_e) \simeq \frac{11\pi}{4\sqrt{2}}\sqrt{\frac{\varepsilon_e}{\varepsilon}}. \tag{5.3}$$

The crucial part of the experiment is an accuracy of measurement of electron polarization before target and after target. If one supposes that spin rotation angle can be measured with accuracy noticeably better than 1/10 then we can put that $(1-r)\varphi_{SQ} = 1/10$. In the gold we find for the energy $\varepsilon = \varepsilon_e = 2.61$ TeV and the magnetic field $H = 4\cdot 10^5$Oe that $1-r = 0.371$, the path of electron in the target is $l \simeq 1$ cm and number of electron traversing the target without energy loss is $N \simeq 3.2\cdot 10^{-5}N_0$. This estimates show that the measurement of the effect found in this paper will be feasible in the not very distant future.

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A Appendix

In the case $\nu_b \ll 1$ (3.7) we present the function $f(\nu)$ (3.8) as

$$f(\tilde{\nu}) = f_1(\tilde{\nu}) + f_2(\tilde{\nu}), \quad \nu = \frac{\tilde{\nu}}{\sqrt{u}},$$
$$f_1 = -2\int_0^\infty \nu \frac{du}{(1+u)^3} \int_0^\infty \frac{\exp(-it)}{\cosh^3 \frac{\nu t}{2}} \left(\sinh \frac{\nu t}{2} - \frac{\nu t}{2}\right) dt,$$
$$f_2 = -\int_0^\infty \nu^2 \frac{du}{(1+u)^3} \int_0^\infty \frac{\exp(-it)}{\cosh^3 \frac{\nu t}{2}} dt. \tag{A.1}$$

In the integral in f_1 the main contribution give $u \sim \nu_b^2 \ll 1$ so that

$$f_1 = -2\tilde{\nu} \int_0^\infty \frac{du}{\sqrt{u}} \int_0^\infty \frac{\exp(-it)}{\cosh^3 \frac{\tilde{\nu} t}{2\sqrt{u}}} \left(\sinh \frac{\tilde{\nu} t}{2\sqrt{u}} - \frac{\tilde{\nu} t}{2\sqrt{u}}\right) dt. \tag{A.2}$$

Substituting variables $x = \sqrt{u}$ and $z = \nu t/2x$ we have

$$f_1 = 2\tilde{\nu}^2 \int_0^\infty \frac{1}{z^2} \frac{\sinh z - z}{\cosh^3 z} dz. \tag{A.3}$$

The function f_2 we present as

$$f = f_2^{(1)} + f_2^{(2)}, \; f_2^{(1)} = \tilde{\nu}^2 \int_0^\infty \frac{du}{u} \left(\frac{1}{1+u} - \frac{1}{(1+u)^3}\right) \int_0^\infty \frac{\exp(-it)}{\cosh^3 \frac{\tilde{\nu} t}{2\sqrt{u}}} t dt$$
$$\simeq -\tilde{\nu}^2 \int_0^\infty \frac{2+u}{(1+u)^3} du = -\frac{3}{2}\tilde{\nu}^2,$$
$$f_2^{(2)} = -\tilde{\nu}^2 \int_0^\infty \frac{du}{u(1+u)} \int_0^\infty \frac{\exp(-it)}{\cosh^3 \frac{\tilde{\nu} t}{2\sqrt{u}}} t dt =$$
$$-\tilde{\nu}^2 \int_0^\infty \frac{dx}{1+x} \int_0^\infty \frac{\exp(-it)}{\cosh^3 \frac{\tilde{\nu} t}{2}\sqrt{x}} t dt. \tag{A.4}$$

Introducing in the last integral the variable $z = \tilde{\nu}^2 t^2 x/4$ we find

$$f_2^{(2)} = -\tilde{\nu}^2 \int_0^\infty \exp(-it) t dt \int_0^\infty \frac{dz}{(z + \tilde{\nu}^2 t^2/4)\cosh^3 \sqrt{z}} \tag{A.5}$$

Integrating in (A.5) by parts over the variable z we find

$$f_2^{(2)} \simeq -\tilde{\nu}^2 \int_0^\infty \exp(-it) \left(-\ln \frac{\tilde{\nu}^2 t^2}{4} + \int_0^\infty \frac{3}{2\sqrt{z}} \ln z \frac{\sinh \sqrt{z}}{\cosh^4 \sqrt{z}} dz \right) tdt$$
$$= 2\tilde{\nu}^2 \left(\frac{i\pi}{2} - \psi(2) - \ln \frac{\tilde{\nu}}{2} + 3 \int_0^\infty \ln x \frac{\tanh x}{\cosh^3 x} dx \right). \qquad (A.6)$$

Combining the results (A.3), (A.4) and (A.7) we obtain

$$f(\tilde{\nu}) = 2\tilde{\nu}^2 \left(\frac{i\pi}{2} + \ln \frac{2}{\tilde{\nu}} - \frac{7}{4} + C + A \right),$$
$$A = \int_0^\infty \frac{1}{\cosh^3 x} \left(\frac{\sinh x - x}{x^2} + 3 \ln x \tanh x \right) dx. \qquad (A.7)$$

In the case $|\tilde{\nu}| \gg 1$ we present the integral in Eq.(3.6) as

$$J(\tilde{\nu}) = \frac{4i}{\tilde{\nu}} \int_0^\infty \frac{\sqrt{u} du}{(1+u)^3} \int_0^\infty \frac{dt}{\cosh^2 t} \exp\left(-i \frac{2\sqrt{u}t}{\tilde{\nu}} \right). \qquad (A.8)$$

To calculate the first three terms of the decomposition of $J(\tilde{\nu})$ over $1/\tilde{\nu}$ one can expands the exponent in the last integral

$$\exp\left(-i \frac{2\sqrt{u}t}{\tilde{\nu}} \right) = 1 - \frac{2i\sqrt{u}t}{\tilde{\nu}} - \frac{2ut^2}{\tilde{\nu}^2}. \qquad (A.9)$$

The next term of this decomposition contains the logarithmic divergence of the integral over u and to calculate the term of the order $1/\tilde{\nu}^3$ one has to use another approach. Using the known integrals

$$\int_0^\infty \frac{tdt}{\cosh^2 t} = \ln 2, \quad \int_0^\infty \frac{t^2 dt}{\cosh^2 t} = \frac{\pi^2}{12},$$
$$\int_0^\infty \frac{\sqrt{u} du}{(1+u)^3} = \frac{\pi}{8}, \quad \int_0^\infty \frac{u^{3/2} du}{(1+u)^3} = \frac{3\pi}{8}, \qquad (A.10)$$

we obtain at $|\tilde{\nu}| \gg 1 (\tilde{\nu} = \sqrt{i}|\tilde{\nu}|)$

$$J(\tilde{\nu}) \simeq \frac{4i}{\tilde{\nu}} \left[\frac{\pi}{8} - \frac{i}{\tilde{\nu}} \ln 2 - \frac{\pi^3}{16\tilde{\nu}^2} \right], \quad \mathrm{Re}J(\tilde{\nu}) \simeq \frac{\pi}{2\sqrt{2}|\tilde{\nu}|} \left(1 - \frac{\pi^2}{2|\tilde{\nu}|^2} \right),$$
$$\mathrm{Im}J(\tilde{\nu}) \simeq \frac{\pi}{2\sqrt{2}|\tilde{\nu}|} \left(1 - \frac{8\sqrt{2}\ln 2}{\pi|\tilde{\nu}|} + \frac{\pi^2}{2|\tilde{\nu}|^2} \right). \qquad (A.11)$$

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