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TUNABLE-OVER-ENERGY SOLENOID-BASED LONGITUDINAL POLARIZATION SCHEME WITH FIXING THE VERTICAL POLARIZATITION AT THE ARCS

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Tunable-over-energy solenoid-based longitudinal polarization scheme with fixing the vertical polarization at the arcs

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Abstract

Principle construction of the solenoid-based kinematic scheme providing a pure longitudinal orientation of the polarization axis at I.P. and simultaneously the vertical one over the main arcs of a storage ring collider with a possibility of tuning over long beam energy range is proposed. Pure vertical direction of polarization at the main arcs significantly decreases a spin orbital coupling and so reduces depolarizing effect of quantum fluctuations. Numerical example of a conceptual application of the proposed longitudinal polarization scheme for the super C-Tau factory project is given in comparison with another scheme variant which is not tunable-over-energy.

1 Introduction

At present, the so-called Central Arc (CA) scheme is under consideration [1, 2, 3] as one of the variants to organize a longitudinal polarization of electrons at the interaction point (I.P.) in the projects of Super c-tau and SuperB factories [4, 5]. This scheme shown in Fig.1 includes two bending magnets and two solenoid insertions symmetrically placed relative to I.P. at the collider sections near the interaction region (IR). At an nominal energy (E_{nom}) the bending magnets each rotate a spin around the vertical axis by an angle of $(2k{+}1)\pi/2$, k is integer. The solenoid field in each insertion is adjusted at that energy to rotate the polarization vector by $\pi/2$ about the particle velocity. All this together provides a stable longitudinal direction of the polarization axis at I.P. and the vertical one at any azimuth beyond the "Central Arc" section with the mentioned bending magnets and insertions. Recently it has been shown [2] for such a scheme that the longitudinal projection of polarization axis at I.P. remains very close to unity over some region around of the nominal energy. Furthermore, a similar picture - quasi-flat dependence with holes near the certain energy points - is repeated when changing energy within a wide range.

Nevertheless, in general, association of the scheme parameters with the nominal beam energy E_{nom} via the Central Arc bend angle may restricts possibilities of the given manner. When the energy differs from the nominal one the polarization axis at the storage ring arcs does not orient definitely along the vertical direction. It results in an increase of the spin-orbit coupling which affects the radiative polarization kinetics reinforcing the depolarization process rate. The super C-Tau and SuperB factory projects assume injection of the polarized beams from a linac in the so-called Trickle Injection mode. The time-average polarization degree of particles in the factory ring will decline if the radiative depolarization rate is notably larger than the particle loss rate due to high luminosity [3, 6].

To avoid such a negative effect we propose to modify the Central Arc scheme by imparting an achromatic feature to it. Toward this end, one more a pair of the bending magnets as well as of the tunable solenoid insertions are added into the Central Arc section. We call it as the "Achromatic Central Arc" (ACA) scheme meaning a possibility to keep the pure longitudinal

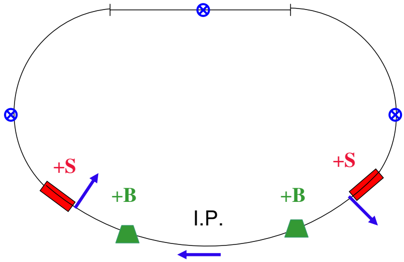

Рис. 1. "Central Arc" longitudinal polarization scheme. Between the solenoids (S) rotating spin through $\pi/2$ about the velocity vector there are two "Central Arc" bends (B). Each of them rotates spin in the horizontal plane through an angle divisible by $\pi/2$. The polarization vector directions at different azimuths are shown by the arrows and the circles.

direction of polarization at I.P. as well as the pure vertical one at the main arcs at different beam energies by tuning the solenoid fields. Note, the term "achromatic" does not concern small deviations of the particle energy from the beam average one. Otherwise, a talk is about fulfillment of the so-called "spin transparency" conditions - the task following the choice and optimization of the kinematic scheme.

In the paper we find the conditions to provide the "achromatic" feature of the modified scheme and present some numerical examples in comparison with the "pure" CA scheme for the super c-tau factory project. These examples include the calculation results on the spin kinematics as well as on the time-average longitudinal polarization degree.

2 Central Arc kinematic scheme

Energy dependence of the polarization axis, the unit vector $\vec{n}$, is calculated using the spin matrix technique [7, 8]. Longitudinal projection of $\vec{n}$ at I.P. of

the CA scheme meant for $E_{nom} = 2500$ MeV and the total CA bend of $\chi = 2 \times 15.9°$ is plotted in Fig.2 in the energy range from 1 to 2.5 GeV interesting in the framework of the super C-Tau factory project [4]. Owing to the scheme symmetry the polarization axis at I.P. can vary only in the vertical plane. At that, its longitudinal projection is close to unity in the extensive islands of energy values decaying in the comparatively narrow ranges around the characteristic values of beam energy E. One of such values nearest to the

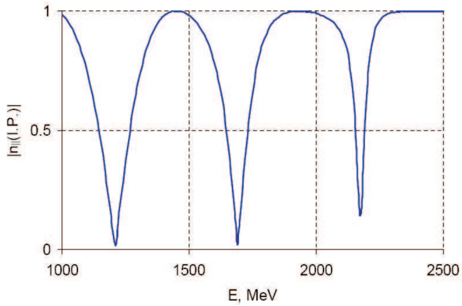

Рис. 2. Modulus of the longitudinal projection of polarization axis at I.P. vs. the beam energy in the case of the CA scheme (E_{nom} = 2500 MeV).

nominal energy almost coincides with a "resonant" energy E = 2203 MeV corresponding with integer value of the parameter $\nu = \gamma a = E/440.65$ (γ and a are the Lorentz-factor and anomalous part of the gyromagnetic ratio, respectively). The latter is a spin precession frequency in units of revolution frequency in a case of usual storage rings with vertical polarization. In the CA case, the spin tune ν_0 defined as a non-integer part of the relative spin frequency can be determined from the equation

$$\cos \pi\nu_0 = \pm \sin \frac{\nu\chi}{2} \sin \left(\pi\nu - \frac{\nu\chi}{2} \right)$$

at a fixed spin rotation angle in each solenoid insertion $\varphi = \pi/2$. Spin tune found from this equation more distinctly differs from the non-integer part of ν with distance from the nominal energy point (see Fig.3). The CA scheme,

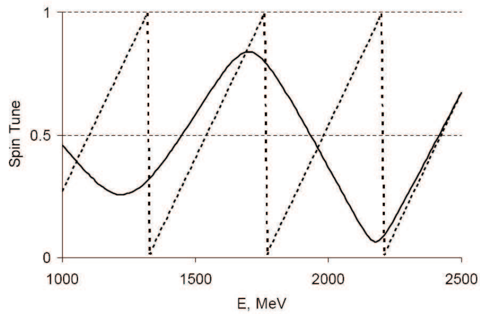

Рис. 3. Spin tune versus the beam energy for the case in Fig.2 (a solid line). For comparison, a non-integer part of the ν-parameter is also plotted (a dash line).

in principle, excludes integer spin resonances for the polarization axis (if the nominal energy is not per se "resonant"): the spin tune ν_0 does not become definitely integer anywhere at energy. It may just approach very close to integer values as seen in Fig.3.

3 Construction of the Achromatic CA scheme

Principle construction of the solenoid-based kinematic scheme providing a pure longitudinal orientation of the polarization axis at I.P. and a vertical one over the main arcs at the arbitrary beam energy value is presented in Fig.4. Here, $\chi_{1,2}$ and $\varphi_{1,2}$ are the velocity rotation angles in two bending magnets and two solenoid insertions, respectively. Successive combination of one bending magnet and one solenoid insertion with the same indexes form a spin rotator sub-system. Two pairs of these sub-systems are placed dissymmetrically in the section with I.P. All this together with the main arcs magnets forms a structure of the ACA kinematic scheme.

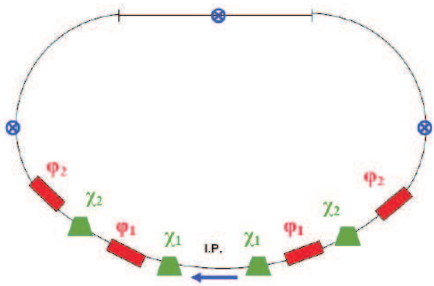

Рис. 4. Achromatic Central Arc longitudinal polarization scheme.

Idea of the achromatic scheme is based on possibility to adjust the polarization axis by tuning the solenoid rotation angles $\varphi_{1,2}$ at fixing the bends $\chi_{1,2}$. Multiplying together the spin rotation matrices one can obtain the condition for a unit longitudinal projection of the polarization axis at I.P. in the form:

$$\sin\varphi_2 = \pm\frac{\cos\nu\chi_1}{\sin\nu\chi_2}. \tag{1}$$

The second equation follows from the requirement to fix a vertical direction of polarization at azimuths beyond the section with the special spin rotators and I.P.:

$$\sin\varphi_1 = \pm\frac{\cos\varphi_2}{\sin\nu\chi_1}. \tag{2}$$

Energy dependence of the equations is in the ν parameter. Equation system (1,2) has real solutions for the unknown quantities $\varphi_{1,2}$ if the condition

$$\left|\frac{\cos\nu\chi_1}{\sin\nu\chi_2}\right| \leq 1 \tag{3}$$

is fulfilled. It means that not every combination of the bends χ_1 and χ_2 suits for construction of the achromatic scheme. Practically, if the bend χ_2 is specified then the χ_1 angle must satisfy the equation $\chi_1 \geq \chi_1^*$ where χ_1^* is a lower limit determined from (3). In Fig.5 the dependence of the lower limit for χ_1 as well as of the sum $\chi_1^* + \chi_2$ on the χ_2 angle from a range $(5 \div 20)^\circ$ is plotted at two values of beam energy: 1500 and 2500 MeV. Signs of both bends are assumed to be positive (like at the main arcs). Following this plot one must choose the χ_1 values lying above the appropriate curve in a case of fixed energy. If the scheme is required to be achromatic in the energy range $E_1 \leq E \leq E_2$ the proper χ_1^* values are above that curve which goes higher than another one. The sum $\chi_1^* + \chi_2$ is constant in a some region of the χ_2 variation. This feature gets broken when the χ_1^* passes through zero (see Fig.5). Note, it is advantageous to have a small sum $\chi_1 + \chi_2$ as compared with 2π (360°) that means a small relative contribution of radiation processes at the corresponding magnets into spin kinetics.

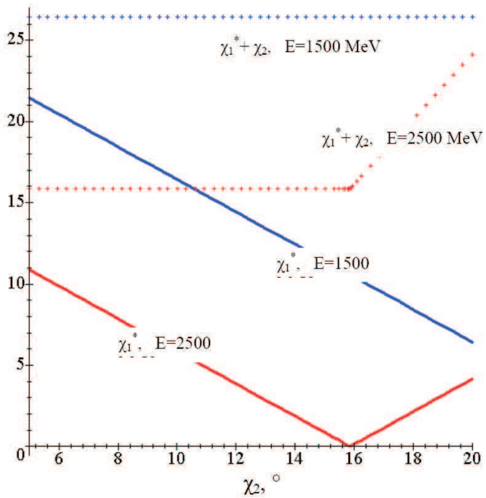

Рис. 5. over limit for the bend angle χ_1^* and the sum $\chi_1^* + \chi_2$ vs. the bend angle χ_2 (both ones are in degree of circle) at two beam energy levels.

Determination of the necessary angles of spin rotation at the solenoid insertions is illustrated by numerical example in Fig.6. Energy dependence of these angles is plotted in the energy region 1000 $\div$ 2500 MeV at $\chi_1 = 15.8^\circ$, $\chi_2 = 12^\circ$. The mentioned energy interval is of interest for physical experiments at the super C-Tau factory [9, 10]. The achromatism conditions (1,2) have been applied only from 1420 to 2500 MeV. Whereas the rotation angle of the insertion 2 reaches 90° at $E = 1420$ MeV, the angle of the insertion 1 becomes zero. Below the 1420 MeV level all the angles keep the same values as at 1420 MeV. Therefore, the kinematic scheme becomes identical to the CA one.

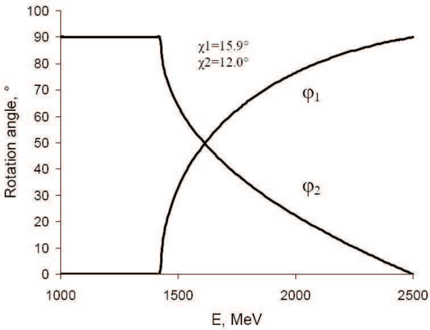

Рис. 6. Spin rotation angles in the solenoid insertions vs. the energy. Above 1420 MeV, the kinematic scheme is of the ACA type. Below 1420 MeV, it turns into CA.

4 Spin kinematics with ACA

Calculation of the kinematic characteristics is made numerically using the spin matrix technique. Longitudinal projection of the polarization axis at I.P. is presented in Fig.7 for the case of the combined ACA scheme at the same parameters as in Fig.6. Corresponding energy dependence of the spin tune is shown in Fig.8. Two conjugate solutions are valid for the tune: ν_0 and $1-\nu_0$. The solution ν_0 is used in the plot when $d\nu_0/dE > 0$. Otherwise, $1-\nu_0$ is used. This helps to see more clear a difference between the required spin tune and the ν-parameter. In theory, the spin tune much closer approaches to integer values at critical energy points as compared with the CA case (see Fig.3) but do not cross them. In practice, one can consider such approachments as complete coincidences. At the same time, the "critical" energies somewhat

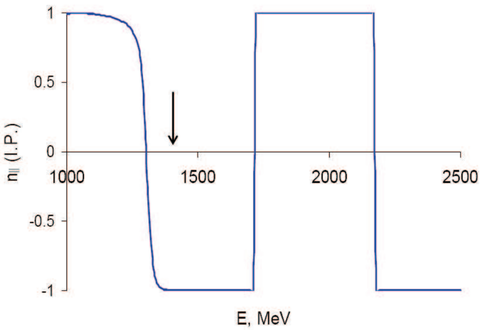

Рис. 7. Longitudinal projection of polarization axis at I.P. vs. the beam energy for the combined ACA scheme. The arrow marks the energy (1420 MeV) below of which the ACA turns into the CA.

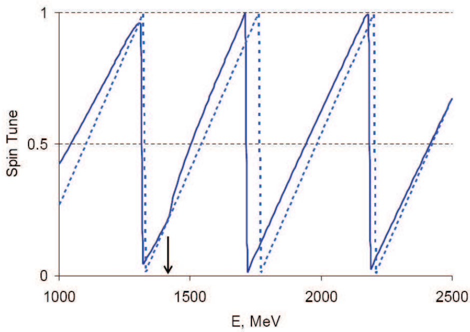

Рис. 8. Spin tune in the combined ACA scheme versus the energy (a solid line). For comparison, a non-integer part of the ν-parameter is plotted like in Fig.3. The arrow shows the energy point where the kinematic scheme is converted.

differ from ordinary values corresponding to integer ν ($E = 2203, 1763, 1322...$ MeV in the concerned range). According to calculation, the longitudinal component of polarization axis at I.P. passes through zero changing a sign just at these critical points. At the energy interval where ACA is used these passes are extremely sharp. Below 1420 MeV the longitudinal component changes around the point of crossing zero rather smoothly. Similar features take a place also with respect to the vertical component of polarization axis at the main arcs (see Fig.9).

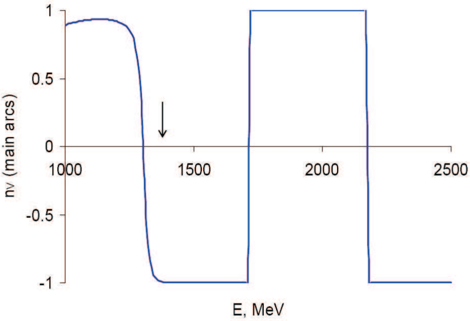

Рис. 9. Vertical projection of polarization axis at the main arcs vs. the beam energy for the combined ACA scheme.

5 Polarization kinetics with CA and ACA

Significant difference between the ACA and CA scheme regarding the energy dependence of the polarization axis results in a corresponding difference of the kinetic parameters of polarization. It is important to estimate this difference by the example of possible longitudinal polarization schemes for the C-Tau factory.

It is assumed that the polarized with a degree of P_0 electrons are injected into the Super C-Tau ring from a linac in the so-called Trickle Injection mode. The frequency $f_i = 1/\tau_i$ of injection per a single bunch is sufficiently high to compensate the particle loss caused mainly by a high luminosity and characterized by beam lifetime τ_l ($\tau_i << \tau_l$). Radiative relaxation time of the injected beam polarization is given by the formula [7]

$$\tau_r = \tau_0 \frac{< |\dot{\vec{v}}|^3 >}{< |\dot{\vec{v}}|^3 [1 - \frac{2}{9}(\vec{n} \cdot \vec{v})^2 + \frac{11}{18}\vec{d}^2] >},$$

where $\vec{v}$ is a velocity vector ($c = 1$); $\vec{v} \times \dot{\vec{v}} \propto \vec{H}_\perp$, $\vec{H}_\perp$ is the transverse magnetic field; the spin-orbit coupling parameter $\vec{d}$ as well as the unit vector $\vec{n}$ are the periodical functions of azimuth; the angular brackets mean averaging over the azimuth. The τ_0 parameter is the radiative polarization rise-time for a given storage ring when all special insertions diverting the polarization vector from the vertical axis are turned off. It can be estimated from

$$\tau_0 = 2.74 \times 10^{-2} \frac{\rho^2 R}{E^5},$$

τ_0 is in hours; ρ, the bend radius of magnets in arcs, and R, the mean machine radius, are in meteres; the beam energy E is in GeV. If radiative relaxation is the depolarizing process ($\tau_r << \tau_0$) and it dominates over other similar effects, for instance, the beam-beam depolarization [6], the polarization degree ($\bar{P}$) averaged over the injection cycle can be found from

$$\bar{P} \approx P_0 \frac{\tau_r}{\tau_r + \tau_l}. \tag{4}$$

It is valid for the Super C-Tau factory project: $\tau_0 \approx= 1.7$ hour at $E = 2.5$ GeV, $\rho = 7$ m, $R = 123$ m and $\tau_l \approx 10$ minute [2].

In the cases under consideration, the radiative spin kinetics is determined basically by quantum fluctuations in the main arcs where the polarization is purely vertical (ACA) or close to that (CA):

$$\tau_r \approx \frac{\tau_0}{1 + \frac{11}{18}\left\langle \vec{d}^2 \right\rangle}. \tag{5}$$

The spin-orbit function $\vec{d}$ can be presented as a sum of two contributions:

$$\vec{d} = \frac{\gamma d\vec{n}}{d\gamma} + \vec{d}_{beta} \tag{6}$$

where the first term is due to polarization axis chromaticity and the second one is due to betatron oscillations. The latter is calculated taking into account focusing features of the specific magnetic structure including the solenoid compensation optics. For generalization, we restrict ourself to consideration only of the contribution from polarization axis chromaticity. Our preceding experience in calculation of the betatron part shows that it is not determinative for the schemes of concerned type with the exception of depolarization at the comparatively narrow spin resonances with betatron frequencies [3].

In Fig.10 we present the calculation results on the time-average longitudinal polarization degree ($< Pn_{||} >$) at I.P. vs. the beam energy for the Super C-Tau factory project with the combined ACA scheme as well as with the CA one. The parameters of schemes are the same as in above numerical examples. The polarization axis $\vec{n}$ at the main arcs is calculated as a function of azimuth and energy using the spin matrix algebra. Then numerical energy differentiation is performed to obtain a squared quantity of a polarization vector chromaticity averaged over the main arcs: $< (\gamma d\vec{n}/d\gamma)^2 >$.

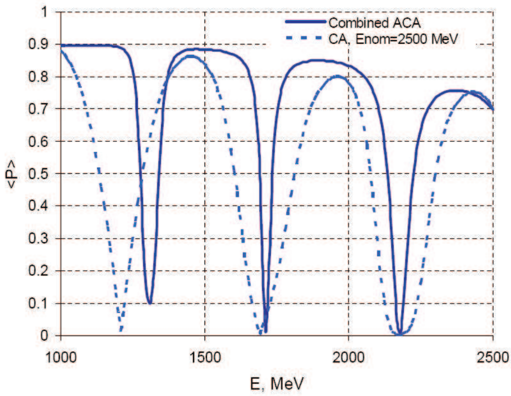

Рис. 10. Time-average longitudinal polarization degree.

6 Discussion

As can be seen from comparison of two dependencies in Fig.10, the width of holes of the polarization degree near the characteristic energy points is several times smaller in the ACA case. Another feature is a difference between the curves in the positions of holes which grows as a distance from the nominal energy increases. It may be important in the viewpoint of a physical experiment program formation at different energy ranges. Contribution of the main arc bending magnets to the radiative polarization kinetics has been considered to estimate the equilibrium longitudinal polarization degree in the Tricle Injection mode. Influence of the "central" bending magnets is neglected. This approximation is based on the fact that a total bend by these magnets of $2 \times (\chi_1 + \chi_2)$ is an order smaller than 2π. Furthermore, their contribution to depolarization rate can be strongly decreased owing to the factor $(H/<H>)^3$ depending on the ratio of the "central arc" magnet field to the average one at the main arcs.

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Перестраиваемая по энергии схема продольной поляризации с использованием соленоидов и сохранением вертикальной поляризации в полукольцах

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