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Non-forward BFKL pomeron at next-to-leading order[1]

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Abstract

The kernel of the BFKL equation for non-zero momentum transfer is found at next-to-leading order. It is presented in various forms depending on the regularization of the infrared singularities in "virtual" and "real" parts of the kernel. The infrared safety of the total kernel is demonstrated and a form free from the singularities is suggested.

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The kernel of the BFKL equation [1] for the case of forward scattering, i.e. for the momentum transfer $t = 0$ and vacuum quantum numbers in the $t-$channel, was found at next-to-leading order (NLO) already five years ago [2]. Unfortunately, the NLO calculation of the kernel for non-forward scattering was not completed till now. We remind that the kernel depends on the representation of the colour group in the t-channel; however for any representation $\mathcal{R}$ it is given by the sum of "virtual" and "real" contributions [3]. The "virtual" contribution is universal (does not depend on $\mathcal{R}$). It is expressed through the NLO gluon Regge trajectory [4] and is known. The "real" contribution is related to particle production in Reggeon-Reggeon collisions and consists of parts coming from one-gluon, two-gluon and quark-antiquark pair production. The first part is expressed through the effective Reggeon-Reggeon-gluon NLO vertex [5]. Apart from a colour coefficient this part is also universal. It was found in Refs. [6] and [7] for the quark and gluon contributions respectively. Each of last two parts for any $\mathcal{R}$ can be presented as a linear combination of two independent pieces, one of which can be determined by the antisymmetric colour octet representation $\mathcal{R} = 8_a$ (we shall call it gluon channel) and the other by the colour singlet representation $\mathcal{R} = 1$ (Pomeron channel). For the case of quark-antiquark production both these pieces are known [6]. Instead, only the piece related to the gluon channel is known for the case of two-gluon production [7].

The only missing piece of the non-forward kernel was then the two-gluon production contribution in the Pomeron channel. We have calculated this contribution and therefore have solved the problem of finding the non-forward kernel at NLO for an arbitrary colour state in the t-channel. Details of the calculation will be given elsewhere. Here we present the NLO kernel for the most important Pomeron channel. Note that for the case of the scattering of physical (colourless) particles only the Pomeron channel exists. Since the quark contribution to the non-forward kernel is known [6] for any $\mathcal{R}$, we shall consider in the following only the gluon contribution, i.e. pure gluodynamics.

Making use of the conventional dimensional regularization with the spacetime dimension $D = 4 + 2\epsilon$, the BFKL equation for the Mellin transform of the Green's function of two Reggeized gluons in the t-channel is written as

$$\omega G\left(\vec{q}_1, \vec{q}_2; \vec{q}\right) = \vec{q}_1^{\,2} \vec{q}_1^{\,\prime\,2} \delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \int \frac{d^{D-2}r}{\vec{r}^{\,2}(\vec{r}-\vec{q})^2} \mathcal{K}\left(\vec{q}_1, \vec{r}; \vec{q}\right) G\left(\vec{r}, \vec{q}_2; \vec{q}\right) \;, \tag{1}$$

where q_i and $q_i^\prime \equiv q_i - q\,,\ (i = 1 \div 2)$ are the Reggeon (Reggeized gluon) momenta, $q \simeq q_\perp$ is the total t-channel momentum, $q^2 \simeq q_\perp^2 = -\vec{q}^{\;2} = t$ and the vector sign is used for denoting the components of momenta transverse to the plane of initial momenta. The kernel

$$\mathcal{K}\left(\vec{q}_1, \vec{q}_2; \vec{q}\right) = \left[\omega\left(-\vec{q}_1^{\,2}\right) + \omega\left(-\vec{q}_1^{\,\prime\,2}\right)\right] \vec{q}_1^{\,2} \vec{q}_1^{\,\prime\,2} \delta^{(D-2)}\left(\vec{q}_1 - \vec{q}_2\right) + \mathcal{K}_r\left(\vec{q}_1, \vec{q}_2; \vec{q}\right) \tag{2}$$

is given by the sum of the "virtual" part, determined by the gluon Regge trajectory $\omega(t)$ (actually the trajectory $j(t) = 1 + \omega(t)$), and the "real" part, related to particle production in Reggeon-Reggeon collisions. In the limit $\epsilon \to 0$ we have [4]

$$\omega(t) = -2\bar{g}_\mu^2\left(\frac{1}{\epsilon} + \ln\left(\frac{-t}{\mu^2}\right)\right) - \bar{g}_\mu^4\left[\frac{11}{3}\left(\frac{1}{\epsilon^2} - \ln^2\left(\frac{-t}{\mu^2}\right)\right) + \left(\frac{67}{9} - 2\zeta(2)\right)\right.$$
$$\left. \times \left(\frac{1}{\epsilon} + 2\ln\left(\frac{-t}{\mu^2}\right)\right) - \frac{404}{27} + 2\zeta(3)\right] \;. \tag{3}$$

Here

$$\bar{g}_\mu^2 = \frac{g_\mu^2 N_c \Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}} \;, \tag{4}$$

g_μ being the renormalized coupling in the $\overline{MS}$ scheme, N_c is the number of colors, $\Gamma(x)$ is the Euler function and $\zeta(n)$ is the Riemann zeta function, $(\zeta(2) = \pi^2/6)$.

The remarkable properties of the "real" part of the kernel, which follow from general arguments, are

$$\mathcal{K}_r(0, \vec{q}_2; \vec{q}) = \mathcal{K}_r(\vec{q}_1, 0; \vec{q}) = \mathcal{K}_r(\vec{q}, \vec{q}_2; \vec{q}) = \mathcal{K}_r(\vec{q}_1, \vec{q}; \vec{q}) = 0 \tag{5}$$

and

$$\mathcal{K}_r(\vec{q}_1, \vec{q}_2; \vec{q}) = \mathcal{K}_r(-\vec{q}_1^{\,\prime}, -\vec{q}_2^{\,\prime}; \vec{q}) = \mathcal{K}_r(-\vec{q}_2, -\vec{q}_1; -\vec{q}) \;. \tag{6}$$

The properties (5) imply that the kernel turns into zero at zero transverse momenta of the Reggeons and appear as consequences of the gauge invariance; in turn the properties (6) are the consequence of cross-invariance.

In pure gluodynamics the "real" part $\mathcal{K}_r$ is given by sum of one-gluon- and two-gluon-production contributions. The first of them differs from the corresponding contribution in the gluon channel only by a colour group coefficient. As for the second one, it occurs to be much more complicated in the Pomeron channel than in the gluon one. The simplicity of the gluon channel is related to the gluon Reggeization. Technically it is determined by the cancellation of contributions of non-planar diagrams due to the colour group algebra. The complexity of contributions of non-planar diagrams is well known since the calculation of the non-forward kernel for the QED Pomeron [8] which was found only in the form of a two-dimensional integral. In QCD the situation is greatly worse because of the existence of cross-pentagon and cross-hexagon diagrams in addition to QED-type cross-box diagrams. It requires the use of additional Feynman parameters. At arbitrary D no integration over these parameters at all can be done in elementary functions. It occurs, however, that in the limit $\epsilon \to 0$ the integration over additional Feynman parameters can be performed, so that the result can be written as a two-dimensional integral, as well as in QED.

Let us present the kernel $\mathcal{K}_r$ in the limit $D = 4 + 2\epsilon \to 4$ as sum of two parts:

$$\mathcal{K}_r = \mathcal{K}_r^{sing} + \mathcal{K}_r^{(reg)} \ . \tag{7}$$

Here the first contains all singularities:

$$\mathcal{K}_r^{sing}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{2\bar{g}_\mu^2 \mu^{-2\epsilon}}{\pi^{1+\epsilon}\Gamma(1-\epsilon)} \left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2} + \vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2} \right) \left\{ 1 + \bar{g}_\mu^2 \left[\frac{11}{3\epsilon} \right.\right.$$

$$\left.\left. + \left(\frac{\vec{k}^{\,2}}{\mu^2} \right)^\epsilon \left\{ -\frac{11}{3\epsilon} + \frac{67}{9} - 2\zeta(2) + \epsilon \left(-\frac{404}{27} + 14\zeta(3) + \frac{11}{3}\zeta(2) \right) \right\} \right] \right\} , \tag{8}$$

where $\vec{k} = \vec{q}_1 - \vec{q}_2 = \vec{q}_1^{\,\prime} - \vec{q}_2^{\,\prime}$. The second, putting $\epsilon = 0$ and $\bar{g}_\mu^2 = \alpha_s(\mu^2)N_c/(4\pi)$, is given by

$$\mathcal{K}_r^{reg}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{\alpha_s^2(\mu^2)N_c^2}{16\pi^3} \left[2(\vec{q}_1^{\,2} + \vec{q}_2^{\,2} - \vec{q}^{\,2}) \left(\zeta(2) - \frac{50}{9} \right) - \frac{11}{3} \left(\vec{q}_1^{\,2} \ln\left(\frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}} \right) \right.\right.$$

$$\left. + \vec{q}_2^{\,2} \ln\left(\frac{\vec{q}_2^{\,2}}{\vec{k}^{\,2}} \right) - \vec{q}^{\,2} \ln\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,4}} \right) - \frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2} - \vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{\vec{k}^{\,2}} \ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) \right) + \vec{q}^{\,2} \left(\frac{1}{2} \ln^2\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) \right.$$

$$+ \ln\left(\frac{\vec{q}_2^{\,2}}{\vec{q}^{\,2}} \right) \ln\left(\frac{\vec{q}_2^{\,\prime 2}}{\vec{q}^{\,2}} \right) + \ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}^{\,2}} \right) \ln\left(\frac{\vec{q}_1^{\,\prime 2}}{\vec{q}^{\,2}} \right) \Bigg) + \ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}} \right) \left(\frac{\vec{q}_1^{\,\prime 2}}{2} \ln\left(\frac{\vec{q}_2^{\,2}}{\vec{k}^{\,2}} \right) \right.$$

$$
-\frac{\vec{q}_2^{\prime 2}}{2}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{k}^{\,2}}\right)-\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2}+\vec{q}_2^{\,2}\vec{q}_1^{\,\prime\,2}}{2\vec{k}^{\,2}}\ln\left(\frac{\vec{q}_1^{\,2}}{\vec{q}_2^{\,2}}\right)+\frac{\vec{q}_1^{\,\prime\,2}(\vec{q}_1^{\,2}-3\vec{q}_2^{\,2})}{2\vec{k}^{\,2}}\ln\left(\frac{\vec{k}^{\,2}}{\vec{q}_2^{\,2}}\right)
$$

$$
+\frac{\vec{q}_2^{\,\prime\,2}(3\vec{q}_1^{\,2}-\vec{q}_2^{\,2})}{2\vec{k}^{\,2}}\ln\left(\frac{\vec{k}^{\,2}}{\vec{q}_1^{\,2}}\right)\Bigg)+\Big(\vec{q}^{\,2}(\vec{k}^{\,2}-\vec{q}_1^{\,2}-\vec{q}_2^{\,2})+2\vec{q}_1^{\,2}\vec{q}_2^{\,2}+\vec{q}_1^{\,2}\vec{q}_1^{\,\prime\,2}+\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}
$$

$$
-\frac{(\vec{q}_1^{\,2}-\vec{q}_2^{\,2})(\vec{q}_1^{\,2}+\vec{q}_2^{\,2})(\vec{q}_1^{\,\prime\,2}-\vec{q}_2^{\,\prime\,2})}{2\vec{k}^{\,2}}-\frac{\vec{k}^{\,2}}{2}(\vec{q}_1^{\,\prime\,2}+\vec{q}_2^{\,\prime\,2})\Bigg)I(\vec{k}^{\,2},\vec{q}_2^{\,2},\vec{q}_1^{\,2})
$$

$$
-2J(\vec{q}_1,\vec{q}_2;\vec{q})-2J(-\vec{q}_2,-\vec{q}_1;-\vec{q})\Bigg]+\left\{\vec{q}_i\longleftrightarrow\vec{q}_i^{\,\prime}\right\}. \tag{9}
$$

In expression (9) two quantities appear, precisely

$$
I(a,b,c)=\int_0^1\frac{dx}{a(1-x)+bx-cx(1-x)}\ln\left(\frac{a(1-x)+bx}{cx(1-x)}\right) \tag{10}
$$

and

$$
J(\vec{q}_1,\vec{q}_2;\vec{q})=\int_0^1 dx\int_0^1 dz\,\Bigg\{\vec{q}_1\vec{q}_1^{\,\prime}\left((2-x_1x_2)\ln\left(\frac{Q^2}{\vec{k}^{\,2}}\right)-\frac{2}{x_1}\ln\left(\frac{Q^2}{Q_0^2}\right)\right)
$$

$$
-\frac{1}{2Q^2}x_1x_2(\vec{q}_1^{\,2}-2\vec{q}_1\vec{p}_1)(\vec{q}_1^{\,\prime 2}-2\vec{q}_1^{\,\prime}\,\vec{p}_2)
$$

$$
+\frac{2}{x_1}\Bigg[\Big(x_2\vec{q}_1\vec{q}_1^{\,\prime}\,\left(\vec{p}_1(\vec{q}_1^{\,\prime}-\vec{p}_2)\right)-\vec{q}_1^{\,\prime\,2}\vec{q}_1\vec{p}_2\Big)\frac{1}{Q^2}
$$

$$
+\Big(z(1-z)\vec{q}_2^{\,\prime\,2}\vec{q}_1\vec{q}_1^{\,\prime}+\vec{q}_1^{\,\prime\,2}(z\vec{q}_1\vec{k}+(1-z)\vec{q}_1\vec{q}_1^{\,\prime})\Big)\frac{1}{Q_0^2}\Bigg]-\frac{1}{Q^2}\Big(\vec{q}_1^{\,\prime\,2}\vec{q}_1\,(\vec{p}_1-2\vec{q}_1^{\,\prime})
$$

$$
+4x_1\vec{q}_1^{\,2}(\vec{q}_1^{\,\prime}\vec{p}_2)+\vec{q}_1^{\,\prime}\vec{q}_1(\vec{q}_1^{\,\prime}\vec{q}_1-\vec{q}_1^{\,\prime}\vec{p}_1-\vec{q}_1\vec{p}_2)+2(\vec{q}_1^{\,\prime}\vec{p}_1)(\vec{q}_1\vec{p}_2)-2(\vec{q}_1^{\,\prime}\vec{p}_2)(\vec{q}_1\vec{p}_1)\Big)
$$

$$
+\vec{q}_1^{\,\prime\,2}\Bigg[\frac{-1}{\mu_2^2Q^2}\left(2\frac{x_2}{x_1}(\vec{q}_1\vec{p}_2)\vec{q}_1^{\,\prime}\vec{k}+x_2(\vec{q}_1^{\,\prime}\vec{p}_2)(\vec{q}_2^{\,2}-\vec{k}^{\,2})+2(\vec{q}_2\vec{p}_2)\vec{q}_1\vec{q}\right)
$$

$$
+\frac{2}{\mu_0^2Q_0^2}\frac{1}{x_1}(\vec{q}_1\vec{p}_0)\vec{q}_1^{\,\prime}\vec{k}-\frac{\vec{q}_1(\vec{q}_1^{\,\prime}+\vec{k})}{x_1}\left(\frac{x_2}{\vec{p}_2^{\,2}}\ln\left(\frac{Q^2}{\mu_2^2}\right)-\frac{1}{\vec{p}_0^{\,2}}\ln\left(\frac{Q_0^2}{\mu_0^2}\right)\right)
$$

$$
+\frac{1}{\vec{p}_2^{\,2}}\left(\frac{1}{\vec{p}_2^{\,2}}\ln\left(\frac{Q^2}{\mu_2^2}\right)+\frac{1}{Q^2}\right)\left(2\frac{x_2}{x_1}(\vec{q}_1\vec{p}_2)(\vec{q}_1^{\,\prime}+\vec{k})\vec{p}_2-2((x_2\vec{q}_1^{\,\prime}+\vec{q}_2)\vec{p}_2)\vec{q}_1\vec{p}_2\right)
$$

$$
-\frac{1}{\vec{p}_0^{\,2}}\left(\frac{1}{\vec{p}_0^{\,2}}\ln\left(\frac{Q_0^2}{\mu_0^2}\right)+\frac{1}{Q_0^2}\right)\left(2\frac{1}{x_1}(\vec{q}_1\vec{p}_0)(\vec{q}_1^{\,\prime}+\vec{k})\vec{p}_0\right)+\frac{(x_2\vec{q}_1^{\,\prime}+\vec{q}_2)\vec{q}_1}{\vec{p}_2^{\,2}}\ln\left(\frac{Q^2}{\mu_2^2}\right)
$$

$$+\frac{\vec{q}_1^{\,2}}{d}\left((\vec{q}_2\vec{k})(\vec{q}_2^{\,\prime}\vec{k})\left(\frac{Q^2}{d}\mathcal{L}-\frac{1}{\vec{k}^{\,2}}\right)+(\vec{q}_2\vec{p}_2)(\vec{q}_2^{\,\prime}\vec{k})\left(\frac{1}{\mu_2^2}-\frac{\mu_1^2}{d}\mathcal{L}\right)+(\vec{q}_2\vec{k})(\vec{q}_2^{\,\prime}\vec{p}_1)\right.$$

$$\left.\left.\left.\times\left(\frac{1}{\mu_1^2}-\frac{\mu_2^2}{d}\mathcal{L}\right)+(\vec{q}_2\vec{p}_2)(\vec{q}_2^{\,\prime}\vec{p}_1)\left(\frac{\vec{k}^{\,2}}{d}\mathcal{L}-\frac{1}{Q^2}\right)+\frac{(\vec{q}_2\vec{q}_2^{\,\prime})}{2}\mathcal{L}\right)\right]\right\}\,. \qquad (11)$$

Here we make use of the following positions:

$$\vec{p}_1 = zx\vec{q}_1 + (1-z)(x\vec{k}-(1-x)\vec{q}_2^{\,\prime}),$$

$$\vec{p}_2 = z((1-x)\vec{k}-x\vec{q}_2)+(1-z)(1-x)\vec{q}_1^{\,\prime};\;\; \vec{p}_1+\vec{p}_2=\vec{k},$$

$$Q^2 = x(1-x)(\vec{q}_1^{\,2}z+\vec{q}_1^{\,\prime 2}(1-z))+z(1-z)(\vec{q}_2^{\,2}x+\vec{q}_2^{\,\prime 2}(1-x)-\vec{q}^{\,2}x(1-x)),$$

$$\mu_i^2 = Q^2+\vec{p}_i^{\,2},\quad \vec{p}_0 = z\vec{k}+(1-z)\vec{q}_1^{\,\prime};\;\; Q_0^2 = z(1-z)\vec{q}_2^{\,\prime\,2},\quad \mu_0^2 = z\vec{k}^{\,2}+(1-z)\vec{q}_1^{\,\prime\,2},$$

$$d = \mu_1^2\mu_2^2-\vec{k}^{\,2}Q^2 = z(1-z)x(1-x)\left((\vec{k}^{\,2}-\vec{q}_1^{\,2}-\vec{q}_2^{\,\prime\,2})(\vec{k}^{\,2}-\vec{q}_1^{\,\prime\,2}-\vec{q}_2^{\,2})+\vec{k}^{\,2}\vec{q}^{\,2}\right)$$

$$+\vec{q}_1^{\,2}\vec{q}_2^{\,2}xz(x+z-1)+\vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,\prime\,2}(1-x)(1-z)(1-x-z),\quad \mathcal{L}=\ln\left(\frac{\mu_1^2\mu_2^2}{\vec{k}^{\,2}Q^2}\right). \quad (12)$$

Note that the integral $I(a,b,c)$ is invariant with respect to any permutation of its arguments, which can be seen from the representation

$$I(a,b,c) = \int_0^1\int_0^1\int_0^1 \frac{dx_1dx_2dx_3\delta(1-x_1-x_2-x_3)}{(ax_1+bx_2+cx_3)(x_1x_2+x_1x_3+x_2x_3)}\,. \qquad (13)$$

In particular, $I(k^2,q_2^2,q_1^2)$ does not change performing the substitution $q_1 \leftrightarrow -q_2$.

As far as the quantity J is concerned, its expression (11) is rather cumbersome. Unfortunately, till now our attempts to find a more simple representation for it have been unsuccessful.

All singularities of $\mathcal{K}_r$ are present only in its first part $\mathcal{K}_r^{sing}$. We recall that the one-gluon- and two-gluon-production contributions to $\mathcal{K}_r$ separately contain first and second order poles at $\epsilon = 0$. When summing these two contributions the pole terms cancel, so that at fixed nonzero $\vec{k}^2$, when the term $\left(\vec{k}^{\,2}/\mu^2\right)^\epsilon$ in expression (8) of the kernel can be expanded in ϵ, the sum is finite at $\epsilon = 0$. However expression (8) is singular at $\vec{k}^{\,2} = 0$ so that, when it is integrated over q_2, the region of so small $\vec{k}^{\,2}$, such that $\epsilon|\ln\left(\vec{k}^{\,2}/\mu^2\right)|\sim 1$, does contribute. Therefore the expansion of $\left(\vec{k}^{\,2}/\mu^2\right)^\epsilon$ is not done in expression (8). Moreover, the terms $\sim\epsilon$ are taken into account in the coefficient of the expression divergent at $\vec{k}^{\,2} = 0$, in order to save all contributions non-vanishing in the limit $\epsilon \to 0$ after integration.

The part $\mathcal{K}_r^{(reg)}$ is finite in the limit $\epsilon = 0$. Moreover, integration of this part in Eq. (1) for the Green's function does not create singularities at $\epsilon = 0$ as well. Indeed, the points $\vec{r} = 0$ and $\vec{r}\,' = 0$, which at first glance could give the singularities in Eq. (1), are not dangerous because of the "gauge invariance" properties (5) of the kernel $\mathcal{K}_r$. It follows from formula (7) that if one of two parts ($\mathcal{K}_r^{(sing)}$ or $\mathcal{K}_r^{(reg)}$) of the kernel possesses these properties, the same is valid for the other. "Gauge invariance" of $\mathcal{K}_r^{(sing)}$ is evident from expression (8), therefore $\mathcal{K}_r^{(reg)}$ also turns into zero at zero Reggeon momenta. It is worthwhile to say that the fulfillment of these properties for $\mathcal{K}_r^{(reg)}$ can be shown directly using the explicit expression (9), although this is far from to be evident.

As we have already seen, at $\epsilon = 0$ divergencies can come from the region of small $\vec{k}$. Nevertheless, it is not difficult to check from expression (9) for $\mathcal{K}_r^{(reg)}$ that non-integrable singularities at $\vec{k} = 0$ are absent.

The total kernel for the Pomeron channel must be infrared safe. Infrared singularities of $\mathcal{K}_r$ must be cancelled by singularities of the gluon trajectory after integration of the total kernel with any function nonsingular at $\vec{k} = 0$. Indeed, one can easily see that this is the case using Eqs. (3) and (8).

It is convenient to present the total kernel in such a form that the cancellation of singularities between real and virtual contributions becomes evident. To this aim let us first switch from the dimensional regularization to the cut-off $\vec{k}^2 > \lambda^2$, with $\lambda \to 0$, which is more convenient for practical purposes. With such regularization we can pass to the limit $\epsilon \to 0$ in the real part of the kernel, so that for its singular part we get

$$\mathcal{K}_r^{sing}(\vec{q}_1, \vec{q}_2; \vec{q}) \;\to\; \mathcal{K}_r^{\lambda}(\vec{q}_1, \vec{q}_2; \vec{q}) = \frac{\alpha_s(\mu^2)N_c}{2\pi^2}\left(\frac{\vec{q}_1^{\,2}\vec{q}_2^{\,\prime\,2} + \vec{q}_1^{\,\prime\,2}\vec{q}_2^{\,2}}{\vec{k}^{\,2}} - \vec{q}^{\,2}\right)$$
$$\times\left\{1 - \frac{\alpha_s(\mu)N_c}{4\pi}\left(\frac{11}{3}\ln\left(\frac{\vec{k}^{\,2}}{\mu^2}\right) - \frac{67}{9} + 2\zeta(2)\right)\right\}\theta((\vec{q}_1 - \vec{q}_2)^2 - \lambda^2)\ . \quad (14)$$

The trajectory must be transformed in such a way that the cut-off regularization yields the same result as the ϵ regularization does:

$$\omega(t) \;\to\; \omega_\lambda(t) = \lim_{\epsilon\to 0}\left(\omega(t) + \frac{1}{2}\int\frac{d^{2+\epsilon}q_2}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}}\mathcal{K}_r^{(1)}(\vec{q}_1, \vec{q}_2; \vec{q})\theta(\lambda^2 - (\vec{q}_1 - \vec{q}_2)^2)\right)$$
$$= -\frac{\alpha_s(\mu^2)N_c}{2\pi}\left\{\ln\left(\frac{-t}{\lambda^2}\right) - \frac{\alpha_s(\mu^2)N_c}{4\pi}\left[\frac{11}{6}\left(\ln^2\left(\frac{-t}{\mu^2}\right)\right.\right.\right.$$

$$
- \ln^2\left(\frac{\lambda^2}{\mu^2}\right)\Bigg) - \left(\frac{67}{9} - \frac{\pi^2}{3}\right)\ln\left(\frac{-t}{\lambda^2}\right) + 6\zeta(3)\Bigg]\Bigg\} \ . \qquad (15)
$$

It is easy to check that by integrating over d^2q_2 any function non-singular for $\vec{k}=0$ with the total kernel (2) at $\omega(t)\to\omega_\lambda(t)$ and $\mathcal{K}_r^{sing}(\vec{q}_1,\vec{q}_2;\vec{q}) \to \mathcal{K}_r^{\lambda}(\vec{q}_1,\vec{q}_2;\vec{q})$ one obtains a λ-independent result in the limit $\lambda\to 0$. Moreover, it is also easy to find a form of the kernel which does not contain λ at all. It is sufficient to find a representation

$$
\omega_\lambda(-\vec{q}_1^{\,2}) = \int d^2q_2 f_\omega(\vec{q}_1,\vec{q}_2)\theta((\vec{q}_1-\vec{q}_2)^2-\lambda^2) \qquad (16)
$$

with a function f_ω such that the non-integrable singularities at $\vec{k}=\vec{q}_1-\vec{q}_2=\vec{q}_1^{\,\prime}-\vec{q}_2^{\,\prime}=0$ are cancelled in the "regularized virtual kernel"

$$
\mathcal{K}_v^{reg}(\vec{q}_1,\vec{q}_2;\vec{q}) = f_\omega(\vec{q}_1,\vec{q}_2) + f_\omega(\vec{q}_1^{\,\prime},\vec{q}_2^{\,\prime}) + \frac{\mathcal{K}_r^{sing}(\vec{q}_1,\vec{q}_2;\vec{q})|_{\epsilon=0}}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}} \ . \qquad (17)
$$

After that we can proceed to the limit $\lambda=0$:

$$
\left(\hat{\mathcal{K}}\Psi\right)(\vec{q}_1) = \int d^2q_2\Bigg\{\mathcal{K}_v^{reg}(\vec{q}_1,\vec{q}_2;\vec{q})\Psi(\vec{q}_1)
$$

$$
+\frac{\mathcal{K}_r^{sing}(\vec{q}_1,\vec{q}_2;\vec{q})|_{\epsilon=0}}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}}\left(\Psi(\vec{q}_2)-\Psi(\vec{q}_1)\right) + \frac{\mathcal{K}_r^{reg}(\vec{q}_1,\vec{q}_2;\vec{q})}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime\,2}}\Psi(\vec{q}_2)\Bigg\} \ . \qquad (18)
$$

Of course, the choice of the function f_ω contains a large arbitrariness. A simple choice is

$$
f_\omega(\vec{q}_1,\vec{q}_2) = -\frac{\alpha_s(\mu^2)N_c}{2\pi^2}\frac{\vec{q}_1^{\;2}}{\vec{k}^{\,2}(\vec{q}_1^{\;2}+\vec{k}^{\,2})}
$$

$$
\times\Bigg\{1-\frac{\alpha_s(\mu)N_c}{4\pi}\Bigg(\frac{11}{3}\ln\left(\frac{\vec{k}^{\,2}}{\mu^2}\right)-\frac{67}{9}+2\zeta(2)
$$

$$
+\left(6\zeta(3)-\frac{11}{3}\zeta(2)\right)\frac{\vec{k}^{\,2}}{(\vec{q}_1^{\;2}+\vec{k}^{\,2})}\Bigg)\Bigg\} \ . \qquad (19)
$$

In a subsequent paper we shall present the results of the investigation of properties of the kernel.

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V.S. Fadin and R. Fiore

Non-forward BFKL pomeron at next-to-leading order

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